Selected Theoretical Topics

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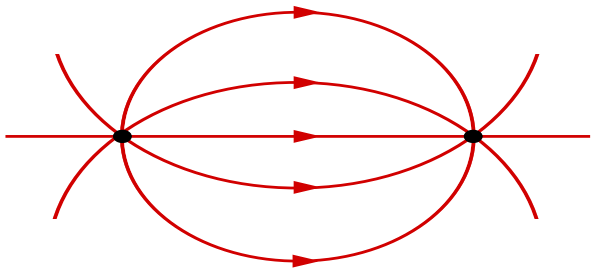
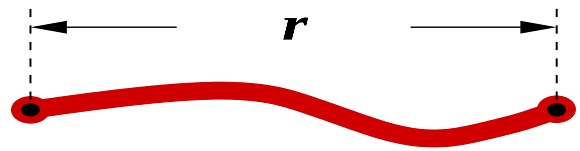
Pure SU(N) gauge theories

Flux distribution in the presence of static color sources



$r < 0.1$ fm



$r >> 1$ fm

perturbative regime

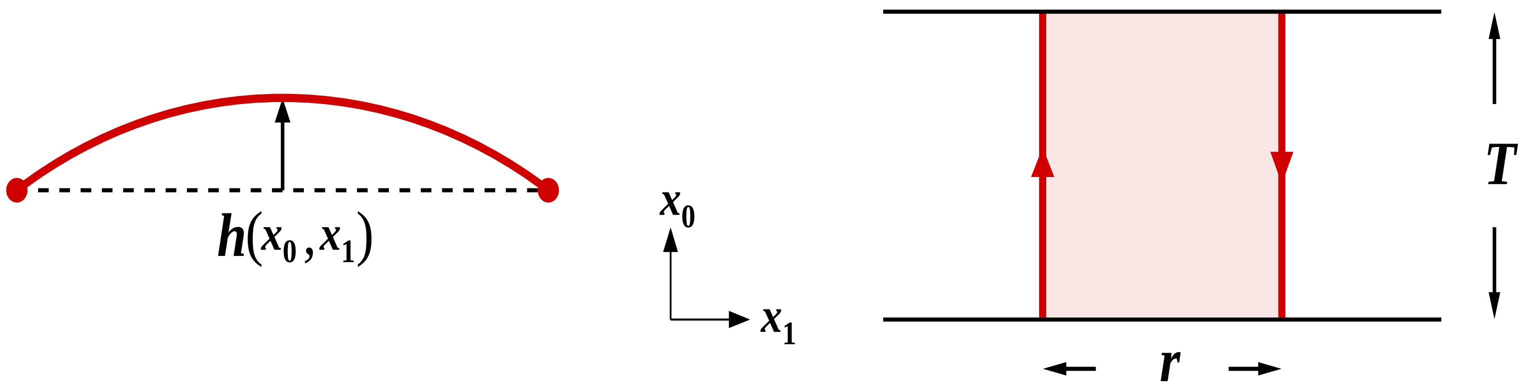
static confinement

scenario of flux tube formation

In presence of dynamical quarks "the tube breaks" due to pair production (not considered in the following) fluctuating string of thickness $1/m_{\mathrm{Glueball}}$

suggests long distance **effective string theory**

displacements: $h_i\,,\ i=1,\dots,d-2\,,\ \ d$: space-time dimension



e.g. for Polyakov loop correlation function:

$$\langle P(r)^* P(0)\rangle \approx \mathrm{e}^{-\sigma r T - \mu T} \times \int_{\mathrm{fluctuations}\ h} \mathrm{e}^{-S_{\mathrm{eff}}}$$

$$S_{\mathrm{eff}} = \int_0^T \int_0^r \mathrm{d}x_0 \mathrm{d}x_1 \Big\{ \tfrac{1}{2}\,(\partial_a h)^2 \quad \text{(free string)}$$

$$+\tfrac{1}{4} c_2\,(\partial_a h \partial_a h)\,(\partial_b h \partial_b h) + \tfrac{1}{4} c_3\,(\partial_a h \partial_b h)\,(\partial_a h \partial_b h) + \dots \Big\}$$

string interactions with couplings c_2, c_3 dimension $[\mathrm{length}]^2$:

Naive model: doesn't take into account
decay of higher excitations $\rightarrow$ lower states + glueballs

But is it basically correct? If so ...

Which string theory? (e.g. ones with additional fermionic modes?)

At which distances does string behavior set in?.....

Lattice gauge theory studies show naive model works well

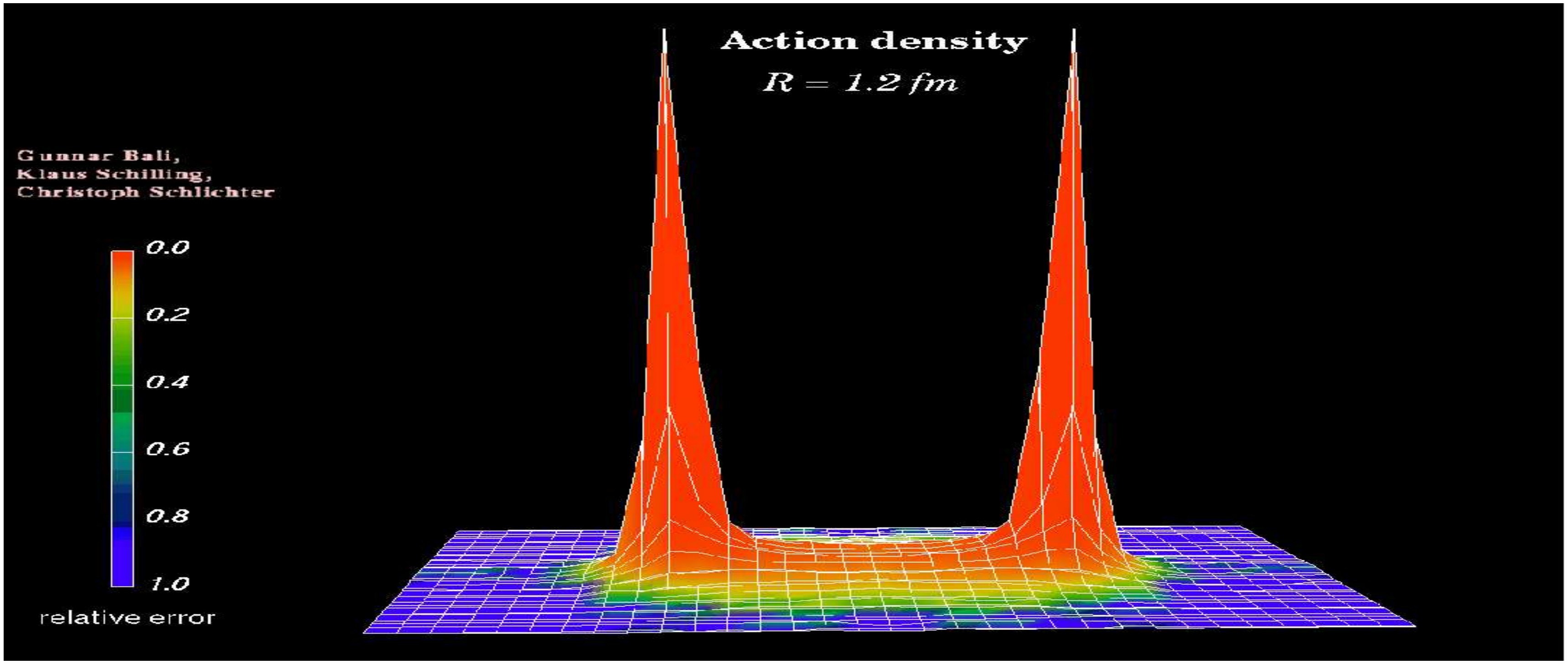
e.g. Width of flux tube:

$$w^2(r/2) \equiv \frac{\int \mathrm{d}^{d-2}\mathbf{x}_\perp \mathbf{x}_\perp^2 \mathcal{E}(r/2,\mathbf{x}_\perp)}{\int \mathrm{d}^{d-2}\mathbf{x}_\perp \mathcal{E}(r/2,\mathbf{x}_\perp)} \sim \frac{(d-2)}{2\pi\sigma} \ln(r) \text{ for } r \rightarrow \infty$$

(prediction Münster,Lüscher,P.W)

$\mathcal{E}$: chromo-electric field energy density distribution

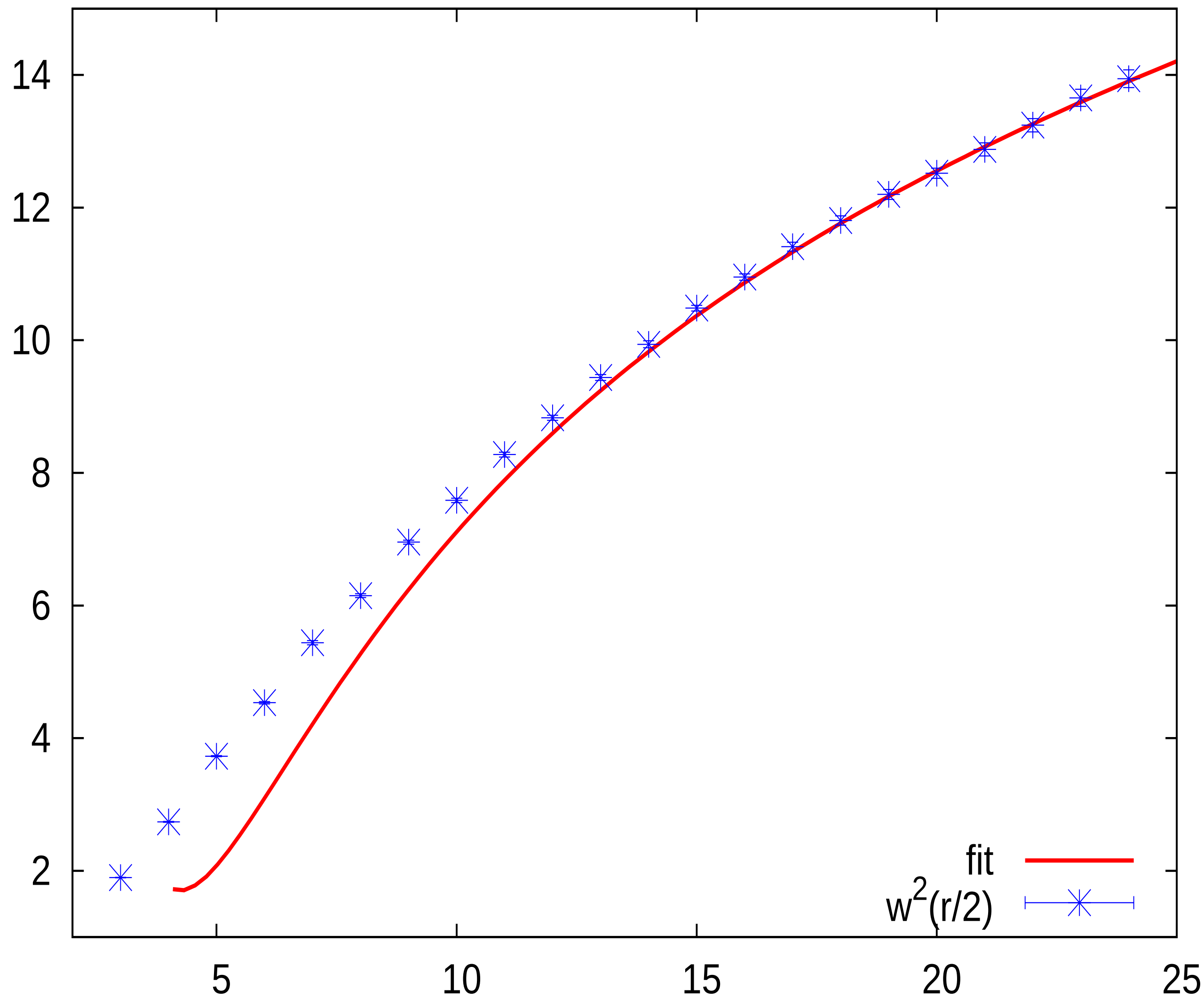
$$\mathcal{E}(x) \propto \langle q\bar{q}|\mathrm{tr}\mathbf{E}^2(x)|q\bar{q}\rangle - \langle q\bar{q}|q\bar{q}\rangle\langle\Omega|\mathrm{tr}\mathbf{E}^2(x)|\Omega\rangle$$



Action density: SU(2) on a 32^4 lattice $a \sim 0.08\mathrm{fm}$

Wuppertal 1994 Collaboration: Bali, Schilling, Schlichter, Wachter very nice data for SU(2) Yang-Mills in $d = 3$ supporting logarithmic growth of the tube width (Gliozzi, Pepe, Wiese 2010)

$w^2(r/2)$



Static potential:

$$V(r) = \sigma r - \frac{(d-2)\pi}{24r} + \left(\frac{\pi}{24}\right)^2 \frac{(d-2)\pi}{2r^3} \{2c_2 + (d-1)c_3\} + \dots$$

In $d = 3$ only one independent coupling $c_2 + c_3$

Open-closed string duality (Lüscher, P.W.):

$$(d-2)c_2 + c_3 = (d-4)/(2\sigma)$$

$\rightarrow$ universal $1/r^3$ term for $d = 3$ $(\star)$

as for Nambu string energy levels (Arvis):

$$E_n = \sigma r \sqrt{1 + \frac{2\pi}{\sigma r^2}\left(n - \frac{d-2}{24}\right)}$$

Caselle and Zago claim evidence for $(\star)$ in 3d Ising,
but not for the universality of the $1/r^5$ term (Aharony, Karzbrun) !!
independent confirmation??

M. Lüscher et al 2008-11

- Applications of Dirac spectral quantities
  - computation of chiral condensate with Wilson fermions
  - singularity free definitions of topological susceptibility

- Applications of gradient flow
  - new update procedures
  - new scales
  - understanding of emergence of topological sectors as $a \to 0$

- Studies of renormalizability of algorithms
  - scaling of autocorrelation times with the lattice spacing a

Consider eigenvalues of Dirac operator in the continuum $i\lambda_k$

spectral density $\quad \rho(\lambda, m) = \frac{1}{V}\sum_k \langle \delta(\lambda - \lambda_k) \rangle$

Banks-Casher relation:

$$\lim_{\lambda\to 0} \lim_{m\to 0} \lim_{V\to\infty} \rho(\lambda, m) = \Sigma/\pi$$

chiral condensate: $\quad \Sigma = -\lim_{m\to 0} \lim_{V\to\infty} \langle \bar{u}u \rangle$

Proposal to look at spectral observables
e.g average number of eigenvalues of $D^\dagger D + m^2$ below M^2:

$$\nu(M, m) = V \int_{-\Lambda}^{\Lambda} \mathrm{d}\lambda\, \rho(\lambda, M)\,, \quad \Lambda = \sqrt{M^2 - m^2}$$

Lüscher, Giusti: $\nu_R(M_R, m_R) = \nu(M, m_q)$ is a renormalized quantity

$\rightarrow$ computation of average spectral density and chiral condensate using Wilson fermions, although these violate chiral symmetry

$$\Sigma = \frac{\pi}{2}\frac{\nu(M,m)}{\Lambda V} + \text{corrections using } \chi\text{PT}$$

$\mathbb{P}_M$: Orthogonal projector to space of eigenvectors of $D^\dagger D + m^2 < M^2$

$$\nu = \langle \mathbb{P}_M \rangle:$$

Topological susceptibility: Formally $\chi_t = \int \mathrm{d}x \, \langle q(x)q(0) \rangle$

singularity free definition: $\chi_t = \frac{\nu}{V}\frac{\langle \mathrm{tr}(\gamma_5 \mathbb{P}_M)\mathrm{tr}(\gamma_5 \mathbb{P}_M)\rangle}{\langle \mathrm{tr}(\gamma_5 \mathbb{P}_M \gamma_5 \mathbb{P}_M)\rangle}$

worries: Autocorrelation time of observables related to topological charge grows $\succeq a^{-5}$ (DelDebbio, Panagopoulos, Vicari) also with dynamical fermions (Sommer, Schaefer, Virotta)

$\rightarrow$ if simulations not sufficiently long, one may totally underestimate the statistical errors

How can this be overcome? (Lüscher, LATT2010)

1: try to enhance transition between topological sectors using trivializing maps $U = \mathcal{F}(V)$ st Jacobian cancels e^{-S}

then algorithm: $U \rightarrow_{\mathcal{F}^{-1}} V \rightarrow_{HMC} V' \rightarrow_{\mathcal{F}} U'$ (not successful yet)

2. use open boundary conditions (in Euclidean time)

Approximate trivializing maps using gradient (Wilson) flow:

$$\frac{\partial}{\partial t} B_\mu = D_\nu G_{\nu\mu}\,, \quad B_\mu|_{t=0} = A_\mu$$

- Correlation functions of B-fields do not require renormalization
- set new scales e.g. $t^2 \langle G^2 \rangle|_{t=t_0} = 0.3$
- understanding of origin of topological sectors in the continuum limit

Knowledge of autocorrelation times important

Regard computer time as extra space-time coordinate s
autocorrelation functions: $\langle\varphi(x,s)\varphi(x',s')\rangle \sim \mathrm{e}^{-|s-s'|/\tau}$

for "renormalizable algorithms" e.g. Langevin (Parisi, Wu '81)
the scaling behavior of τ as a function of a is predictable:

$\tau \sim a^{-2}\ln^{\omega} a\,,$ Zinn-Justin; Zwanziger, Baulieu (2000)

Hybrid Monte Carlo algorithm (Duane,Kennedy,Pendleton,Roweth'87)
is present algorithm of choice; for free fields $\tau \sim a^{-1}$

HMC is not a renormalizable algorithm (Lüscher, Schaefer 2011)

HMC may be in same universality class as Langevin

Seek algorithms which scale better than a^{-2}

New algorithms: e.g. the Worm Algorithm

introduced by Prokof'ev, Svistunov, Tupitsyn (1998)

recent developments: Chandrasekharan LATT2008, Wolff LATT2010

Based on strong coupling expansion

e.g. Ising model: $S = -\beta \sum_{<x,y>} \sigma(x)\sigma(y)$

2–point fn: $\langle \sigma(u)\sigma(v) \rangle = Z_2(u,v)/Z_0$

$$Z_2(u,v) = 2^{-V} \sum_{\sigma(x)=\pm 1} \mathrm{e}^{-S} \sigma(u)\sigma(v)$$

expand $\mathrm{e}^{\beta\sigma(x)\sigma(y)} = \sum_{k=0} \frac{\beta^k}{k!} \sigma(x)^k \sigma(y)^k$ for each link $< x, y >$

summing spins: $Z_0 = \sum_{g \in \mathcal{G}_0} \beta^{\sum_l k(l)} W[k] \,, \quad W[k] = \prod_l \frac{1}{k(l)!}.$

g: Graph with $k(l)$ lines on a link l

class $\mathcal{G}_0$ has even number of lines ending at each point

similarly $\quad Z_2 = \sum_{g \in \mathcal{G}_2(u,v)} \beta^{\sum_l k(l)} W[k]$

$\mathcal{G}_2(u,v)$: odd number of lines ending at u, v, even elsewhere

Not all graphs computed - instead a MC procedure samples a sufficiently important subset of high order terms

production of independent long distance correlated configurations $\rightarrow$ need to efficiently pass between relevant strong coupling graphs

reduces critical slowing down & in some cases no sign problems!

applications e.g. to QCD at finite density?

Consider O(n) Nienhuis "action":

$$e^{-S} = \prod_{<x,y>} (1 + \beta \sigma_x \cdot \sigma_y)$$

Factor not positive for $\beta > 1 \rightarrow$ not treatable by usual methods

this sign problem not seen using worm algorithm (Wolff, 2010)

Measured step scaling function for $n = 3$ at $u = 1.05$...

is equal to analytically known result of Balog and Hegedus

Same universality class as the standard action? (Nienhuis, 1998)

Topological Actions: in the sense that they are invariant under small changes of the fields

e.g. for the O(n) non-linear 2d σ–model:

$S = \infty$ if $\sigma_x \cdot \sigma_{x+\hat{\mu}} < 1 - \delta$, $\quad S = 1$ otherwise

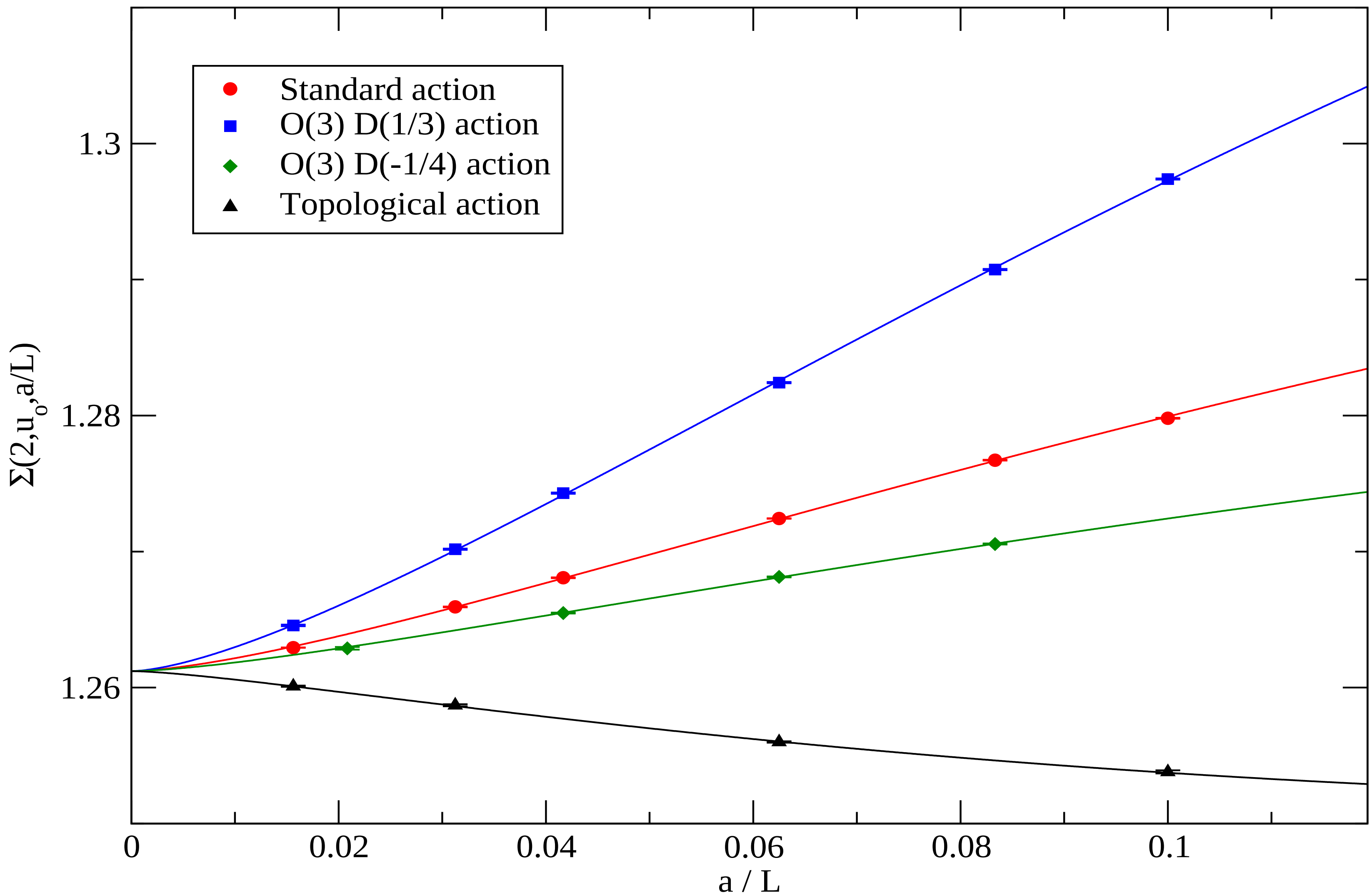
first considered by Patrascioiu, Seiler; Hasenbusch,

now revisited by Bietenholz, Gerber, Pepe, Wiese arXiv:1009.2146

No classical continuum limit & no known perturbative treatment,

but seem in the same universality class as the standard action!

O(3) σ–model step scaling function (Bietenholz, Gerber, Pepe, Wiese)



→ Are universality classes much larger than expected??

Implications? Possible applications to Dirac operators?