Japan Days

Wuppertal, 02.05.2011

Status of the QCD phase diagram from the lattice

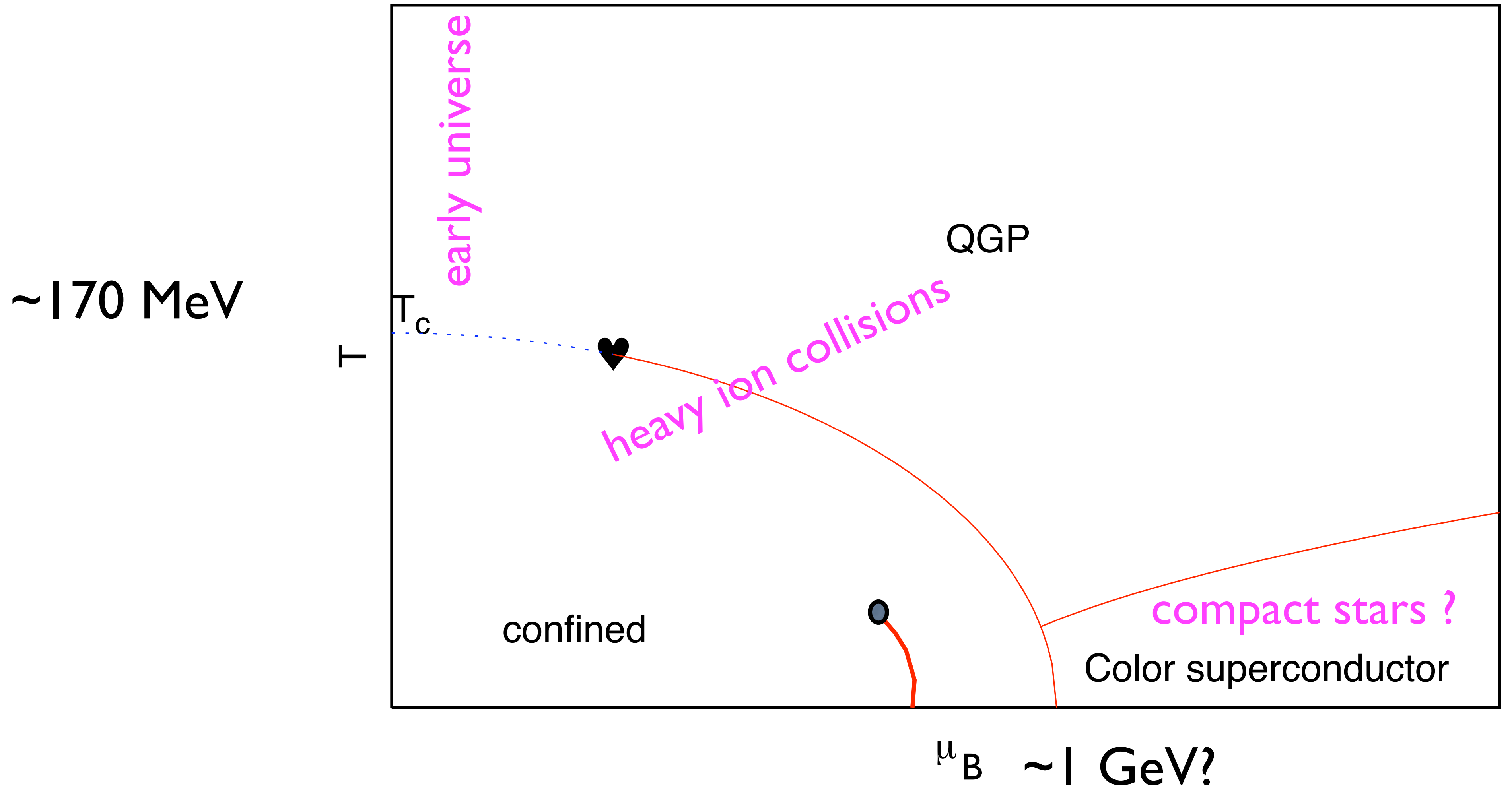
HIC for FAIR
Helmholtz International Center

Owe Philipsen

GOETHE UNIVERSITÄT
FRANKFURT AM MAIN

- Is there a critical end point in the QCD phase diagram?
- Is it connected to a chiral phase transition?
- Imaginary chemical potential: rich phase structure, benchmark for models!

QCD phase diagram: theorist’s conjectures



QGP and colour SC at asymptotic T and densities by asymptotic freedom!

Until 2001: no finite density lattice calculations, sign problem!

Expectation based on models: NJL, NJL+Polyakov loop, linear sigma models,
random matrix models, ...

Theory: how to calculate p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

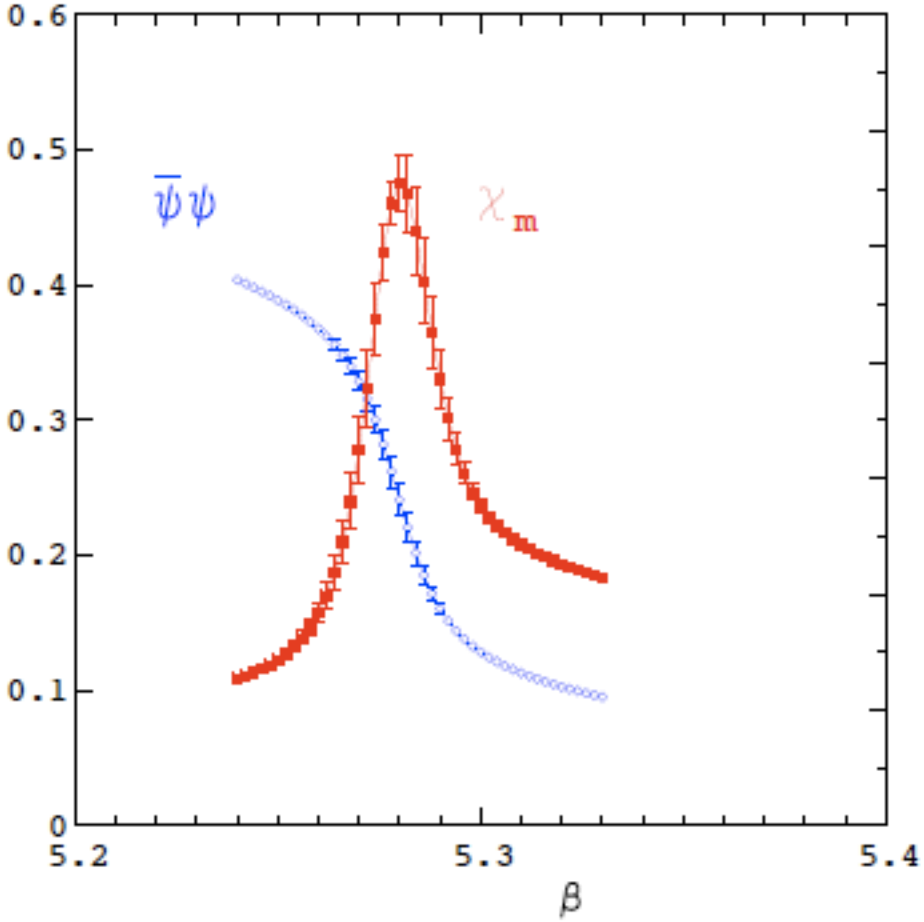
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz. T

only pseudo-critical on finite V!

Order of transition:

finite volume scaling

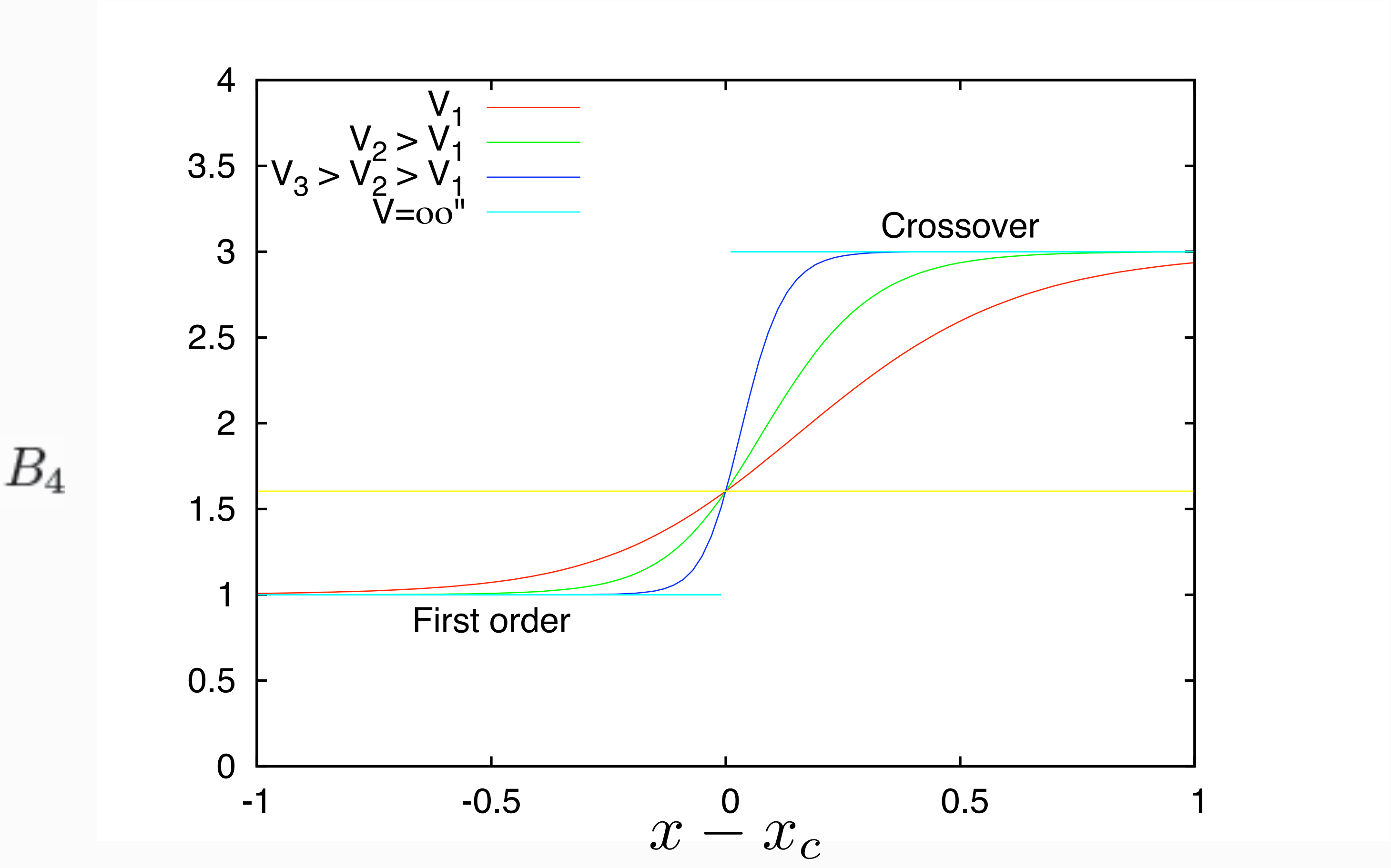
$$\chi_{max} \sim V^{\sigma}$$

$\sigma = 1$	1st order
$\sigma =$ crit. exponent	2nd order
$\sigma = 0$	crossover

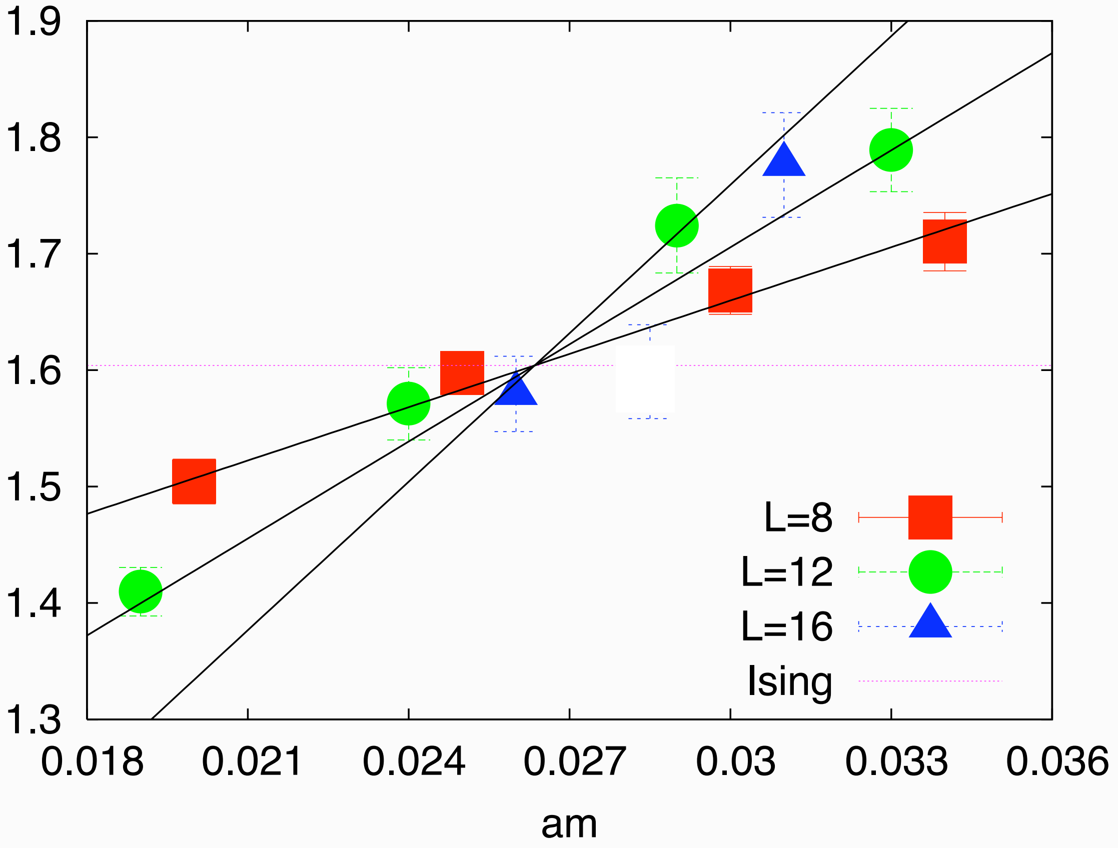
How to identify the order of the phase transition

$$B_4(\bar{\psi}\psi) \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first} - \text{order} \\ 3 & \text{crossover} \end{cases}$$

$\mu = 0:$ $$B_4(m, L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$

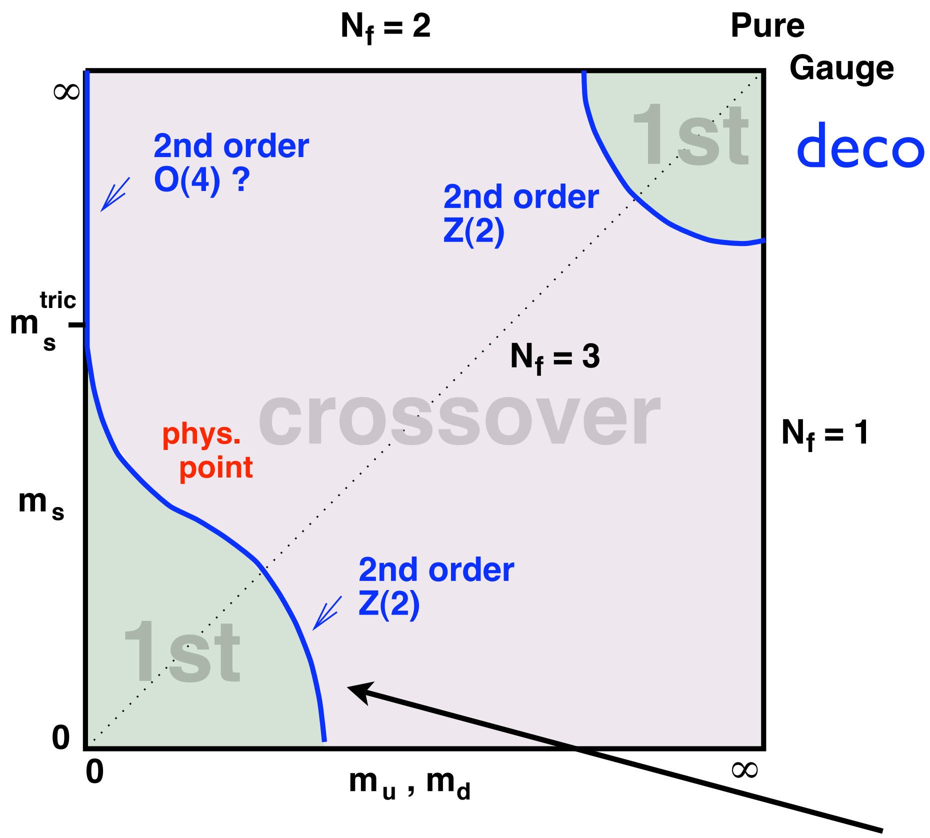


parameter along phase boundary, $T = T_c(x)$

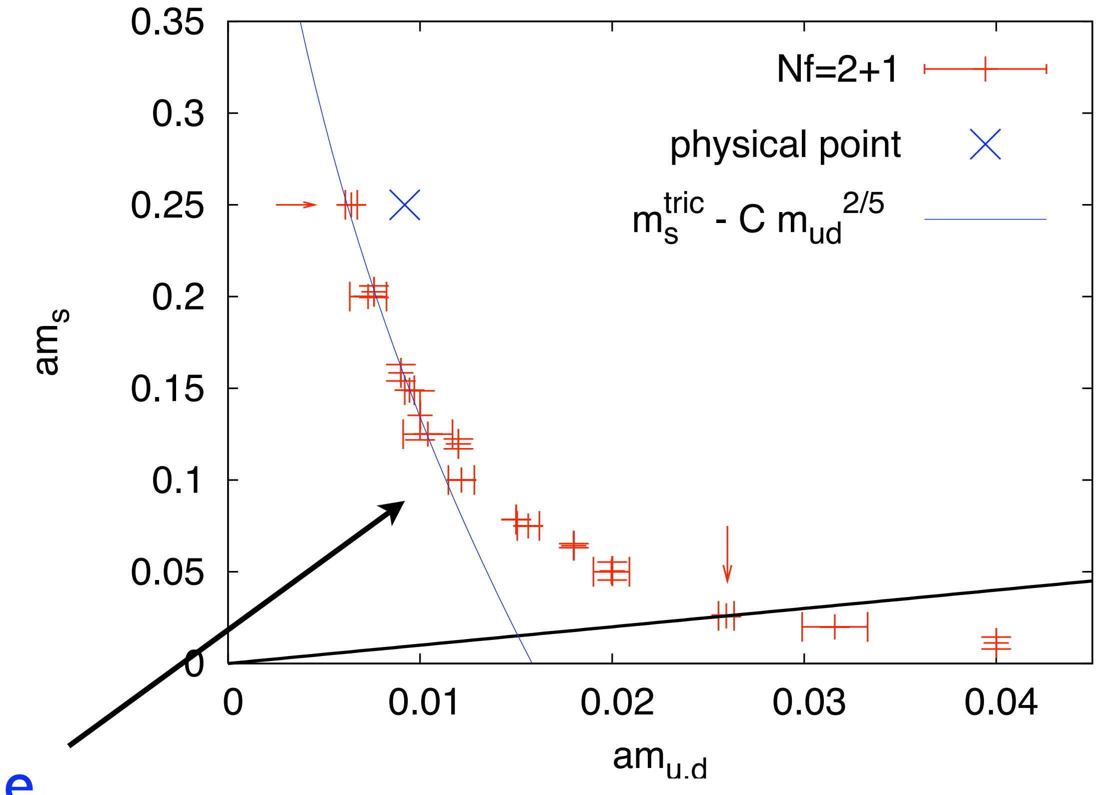


Order of p.t., arbitrary quark masses $\mu = 0$

chiral p.t.



deconf. p.t.



chiral critical line

- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\rm tric} \sim 2.8T$
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07

The ‘sign problem’ is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\not{D}(\mu)^\dagger = \gamma_5 \not{D}(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

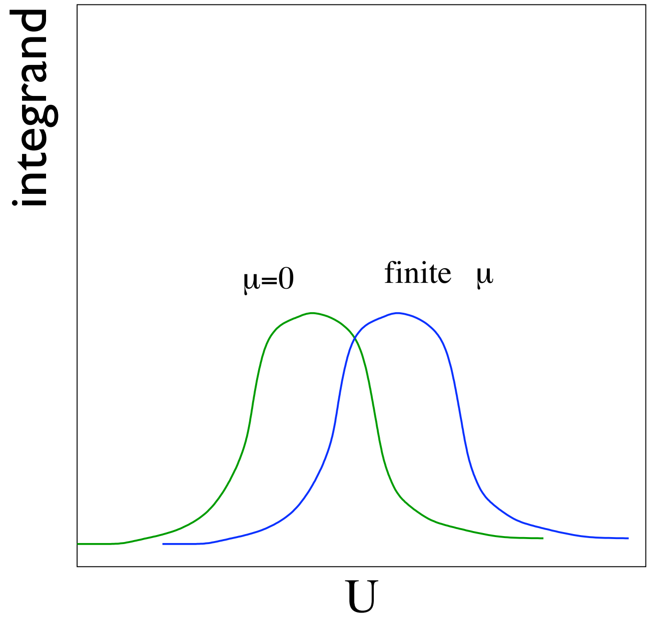
$\Rightarrow$real positive for SU(2), $\mu = i\mu_i$

$\Rightarrow$real positive for $\mu_u = -\mu_d$

N.B.: all expectation values real, imaginary parts cancel,
but importance sampling config. by config. impossible!

Same problem in many condensed matter systems!

Finite density: methods to evade the sign problem



- Reweighting: $Z = \int DU \; \det M(0) \frac{\det M(\mu)}{\det M(0)} \, \mathrm{e}^{-S_g}$

$$\uparrow \qquad \uparrow$$

~exp(V) statistics needed, overlap problem

use for MC

calculate

Optimal: use |det| in measure, reweight in phase

- Taylor expansion:

$$\langle O\rangle(\mu) = \langle O\rangle(0) + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

coefficients one by one, convergence?

- Imaginary $\mu = i\mu_i$: no sign problem, fit by polynomial, then analytically continue

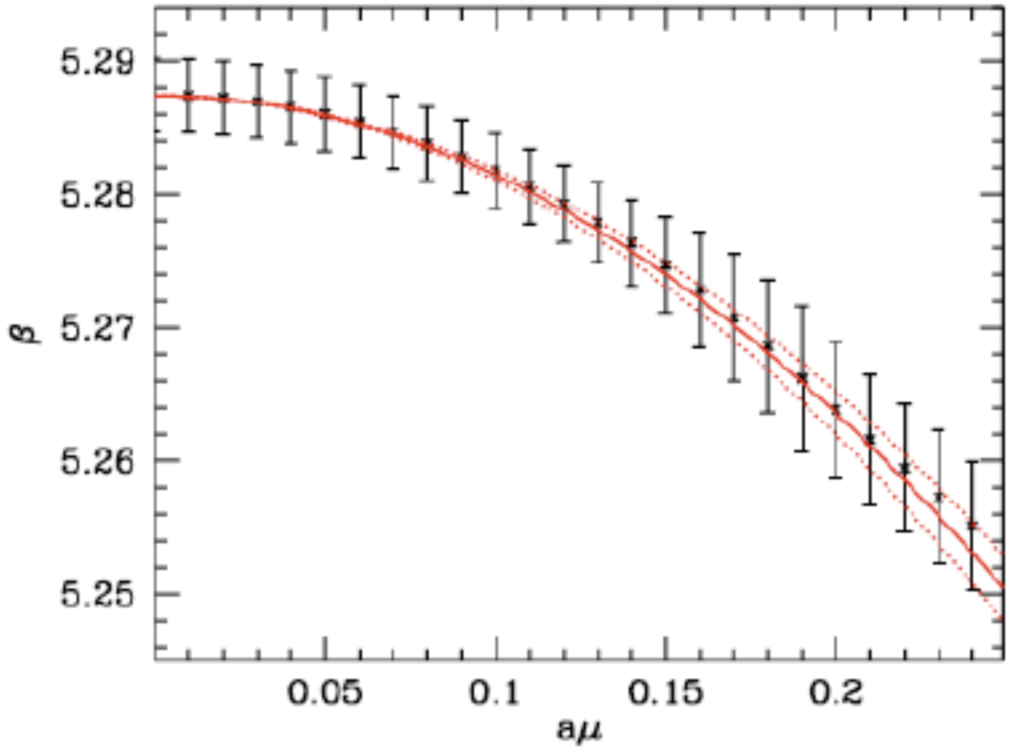
$$\langle O\rangle(\mu_i) = \sum_{k=0}^{N} c_k \left(\frac{\mu_i}{\pi T}\right)^{2k}, \qquad \mu_i \to -i\mu$$

requires convergence for analytic continuation

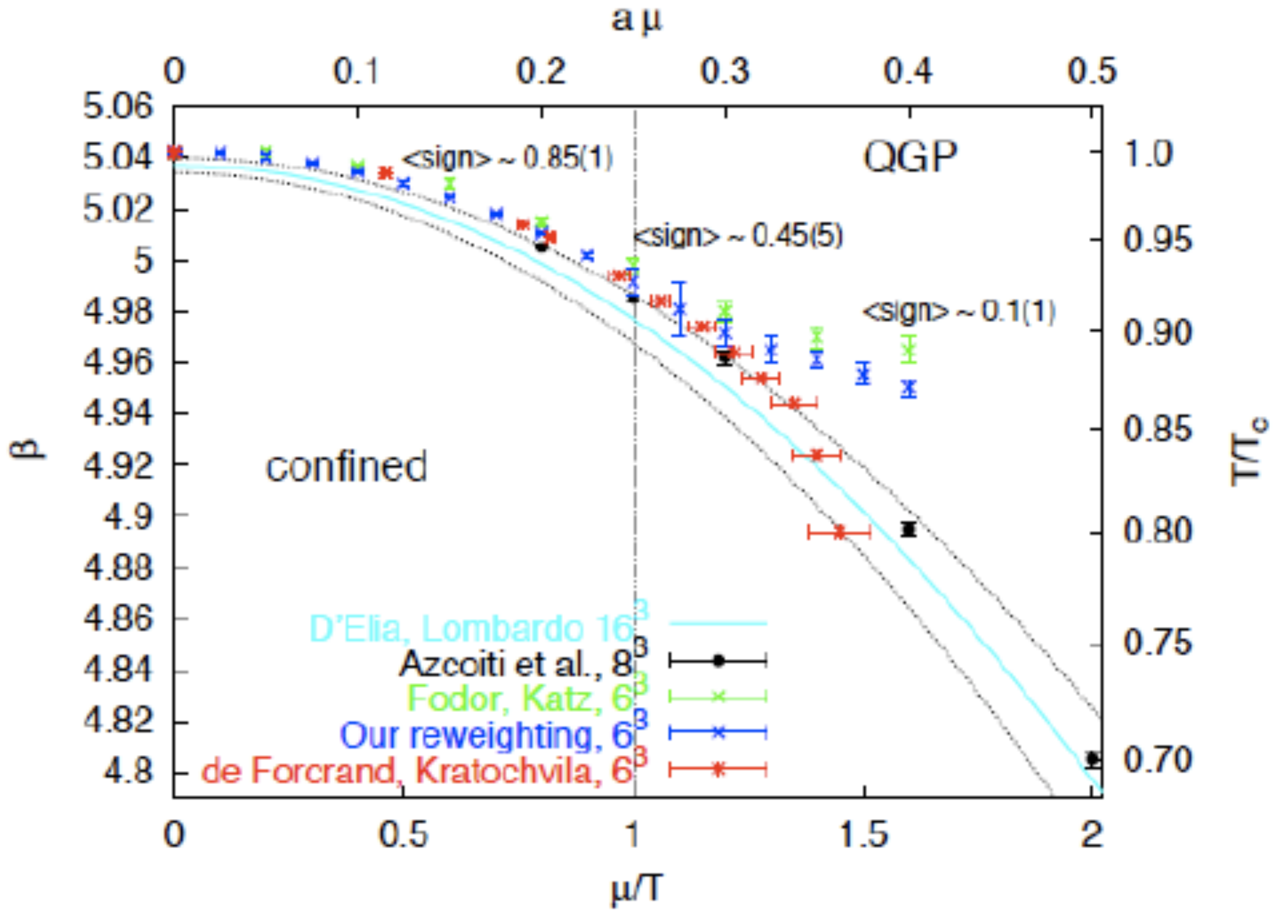
All require $\mu/T < 1$!

Test of methods: comparing $T_c(\mu)$

Reweighting vs. imag. μ (FK, FP)

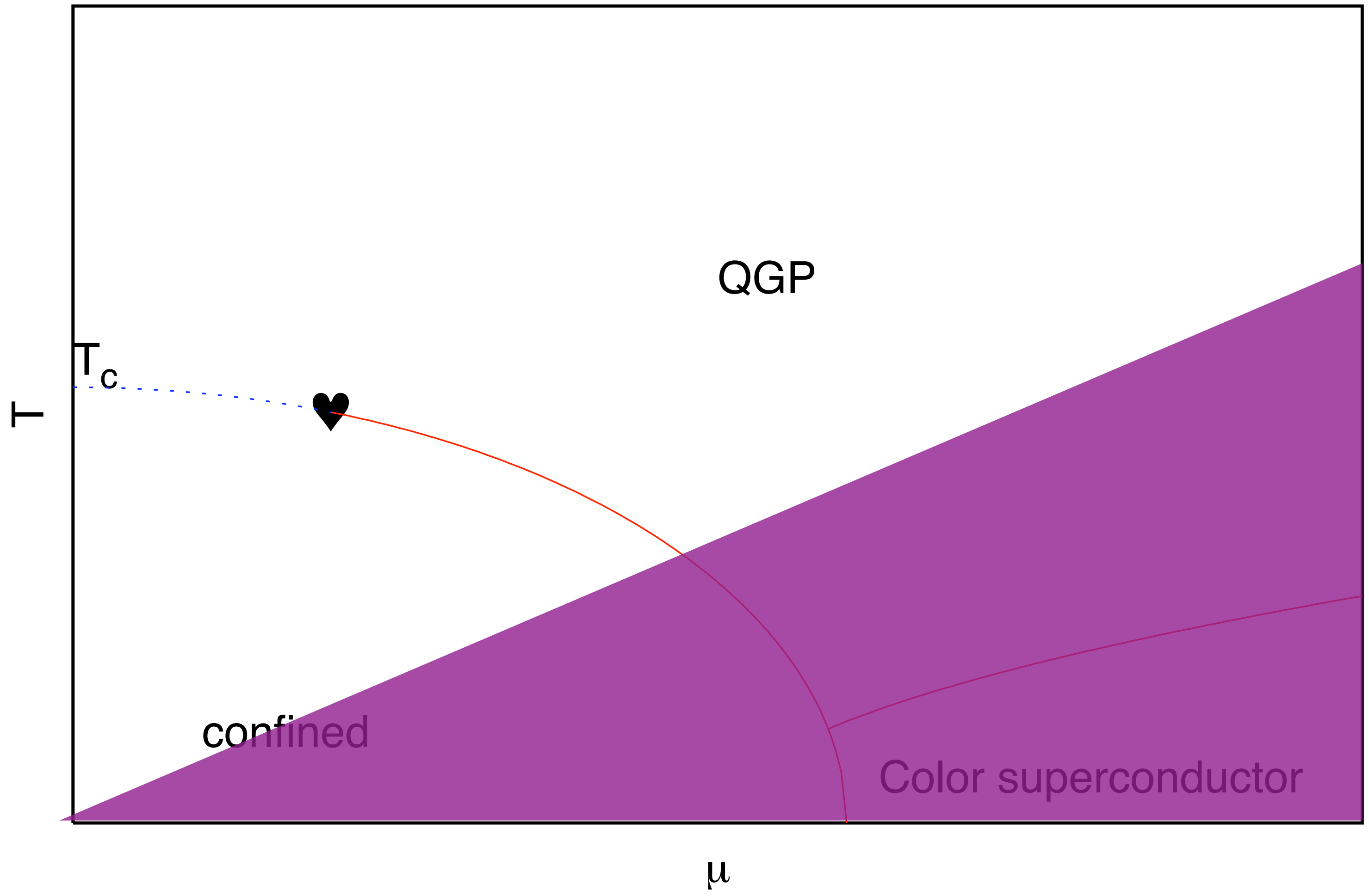


Rew., imag. μ, canonical ensemble ...



All agree on $T_0(m,\mu)$!!! $(\mu/T \lesssim 1)$

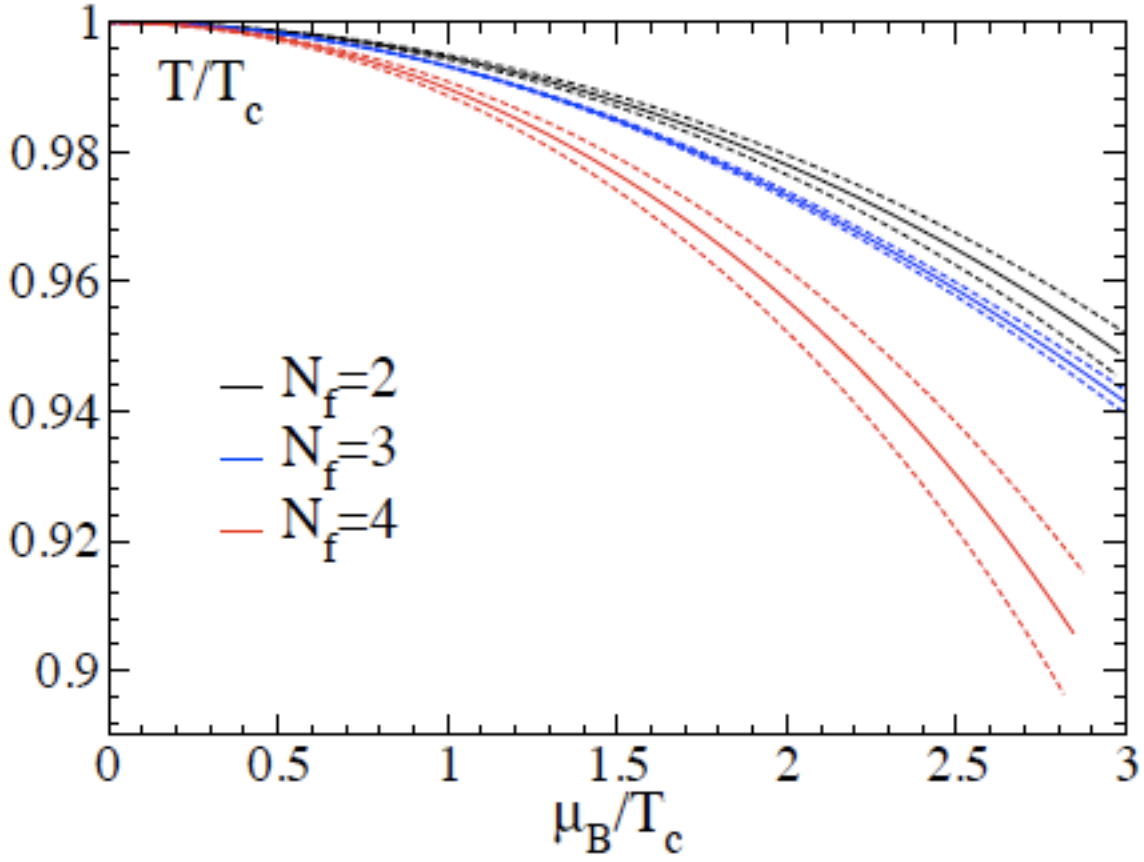
The calculable region of the phase diagram



- need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control

The (pseudo-) critical temperature

$$\frac{T_c(\mu)}{T_c(0)} = 1 - \kappa(N_f, m_q)\left(\frac{\mu}{T}\right)^2 + \ldots$$



- Curvature rather small
- $\kappa \propto \frac{N_f}{N_c}$ Toublan 05

de Forcrand, O.P. 03
D'Elia, Lombardo 03

Pseudo-critical temperature

- Curvature of crit. line from Taylor expansion
  2+1 flavours, Nt=4, 8 improved staggered
- Extrapolation to chiral limit assuming
  O(4),O(2) scaling of magn. EoS
- $\kappa(\bar{\psi}\psi) = 0.059(2)(4)$

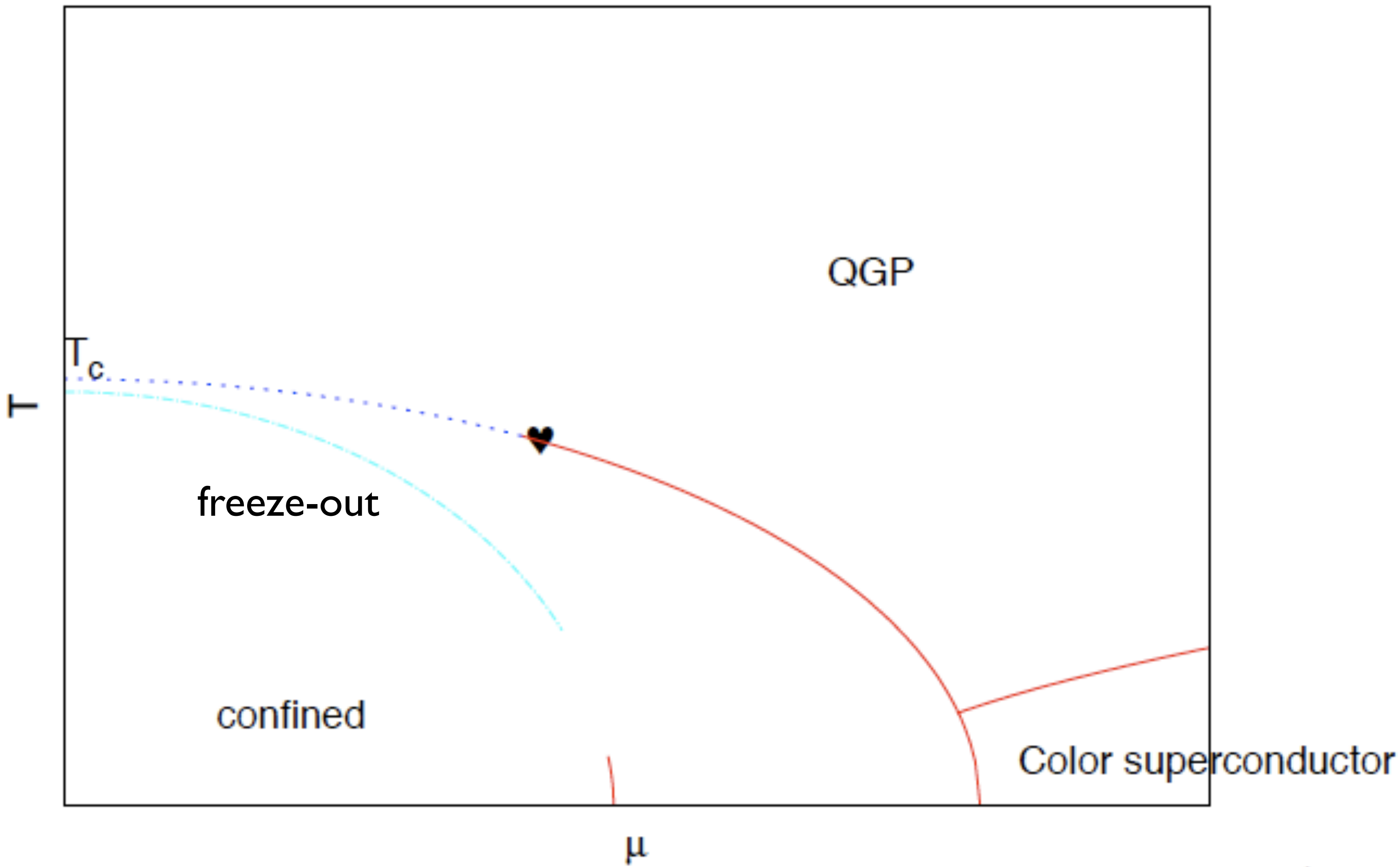
hotQCD 11

Endrödi et al. 11

- Curvature of crit. line from Taylor expansion
  2+1 flavours, Nt=6,8,10 improved staggered
- Observables $\bar{\psi}\psi_r, \chi_s$
- Continuum extrapolation:

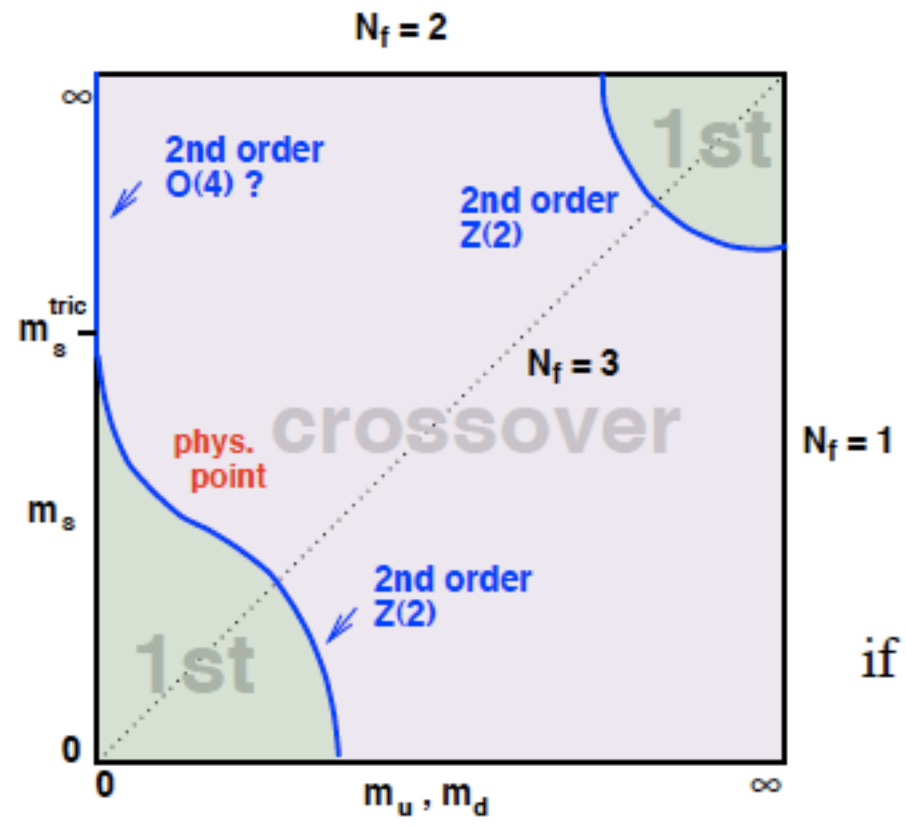
$\kappa^{(\bar{\psi}\psi_r)} = 0.0066(20)$ $\kappa^{(\chi_s/T^2)} = 0.0089(14)$

Comparison with freeze-out curve

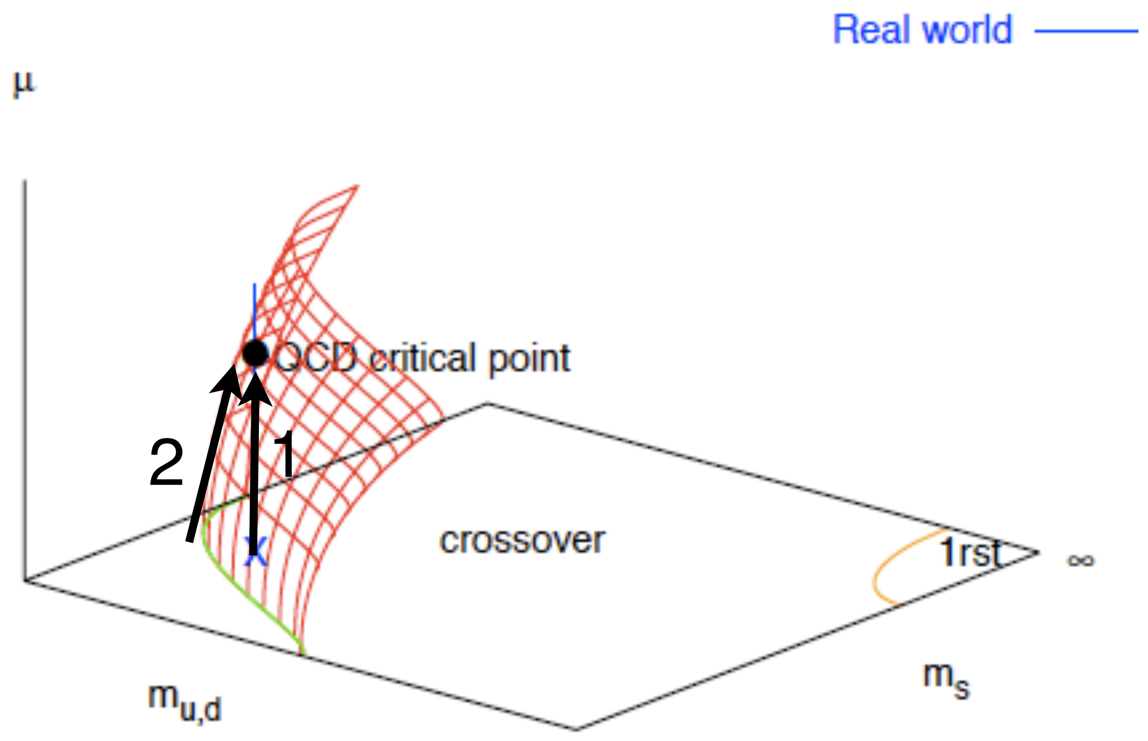


$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2 T_c}{d\mu^2}\right|_{\mu=0}$)

Much harder: is there a QCD critical point?



$\underbrace{\longrightarrow}_{\text{if } chiral \text{ CEP}}$



Two strategies:

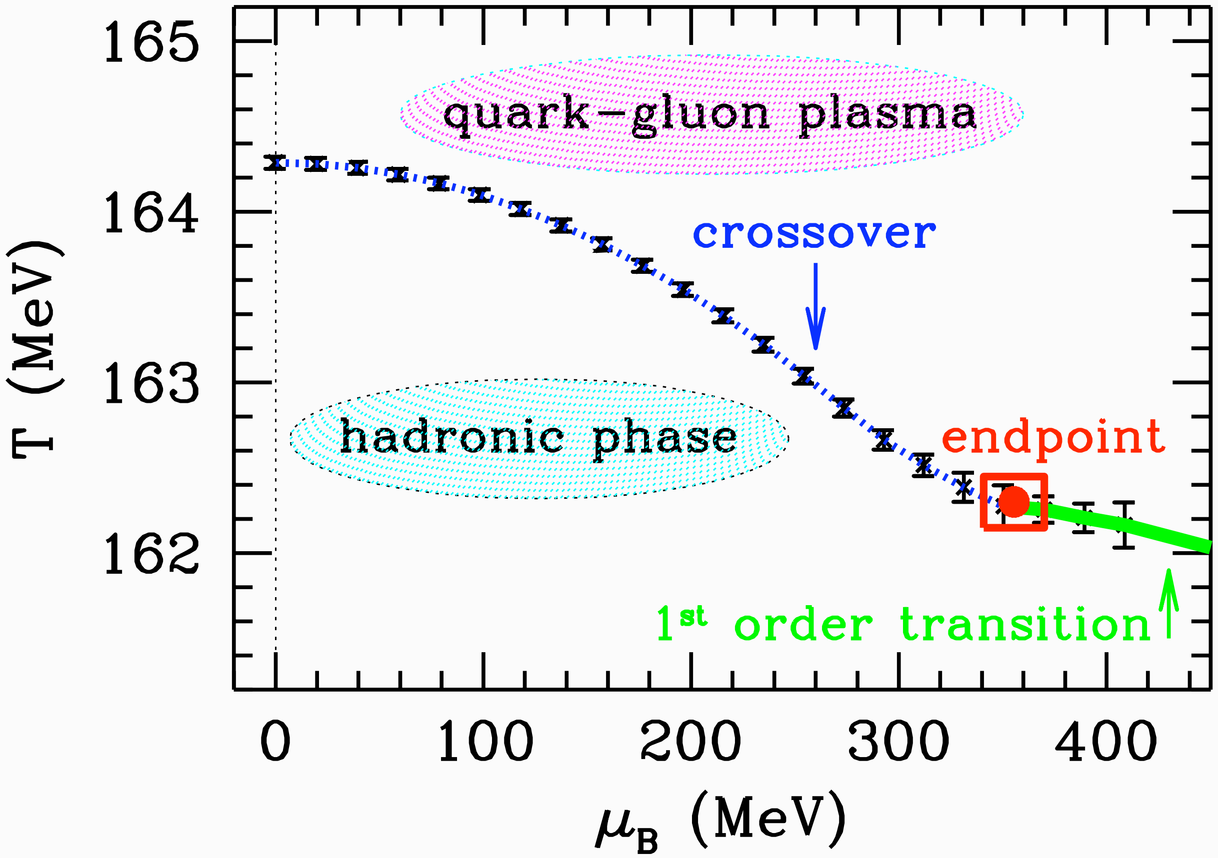
1 follow vertical line: $m = m_{\rm phys}$, turn on μ

2 follow critical surface: $m = m_{\rm crit}(\mu)$

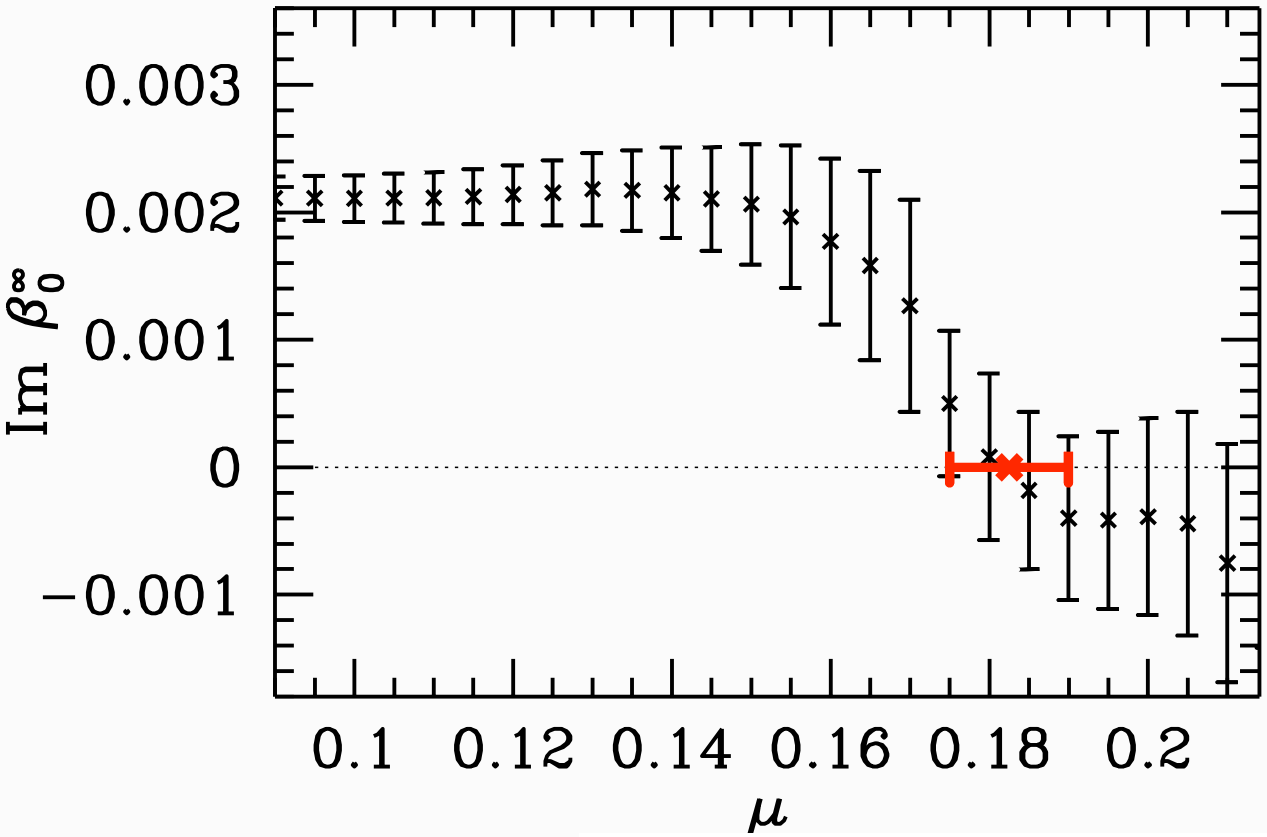
Approach 1a: CEP from reweighting

Fodor, Katz 04

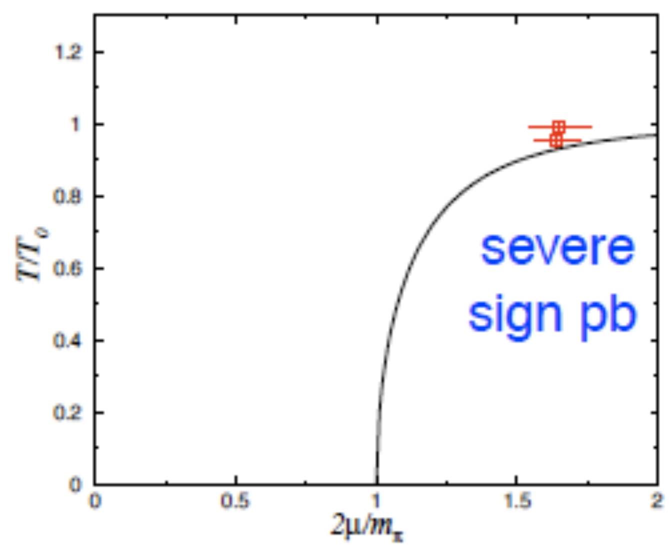
$N_t = 4, N_f = 2+1$ physical quark masses, unimproved staggered fermions



Lee-Yang zero:



$$(\mu_E^q, T_E) = (120(13), 162(2)) \text{ MeV}$$



severe sign pb

abrupt change: caused by baryon or pion condensation?

Splittorf 05, Stephanov 08

Approach 1b: CEP from Taylor expansion

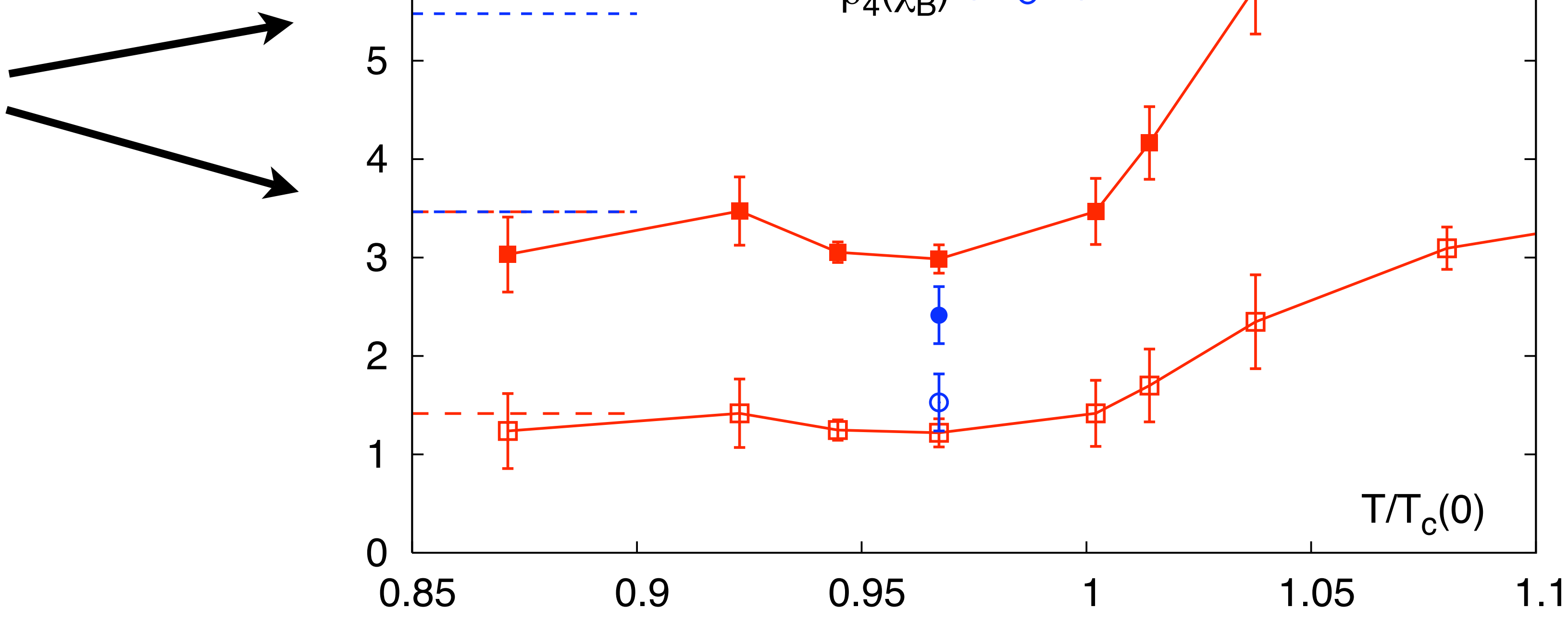
$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Nearest singularity=radius of convergence

$$\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}, \quad \lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$$

Different definitions agree only for $n \to \infty$
not n=1,2,3,...
control of systematics?

Hadron resonance gas



C.Schmidt, hotQCD 09

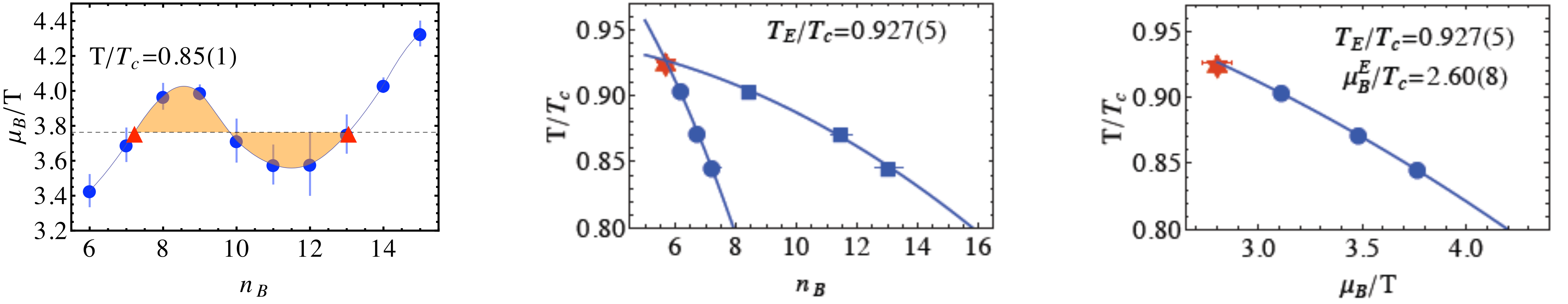
Radius of convergence necessary condition for CEP, but can it proof its existence?

Approach 1c: canonical ensemble

$$Z_C(V,T,k) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, e^{-ik\phi} Z(V,T,\mu)|_{\mu=i\phi T} \qquad \langle\mu\rangle_{n_B} = \frac{F(n_B+1)-F(n_B)}{(n_B+1)-n_B}$$

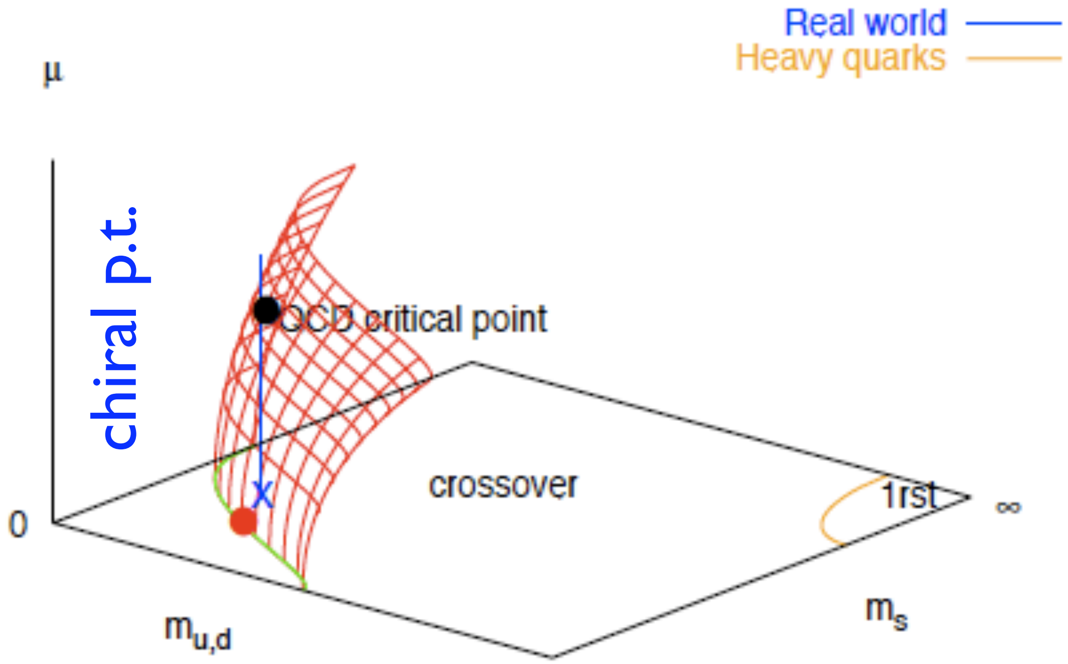
Alexandru, Li, Liu 11 Wilson clover fermions $6^3\times 4, m_\pi \sim 700-800\mathrm{MeV}$

Maxwell construction on finite volumes:

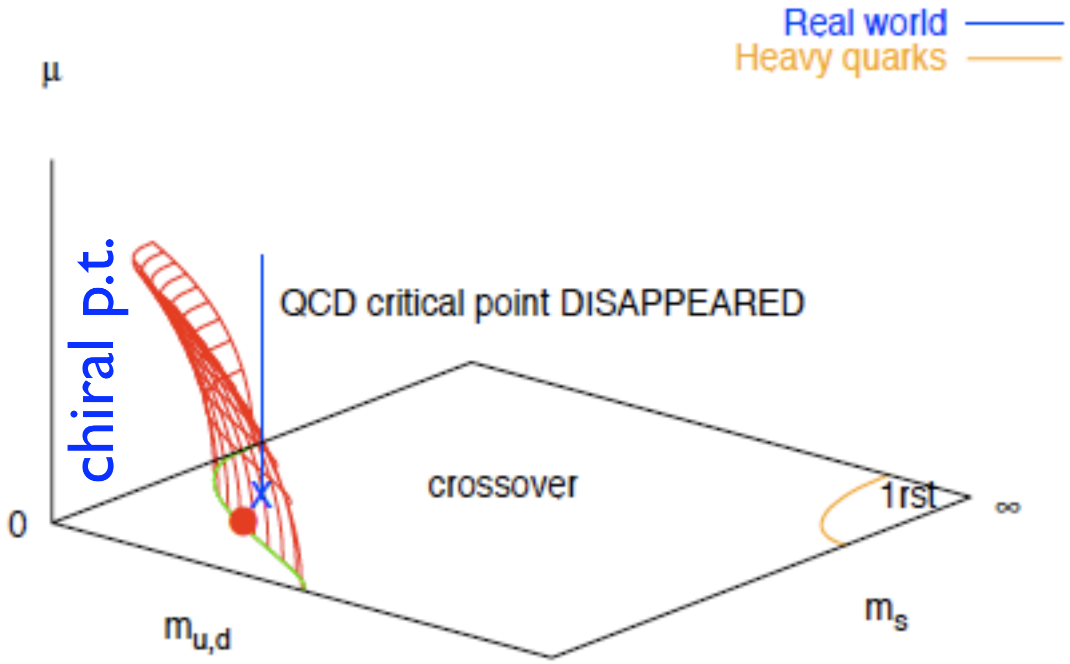


No thermodynamic limit yet, heavy pions

Approach 2: follow chiral critical line → surface



$c_1 > 0$



$c_1 < 0$

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

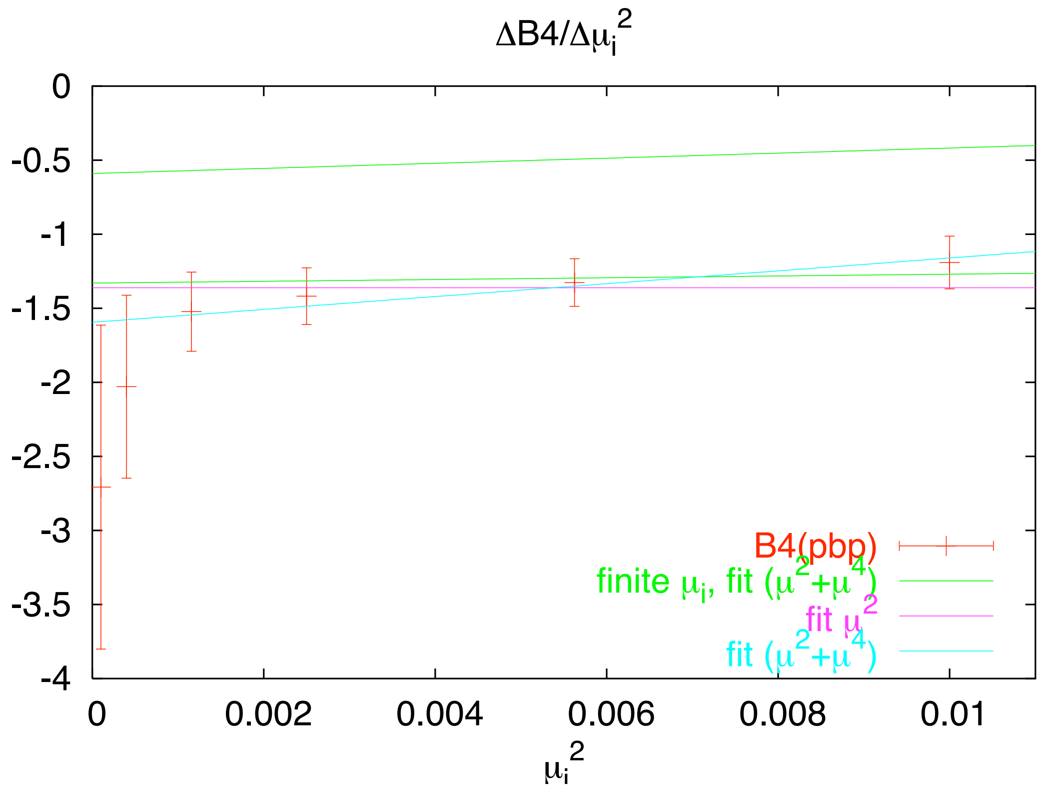
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0, T = T_c$
   known universality class: $3d$ Ising

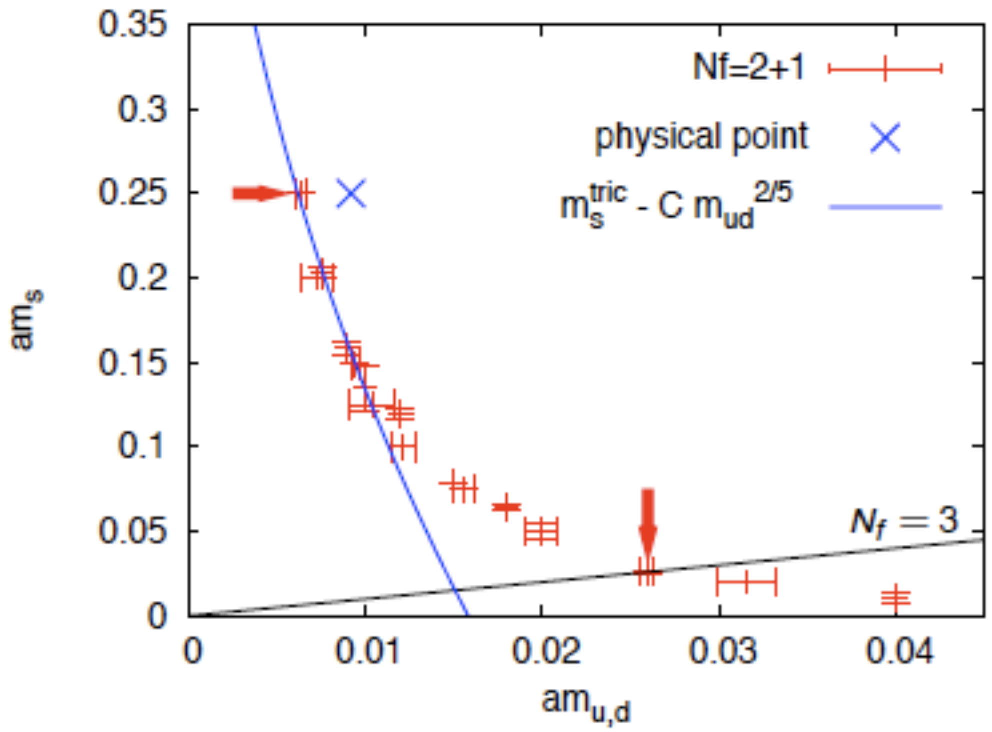
2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Curvature of the chiral critical surface





Nf=3: a) fit to imaginary chemical potential
b) calculation of coefficient by finite differences

consistent $8^3 \times 4$ and $12^3 \times 4$, $\sim 5 \times 10^6$ traj.

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 \underbrace{- 47(20)\left(\frac{\mu}{\pi T}\right)^4}_{\text{8th derivative of P}} - \ldots$$

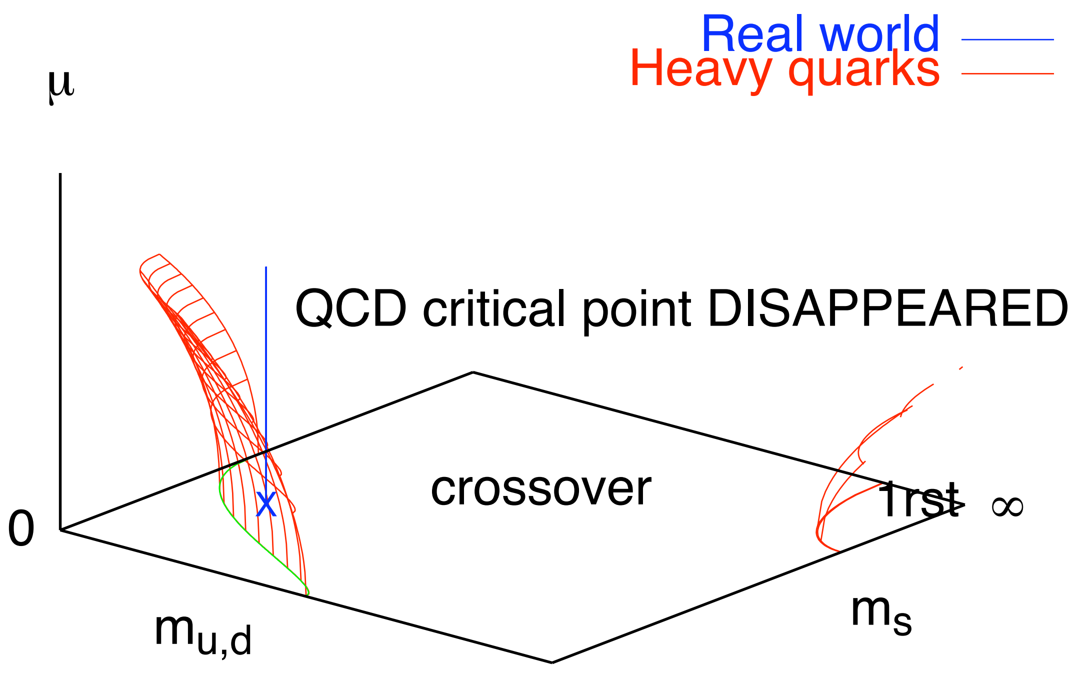
$16^3 \times 4$, Grid computing, $\sim 10^6$ traj.

$$\frac{m_c^{u,d}(\mu)}{m_c^{u,d}(0)} = 1 - 39(8)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

Importance of higher order terms ?

de Forcrand, O.P. 08,09

On coarse lattice exotic scenario: no chiral critical point at small density



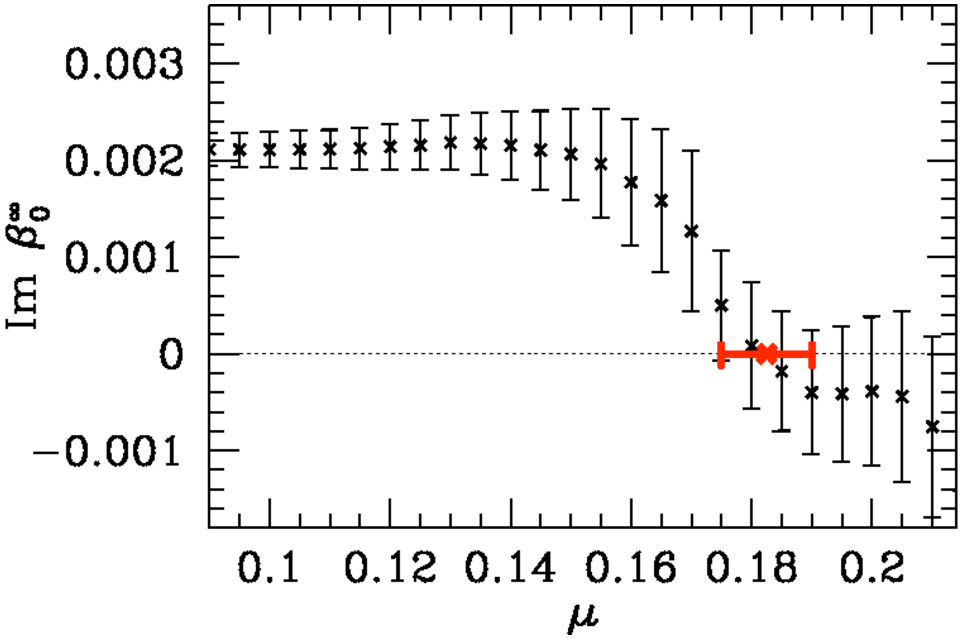
Weakening of p.t. with chemical potential also for:

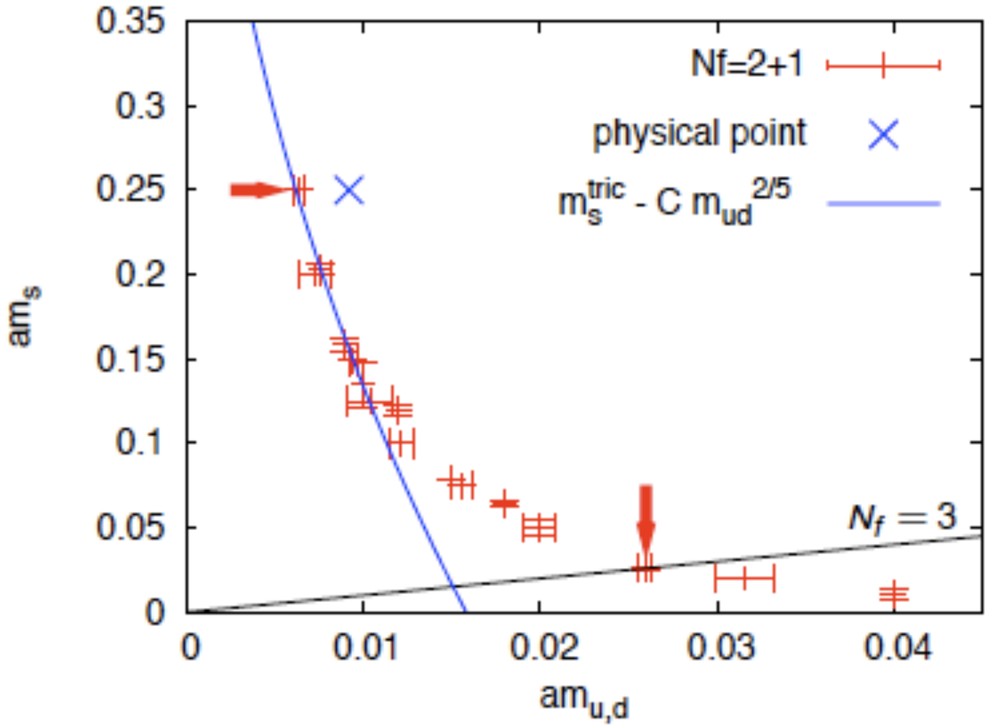
-Heavy quarks	de Forcrand, Kim, Takaishi 05
-Light quarks with finite isospin density	Kogut, Sinclair 07
-Electroweak phase transition with finite lepton density	Gynther 03

Contradiction between Nt=4 results?

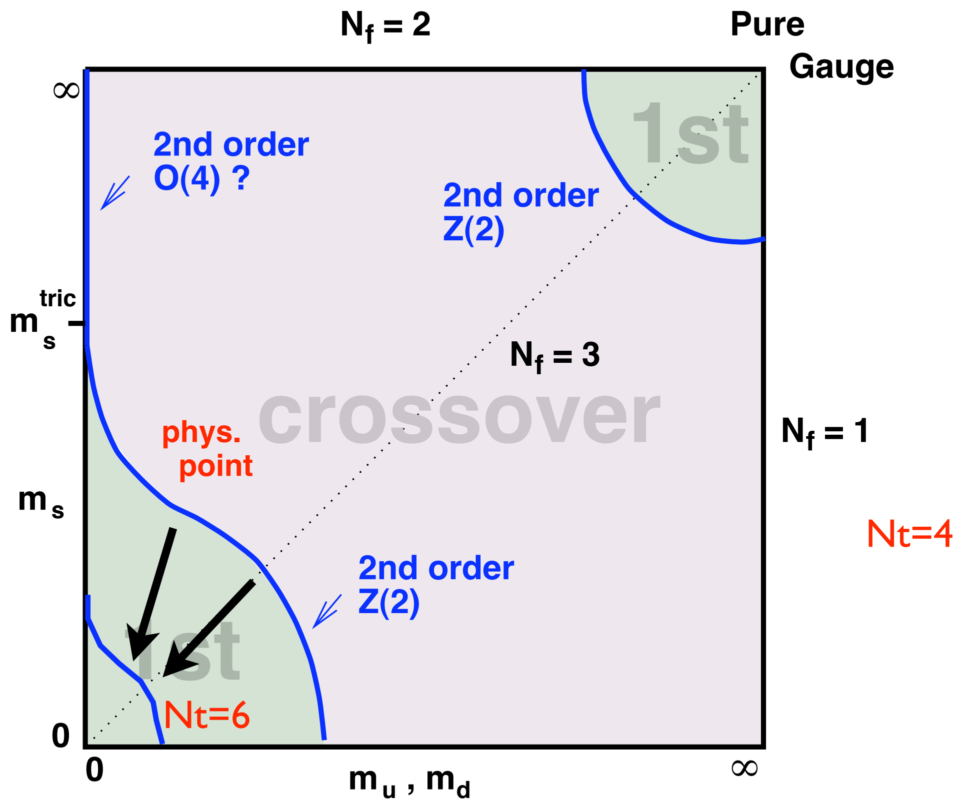
No: consistent initial weakening of transition;

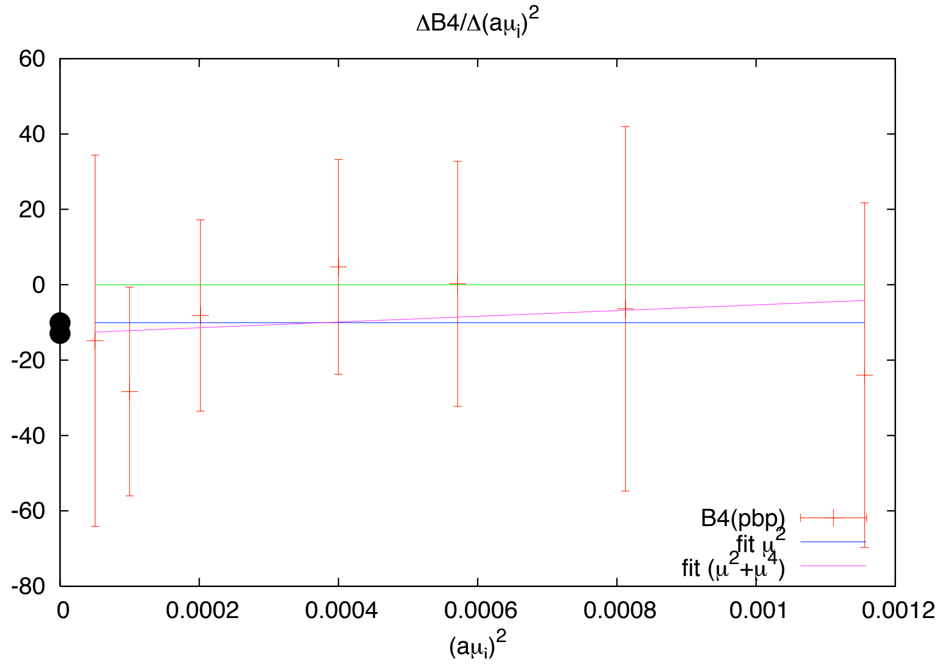
Possible sources for discrepancy:
renormalization effect/ importance of higher order terms, pion condensation





Towards the continuum: $N_t = 6, a \sim 0.2 \text{ fm}$





$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \qquad N_f = 3$$

de Forcrand, Kim, O.P. 07
Endrödi et al 07

- Physical point deeper in crossover region as $a \to 0$
- Cut-off effects stronger than finite density effects!
- Preliminary: curvature of chiral crit. surface remains negative de Forcrand, O.P. 11
- No chiral critical point at small density

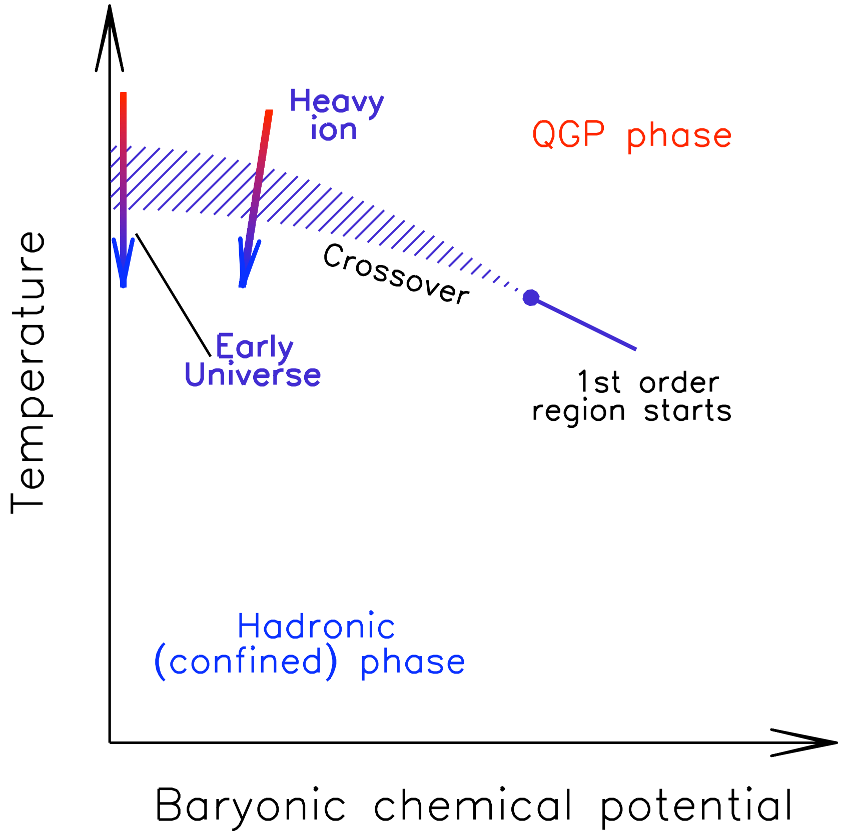
Same statement with different methods

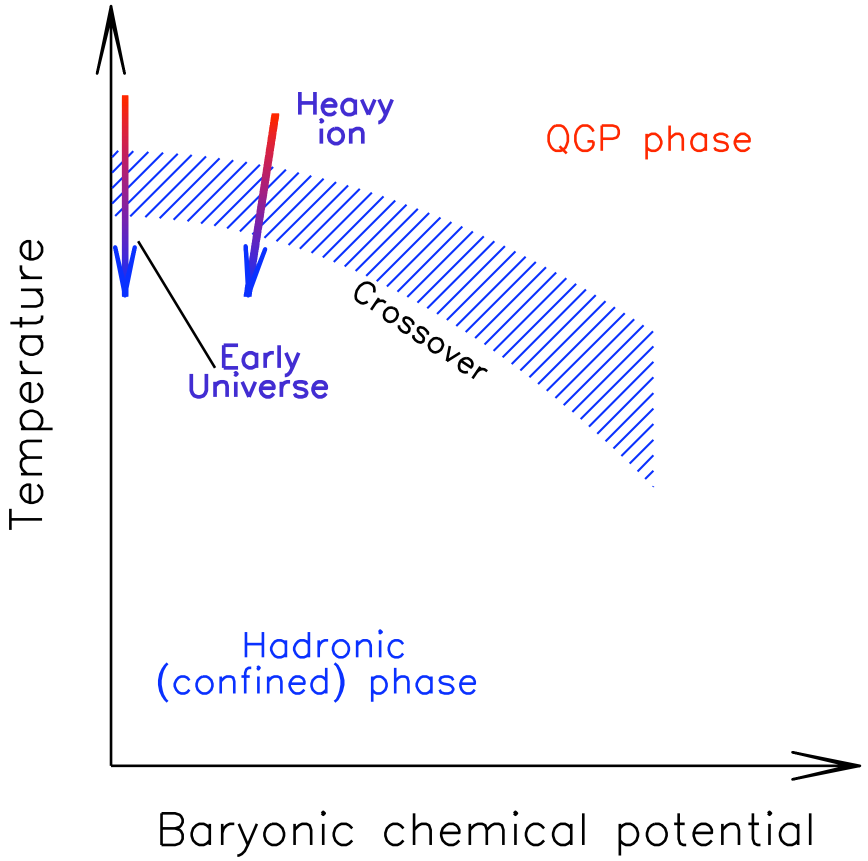
Study suitably defined width of crossover region

$$\frac{1}{W}\frac{\partial W}{\partial(\mu^2)} = -\left.\frac{1}{T_c}\frac{\partial \kappa}{\partial T}\right|_{T=T_c}$$

strengthening of transition

Endrödi et al., 11 find
weakening of crossover
continuum extrapolated Nt=6,8,10



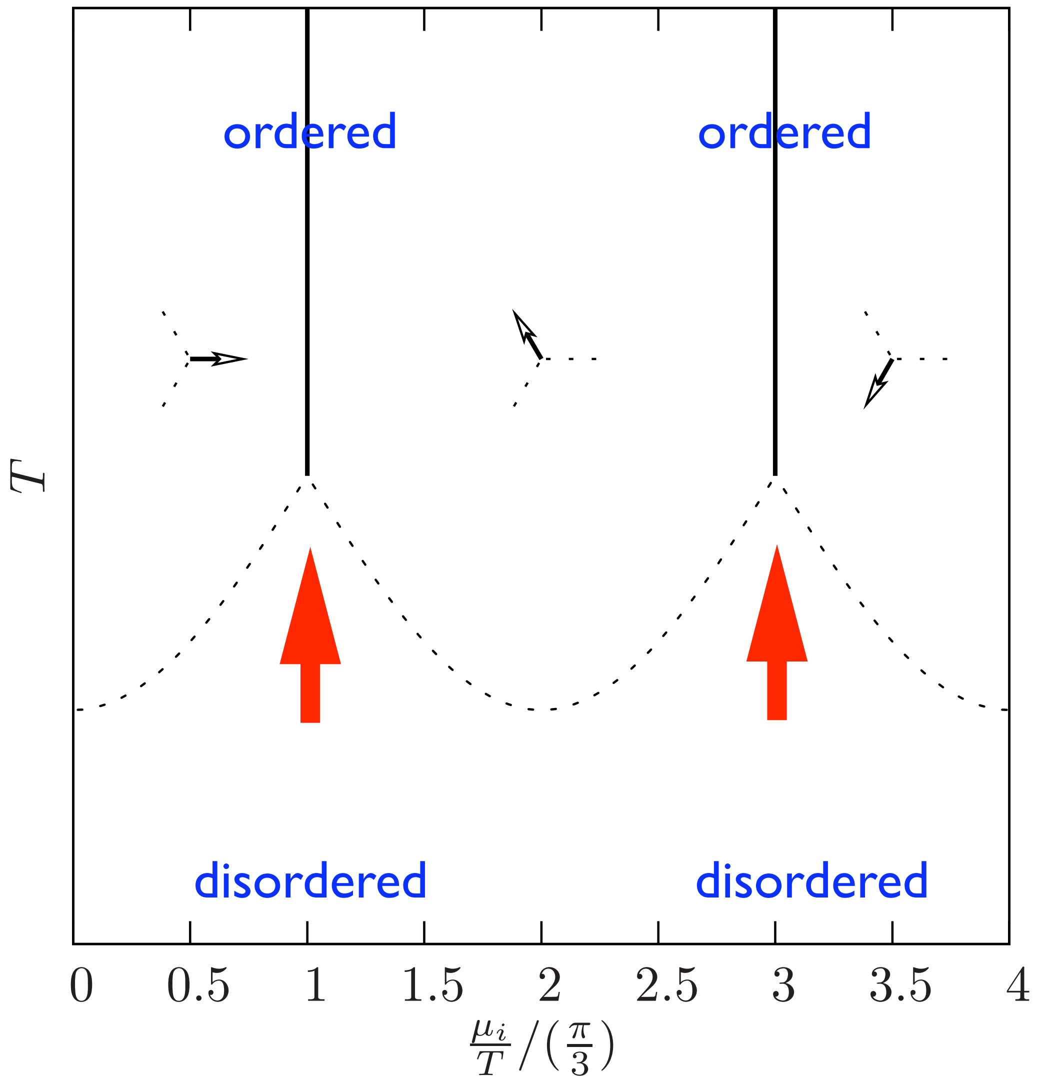


Understanding the curvature from imaginary μ

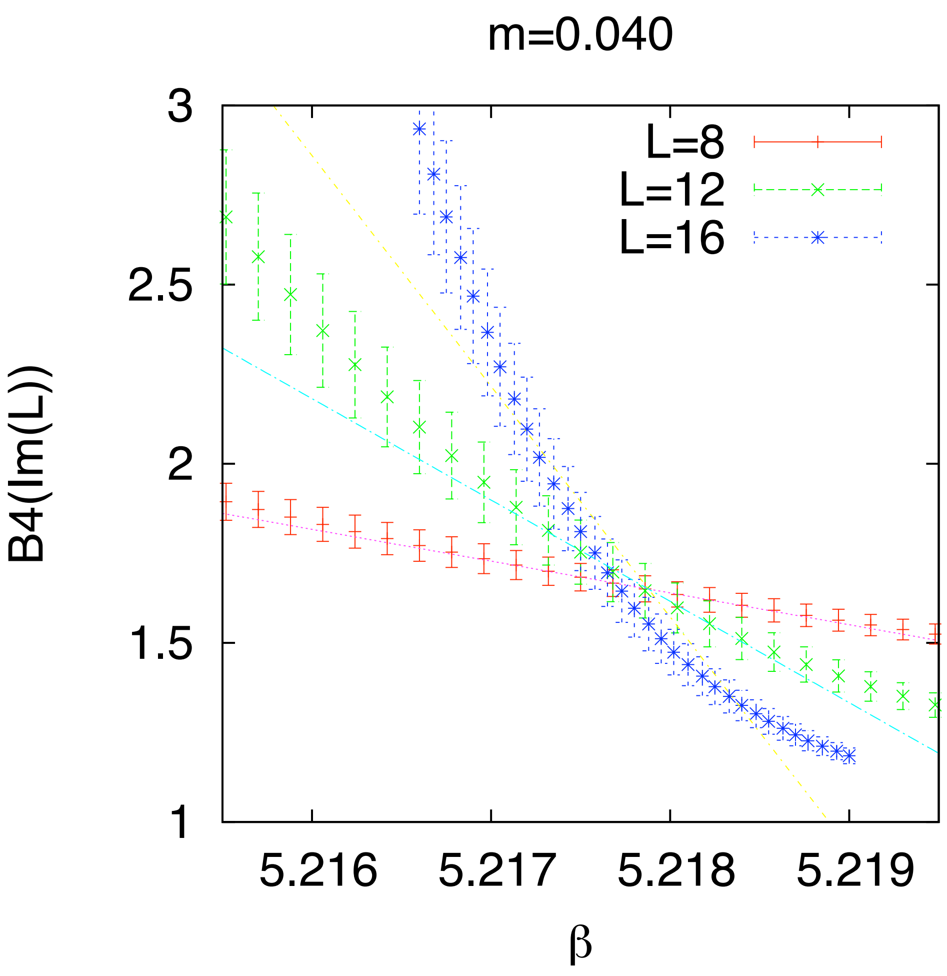
Nf=4: D'Elia, Di Renzo, Lombardo 07   Nf=2: D'Elia, Sanfilippo 09   Nf=3: de Forcrand, O.P. 10

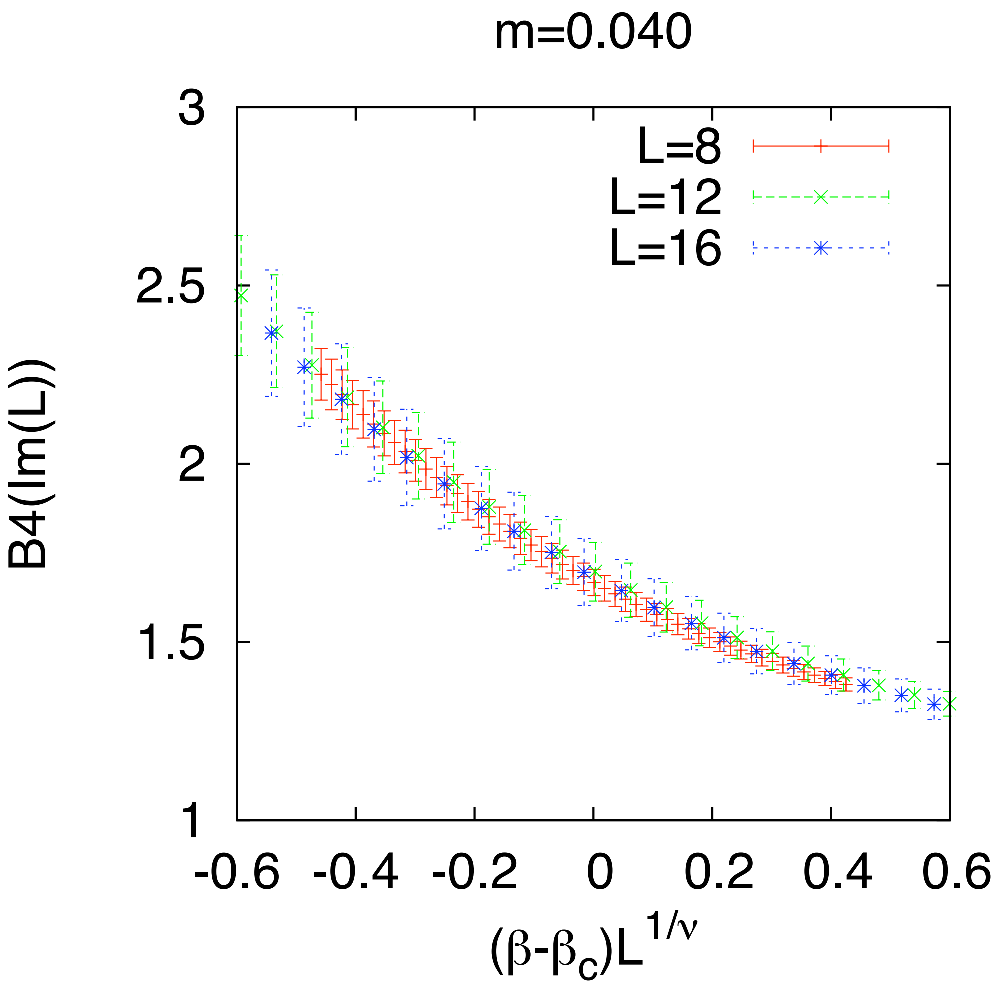
Strategy: fix $\frac{\mu_i}{T} = \frac{\pi}{3}, \pi$, measure Im(L), order parameter at $\frac{\mu_i}{T} = \pi$

determine order of Z(3) branch/end point as function of m



Results:





$\nu = 0.33$

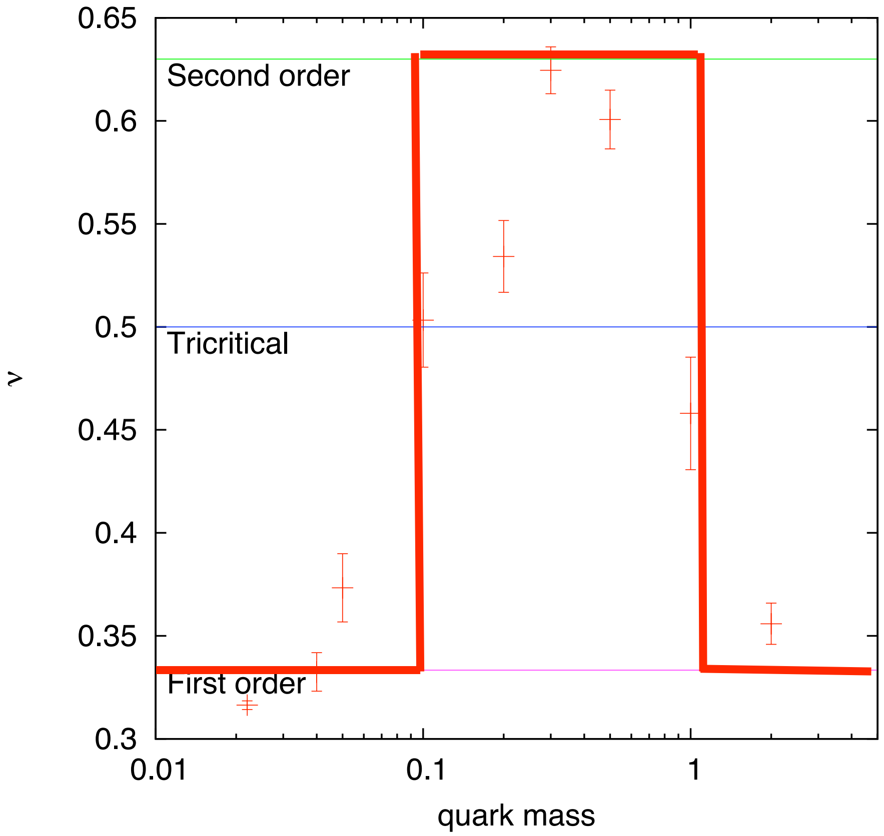
$$B_4(\beta, L) = B_4(\beta_c, \infty) + C_1(\beta - \beta_c)L^{1/\nu} + C_2(\beta - \beta_c)^2 L^{2/\nu} \dots$$

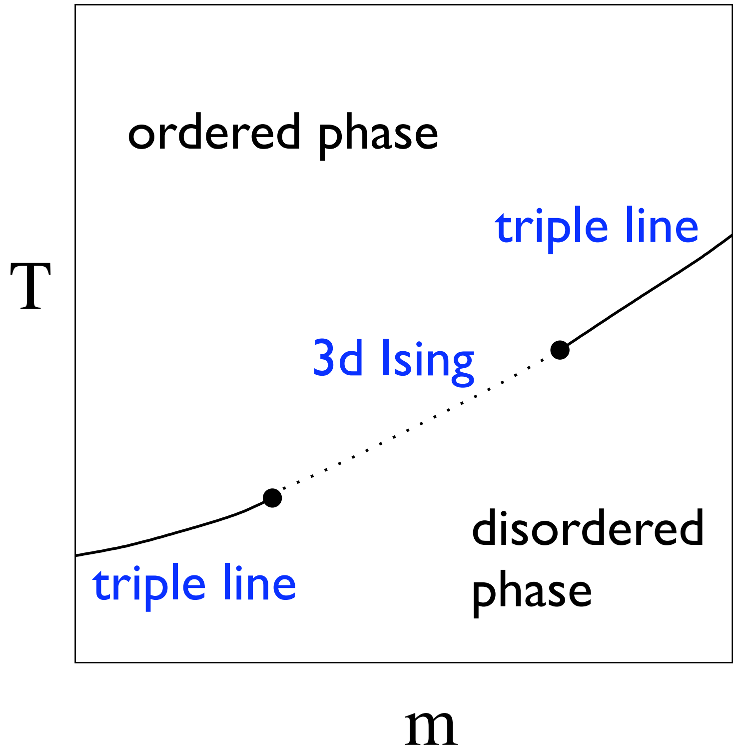
B4 at intersection has large finite size corrections (well known), ν more stable

Scaling of Binder cumulant: $\nu = 0.33, 0.5, 0.63$

for 1st order, tri-critical, 3d Ising

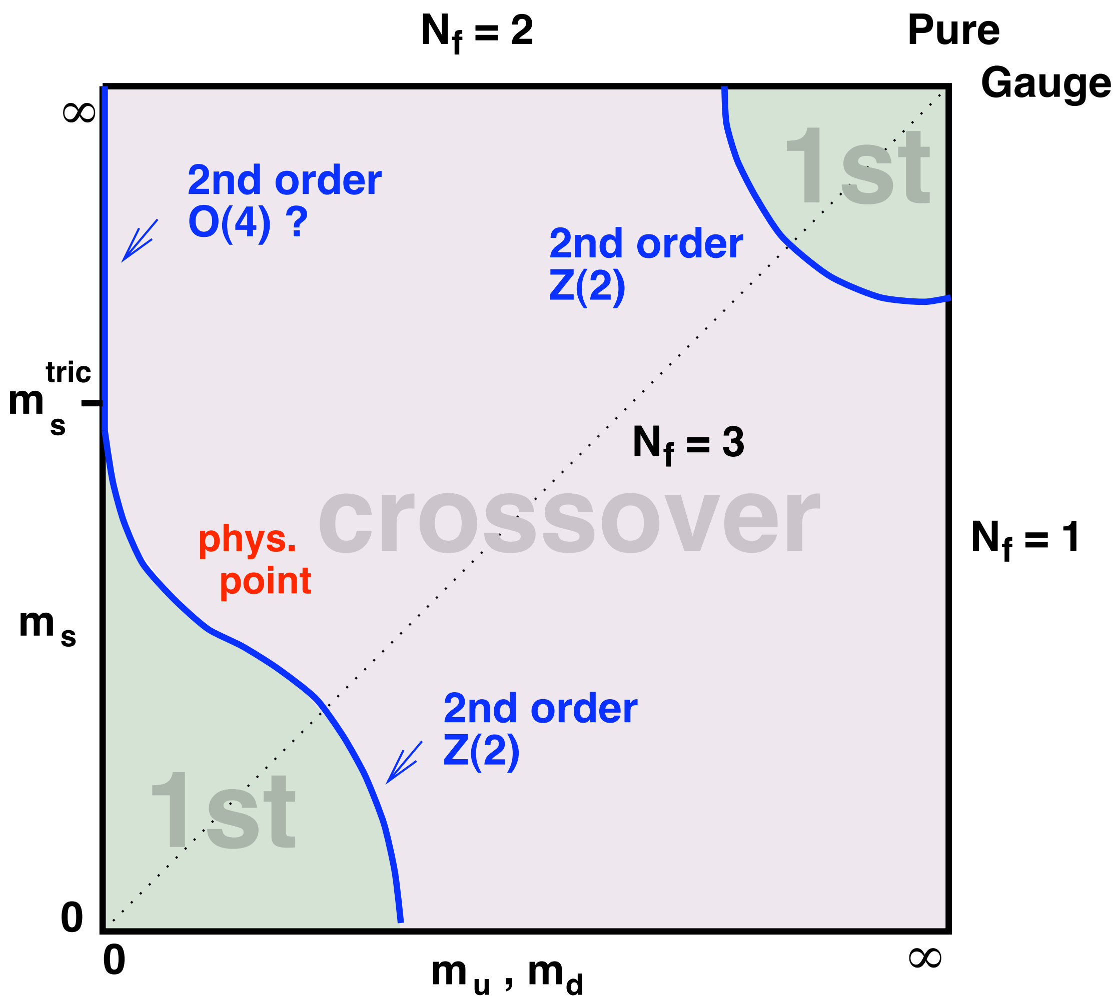
Phase diagram at fixed $\frac{\mu_i}{T} = \frac{\pi}{3}, \pi$



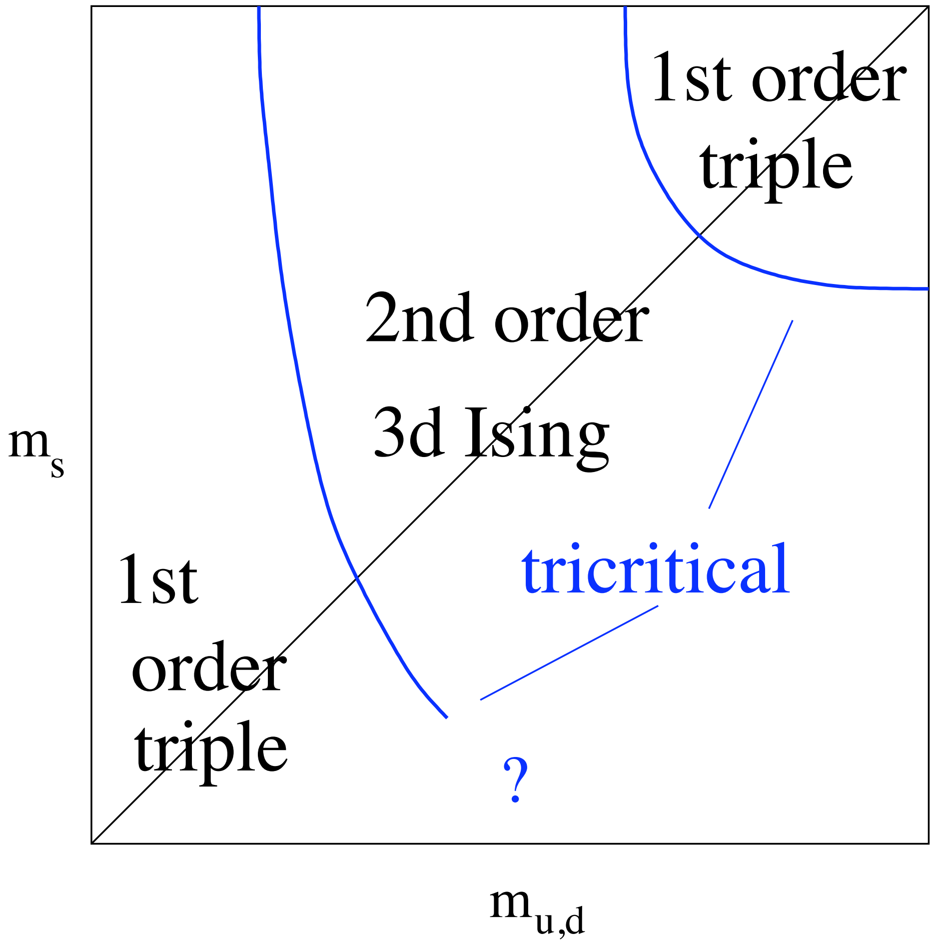


On infinite volume, this becomes a step function, smoothness due to finite L

Critical lines at imaginary μ



$$\mu = 0$$



$$\mu = i\frac{\pi T}{3}$$

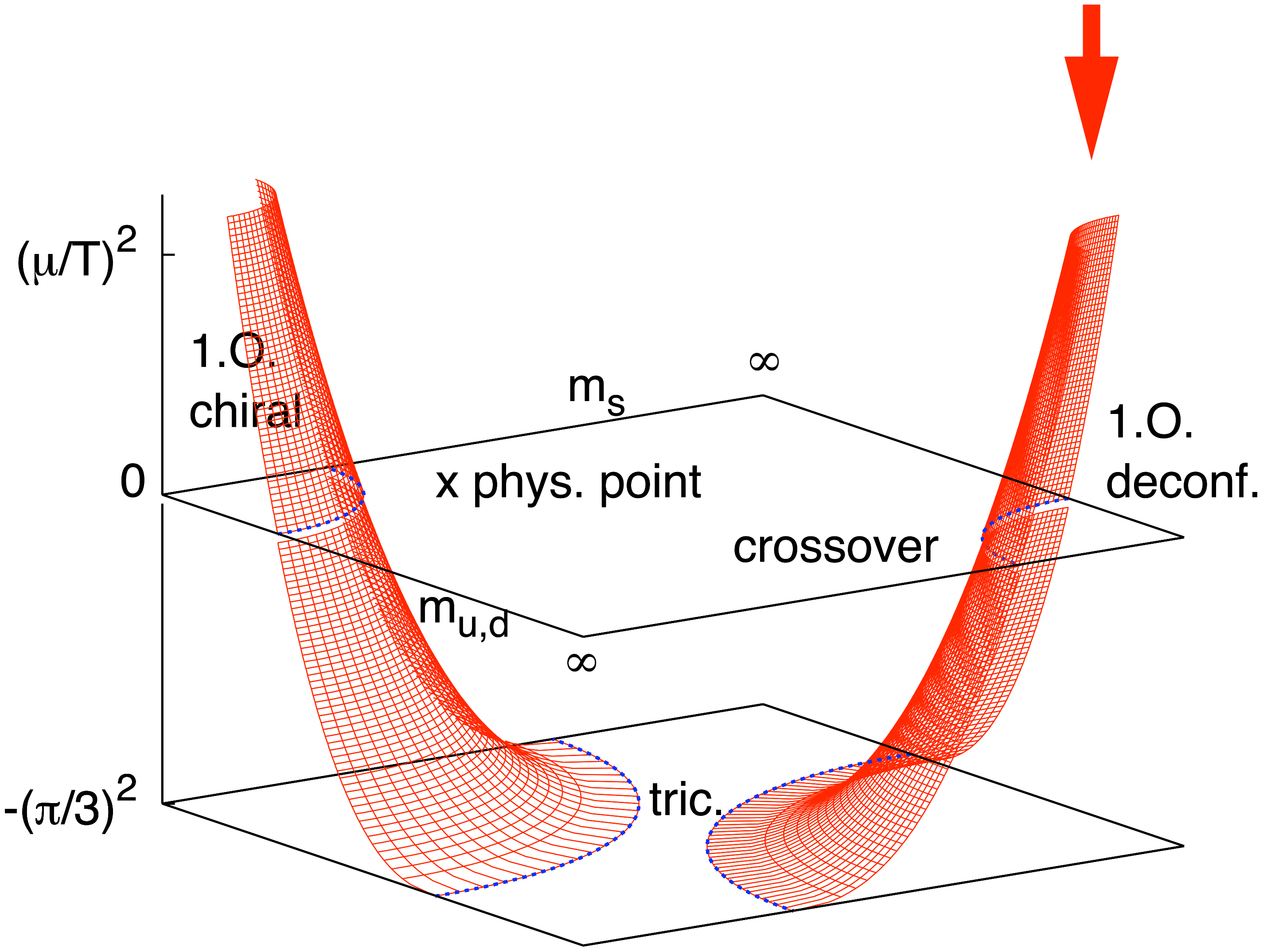
-Connection computable with standard Monte Carlo!

-Here: heavy quarks in eff. theory

Critical surfaces

de Forcrand, O.P. 10

shape, sign of curvature determined by tric. scaling!



Similar chiral crit. surface: tric. line renders curvature negative!

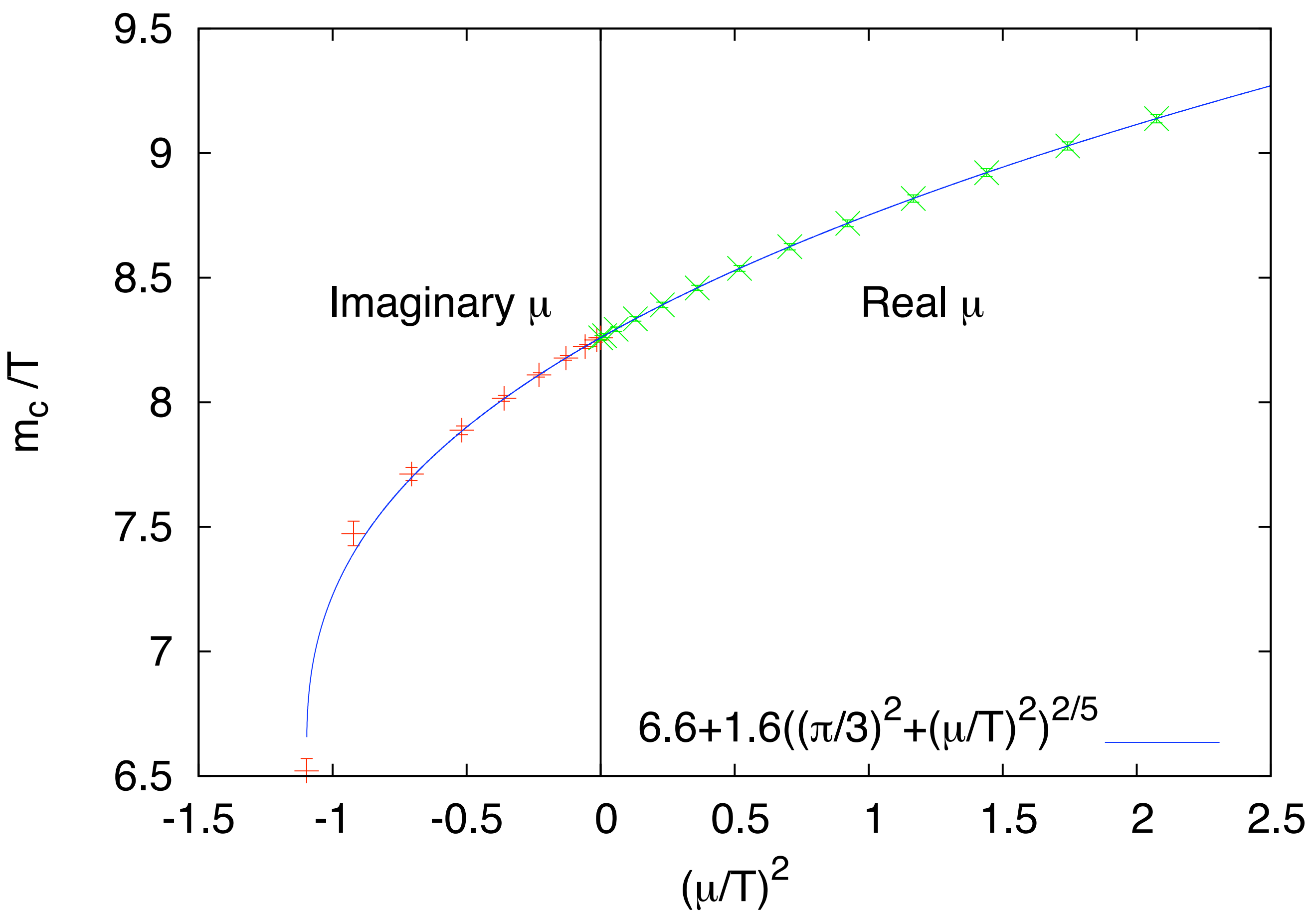
$m \to \infty$: QCD$\to$ theory of Polyakov lines $\longrightarrow$ universality class of 3d 3-state Potts model

(3d Ising, Z(2))

small μ/T: sign problem mild, doable for real μ!

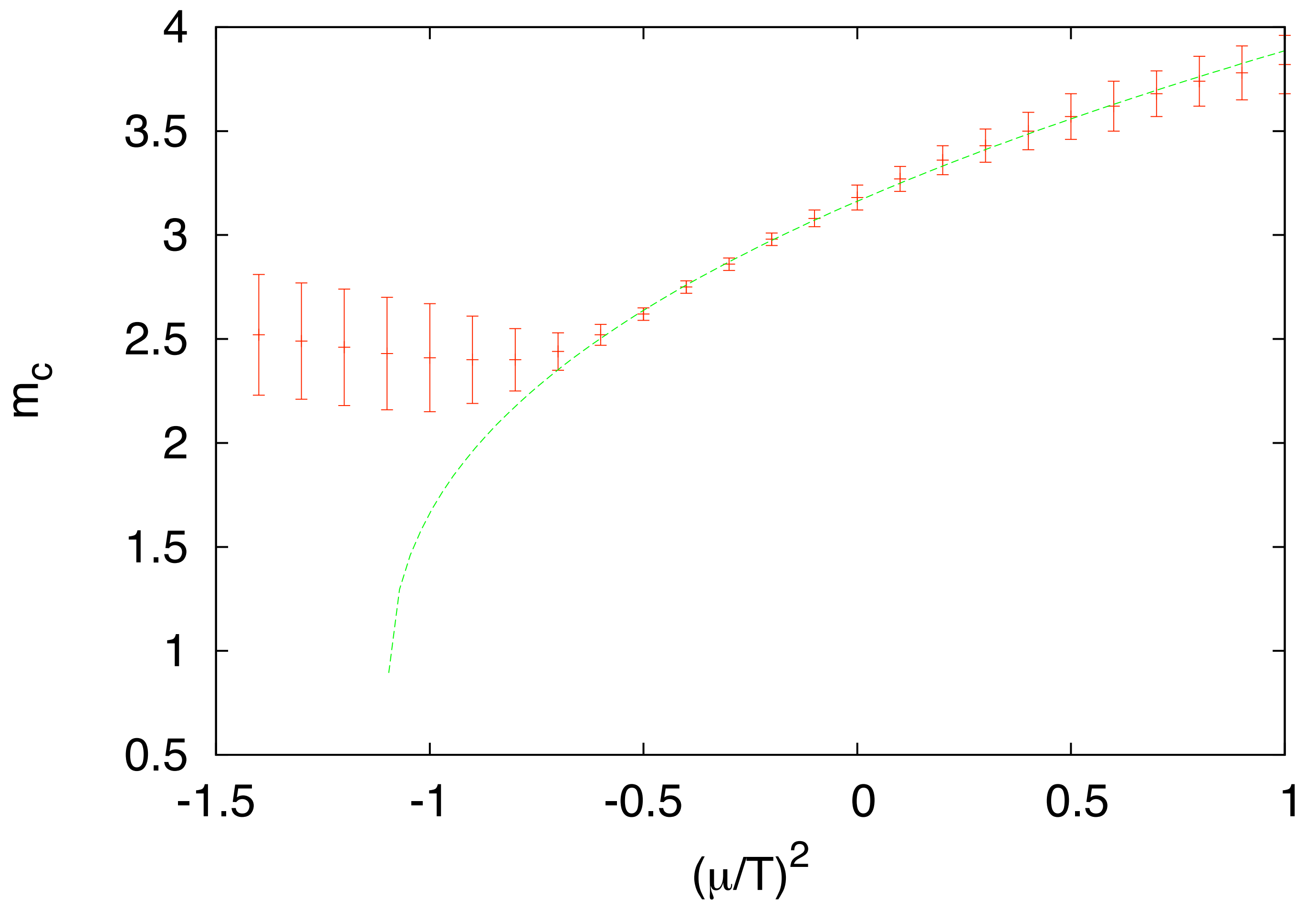
de Forcrand, Kim, Kratochvila, Takaishi

Potts:



QCD, Nt=1, strong coupling series:

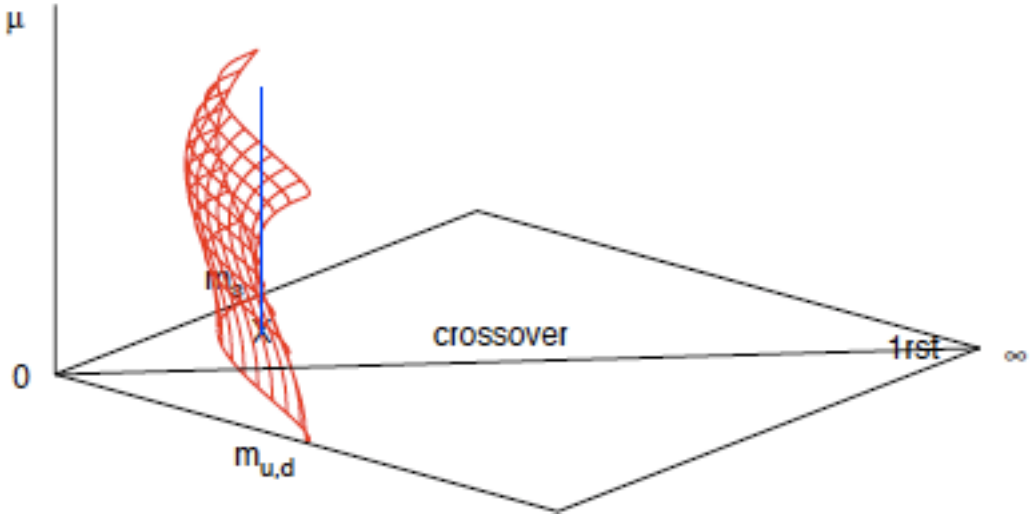
Langelage, O.P. 09



tri-critical scaling: exponent universal

Conclusions

- Reweighting, Taylor, canonical: indications for critical point on coarse lattices
- Chiral crit. surface, deconfinement crit. surface:
  Transitions weaken with chemical potential, decreasing lattice spacing
- No chiral critical point for $\mu/T \lesssim 1$
- Still possible: chiral critical point at large chemical potential
  non-chiral critical point(s)



?

cf. Ejiri 0812.1534 → $\left(\frac{\mu}{T}\right)^{\mathrm{CEP}} \sim 2.4$

The nature of the transition for phys. masses

Aoki et al. 06

...in the staggered approximation...in the continuum...is a crossover!