Aspects of HQET on the lattice

Rainer Sommer

Japan Wochen an Deutschen Hochschulen, May 2011, Bergische Universität, Wuppertal

Introduction: Particle Physics

- **Observations** ($e, \mu, \ldots Z, \ldots t$, Lorenz invariance ...)

  + Principles (Unitarity, Causality, Renormalizability)

  + theory calculations including lattice QCD (spectrum, F_π)

- **Standard Model of Particle Physics**

  local Quantum Field Theory (gauge theory)
  QED + Salam-Weinberg + QCD + GR

Introduction: the successfull Standard Model

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- very constrained: 3 coupling constants
- enormous predictivity

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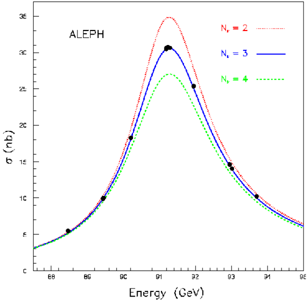

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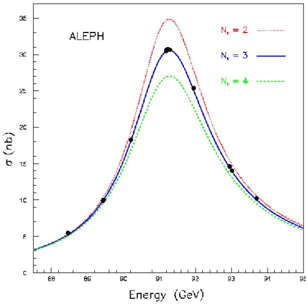

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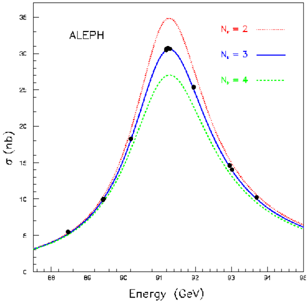
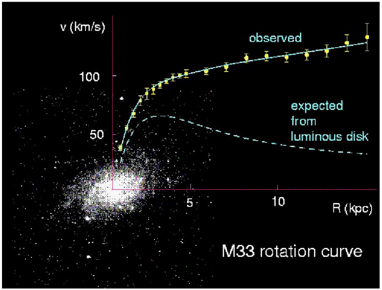


- too successfull (all particle physics experiments match)

Introduction: the incomplete Standard Model

But from other sources we know that there are missing pieces

- dark matter
- too little CP-violation for the observed matter / antimatter asymmetry

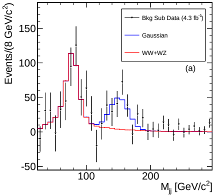


- There is an intense search for deviations from the Standard Model in particle physics experiments

Two Frontiers

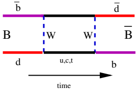
to search for missing pieces

- High Energy
  - Tevatron
  - LHC

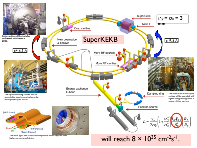


[CDF: arXiv:1104.0699]

- High Intensity
  - virtual (quantum) effects



  - less tested interactions



[Yutaka Ushiroda, May 2008]

High Intensity Frontier

Less tested interactions: quark-flavour changing interactions

$$\mathcal{L}_{\rm int} = \ldots g_{\rm weak} W_\mu^+ \bar{U} \gamma_\mu (1-\gamma_5) D' \ldots$$

- B-decays

$$D' = \underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{weak int.}} = V_{\rm CKM} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{strong int.}} = V_{\rm CKM}\, D$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

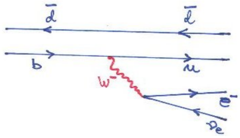
Confinement: V_{ij} are *not* directly measurable.
QCD matrix elements (or assumptions/approximations) are needed.

b to u transitions

- “clean” transitions: $B = b\bar{u} \rightarrow W \rightarrow l\nu$

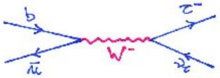
1. inclusive: $B \rightarrow X_u l\nu$
   optical theorem + heavy quark expansion
   $\rightarrow$ perturbatively calculable: (accuracy?)
   double expansion in $\alpha_s(m_{\rm b}) \approx 0.2$, $\Lambda_{QCD}/m_{\rm b} \approx 0.1$
2. semileptonic: $B \rightarrow \pi l\nu$

   (three-body, form factor)



3. leptonic: $B \rightarrow l\nu$

   (decay constant)



b to u transitions

- $V_{\rm ub}$ "puzzle"

b to u transitions

- V_{ub} "puzzle"

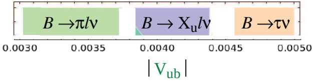
G. Isidori – *Quark flavour mixing with right-handed currents* — *Euroflavour2010, Munich*

Motivation

Exp. side: RH currents provide a natural solution to the "V_{ub} puzzle"

$$\mathrm{B}(B \to \pi\, l\nu) \propto |V_{ub}|^2$$
$$\mathrm{B}(B \to \tau\nu) \propto |V_{ub}|^2$$
$$\mathrm{B}(B \to X_u l\nu) \propto |V_{ub}|^2$$

Within SM →



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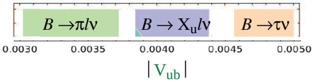
Lattice $\longrightarrow$ ME ME

Within SM $\longrightarrow$



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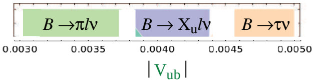

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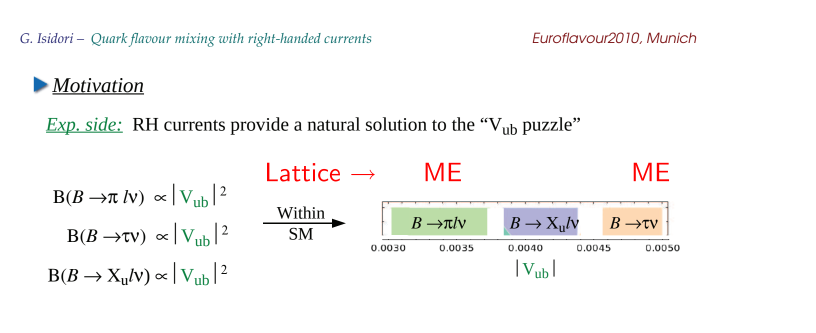

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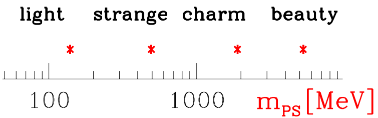
- **HQET on the lattice**

The challenge of B-physics on the lattice

multiple scale problem
always difficult
for a numerical treatment



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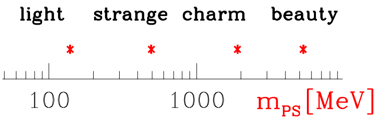

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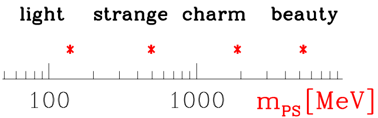

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$$L^{-1} \ll m_\pi, \ldots, m_{\rm D}, m_{\rm B} \ll a^{-1}$$

$$\mathrm{O}(\mathrm{e}^{-Lm_\pi}) \qquad\qquad m_{\rm D}\,a \lesssim 1/2$$

$$\downarrow \qquad\qquad \downarrow$$

$$L \gtrsim 4/m_\pi \sim 6\,\mathrm{fm} \qquad\qquad a \approx 0.05\,\mathrm{fm}$$

$$L/a \gtrsim 120$$

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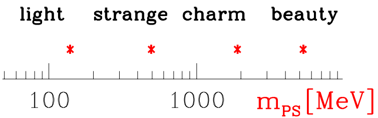

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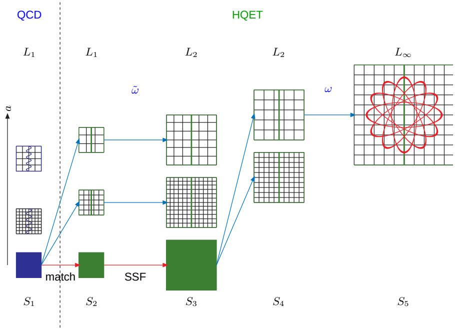
beauty not yet accomodated: effective theory, $\Lambda_{\rm QCD}/m_{\rm b}$ expansion

Non-perturbative Heavy Quark Effective Theory

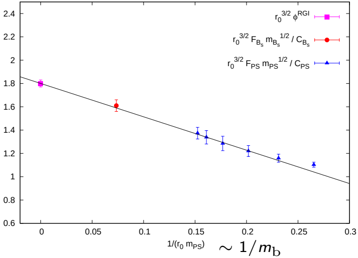
- Systematic expansion in $\Lambda/m_{\rm b} \approx 1/10$
- Non-perturbative implementation including 1st order corrections: NIC group
  [Heitger & S., 2003] . . .
  [B. Blossier, M. Della Morte, N. Garron, G. von Hippel, T. Mendes, H. Simma, R. S., 2010]



B physics

$B_s \to \mu^+\mu^-$ at LHCb: sensitive to SUSY contributions
NP matrix element: $F_{\mathrm{B_s}}$

First lattice computation of $1/m_{\mathrm{b}}$ correction in HQET [ALPHA Collaboration 2010]



quenched

Beyond the classical theory: Renormalization and Matching at leading order in $1/m$

a matrix element of A_0:

QCD	HQET in static approx.
$Z_{\rm A}\,\langle f\vert A_0(x)\vert i\rangle_{\rm QCD}$	$Z_{\rm A}^{\rm stat}(\mu)\langle f\vert A_0^{\rm stat}(x)\vert i\rangle_{\rm stat}$
$\Phi^{\rm QCD}(m)$	$\Phi(\mu)$

- m: mass of heavy quark (b) in some definition (all other masses zero for simplicity)
- μ: arbitrary renormalization scale
- matching (equivalence):

$$\Phi^{\rm QCD}(m) = \widetilde{C}_{\rm match}(m,\mu) \times \Phi(\mu) + {\rm O}(1/m)$$
$$\widetilde{C}_{\rm match}(m,\mu) = 1 + c_1(m/\mu)\bar{g}^2(\mu) + \ldots$$

Physical observables, such as $F_{\rm B_s}$, are independent of renormalization scheme, scale.
$\Rightarrow$ switch to Renormalization Group Invariants

Better: change to RGI's

see e.g. [R.S., arXiv:1008.0710]

$$\Phi_{\rm RGI} = \exp\left\{ -\int^{\bar g(\mu)} {\rm d}x\, \frac{\gamma(x)}{\beta(x)} \right\} \Phi(\mu)$$

β : beta-fct

$$\equiv \left[2b_0 \bar g(\mu)^2\right]^{-\gamma_0/2b_0} \exp\left\{ -\int_0^{\bar g(\mu)} {\rm d}x \left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x} \right] \right\} \Phi(\mu)$$

$$\Phi^{\rm QCD} = C_{\rm PS}(M/\Lambda) \times \Phi_{\rm RGI}$$

γ : AD in HQET

$$C_{\rm PS}(M/\Lambda) = \exp\left\{ \int^{g_\star(M/\Lambda)} {\rm d}x\, \frac{\gamma_{\rm match}(x)}{\beta(x)} \right\}$$

with

Λ : Lambda-para

$$\frac{\Lambda}{M} = \exp\left\{ -\int^{g_\star(M/\Lambda)} {\rm d}x\, \frac{1-\tau(x)}{\beta(x)} \right\}, \qquad \rightarrow \qquad g_\star(M/\Lambda)$$

M : RGI quark mass

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Matching and RGI's

$$\frac{M}{\Phi}\frac{\partial\Phi}{\partial M}\bigg|_{\Lambda} = \frac{M}{C_{\rm PS}}\frac{\partial C_{\rm PS}}{\partial M}\bigg|_{\Lambda} = \frac{\gamma_{\rm match}(g_\star)}{1-\tau(g_\star)}\,, \quad g_\star = g_\star(M/\Lambda)\,.$$

and with

$$\gamma_{\rm match}(g_\star) \overset{g_\star\to 0}{\sim} -\gamma_0 g_\star^2 - \gamma_1^{\rm match} g_\star^4 + \ldots\,, \qquad \beta(\bar g) \overset{\bar g\to 0}{\sim} -b_0\bar g^3 + \ldots$$

we can give the leading large mass behaviour

$$C_{\rm PS} \quad \overset{M\to\infty}{\sim} \quad (2b_0 g_\star^2)^{-\gamma_0/2b_0} \sim [\log(M/\Lambda)]^{\gamma_0/2b_0}$$

The present knowledge

For $\gamma_{\rm match}$ at l loops need
$\gamma_{\overline{\rm MS}} = \gamma$: l loops;
$C_{\rm match}(g_\star)$: $l-1$ loops

γ_0 [Shifmann& Voloshin; Politzer& Wise]

...

$\gamma_{\overline{\rm MS},2}$ [Chetyrkin & Grozin, 2003]

$C_{\rm match}(g_\star)$ to 3 loops [Bekavac, S. et al, 2009]

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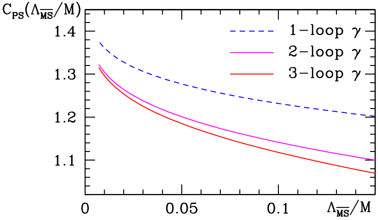
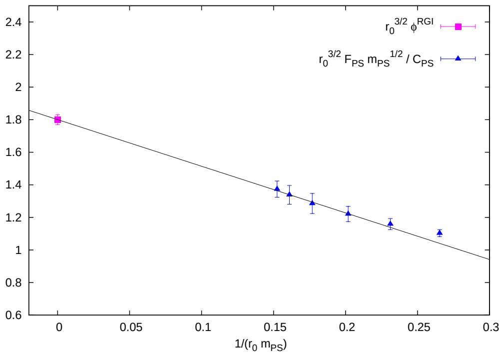
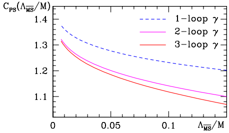


$C_{\rm match}(g_\star)$ to 3 loops [Bekavac, S. et al, 2009]

An application





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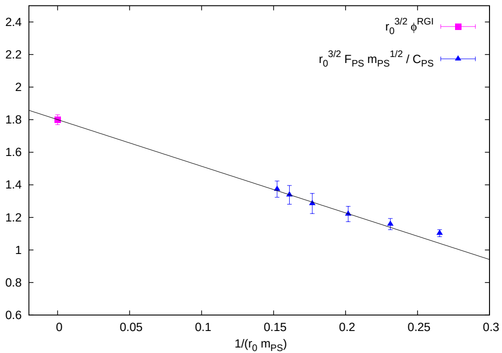
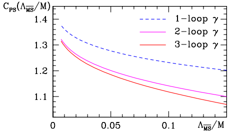




this looks good; one may interpolate to the physical point ... but

What is the accuracy of perturbation theory?

$C_{\rm match}(g_\star)$ to 3 loops [Bekavac, S. et al, 2009] also for various bilinears

$\gamma_{\rm match}$

$$\mathcal{O}_\Gamma = \overline{\psi}_{\rm l}(x)\Gamma\psi_{\rm h}(x)$$

Γ	notation
$\gamma_0\gamma_5$	A_0
γ_5	P
γ_k	V_k
γ_{kl}	T

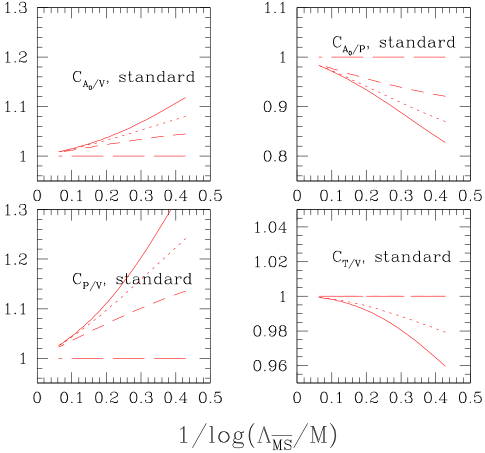
$$\Phi_\Gamma^{\rm QCD} = C_{\rm match}^\Gamma(g_\star) \times \Phi(\mu) = C_{\rm match}(g_\star) \exp\left\{\int^{g_\star} {\rm d}x \frac{\gamma(x)}{\beta(x)}\right\} \Phi_{\rm RGI}^\Gamma$$

$$\equiv \exp\left\{\int^{g_\star} {\rm d}x \frac{\gamma_{\rm match}^\Gamma(x)}{\beta(x)}\right\} \Phi_{\rm RGI}$$

$\gamma_{\rm match}^\Gamma$: 3-loops $\qquad \gamma_{\rm match}^\Gamma - \gamma_{\rm match}^{\Gamma'}$: 4-loops

[chiral symmetry of light quarks ($N_{\rm light} > 1$): $\gamma_{\rm match}^{\Gamma\gamma_5} = \gamma_{\rm match}^\Gamma$]

Compare different orders



We actually show $C_{\Gamma/\Gamma'} = C^{\Gamma}_{\rm match}(m,\mu)/C^{\Gamma'}_{\rm match}(m,\mu)$

B-physics: $\Lambda_{\overline{\rm MS}}/M_{\rm b} \approx 0.04$
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm b}) \approx 0.3$

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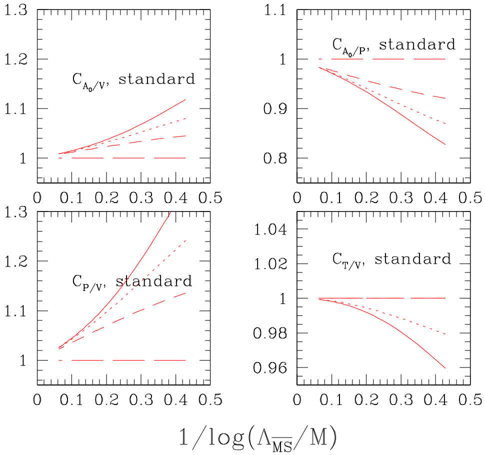

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Perturbation theory is badly behaved
for charm quarks very badly
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm c}) \approx 0.5$

Different orders of PT

the normal behavior for one-scale quantities is

$$\mathcal{O} = o_0 + o_1\,\alpha + o_2\,\alpha^2 + \ldots \qquad \alpha = \bar{g}^2/(4\pi)$$
$$|o_i| \lesssim 1$$

(o_0 suitably normalized)

examples: $\mu \frac{\partial \bar{g}}{\partial \mu} = -\bar{g}\left\{ b_0\alpha + b_1\alpha^2 + \ldots \right\}$

$$\frac{\mu}{\bar{m}}\frac{\partial \bar{m}}{\partial \mu} = -d_0\,\alpha - d_1\alpha^2 + \ldots$$
$$\frac{\mu}{\Phi}\frac{\partial \Phi}{\partial \mu} = \gamma = -\gamma_0\alpha - \gamma_1\alpha^2 + \ldots$$

($N_\mathrm{f} = 3$)

$\overline{\mathrm{MS}}$	b_i	0.71620	0.40529	0.32445	0.47367
	d_i	0.63662	0.76835	0.80114	0.90881
	γ_i	-0.31831	-0.26613	-0.25917	

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but mass-dependence (matching anomalous dimensions):

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$A_0,\ \gamma_i$	-0.31831	-0.57010	-0.94645	
$V_0,\ \gamma_i$	-0.31831	-0.87406	-3.12585	
$\ldots$				
$A_0/V,\ \gamma_i$	0	0.30396	2.17939	14.803

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Perturbation theory is ill behaved (applicable at very small α)

Changing the scale

we had

$$\Phi^{\rm QCD}(m) \quad = \quad \widetilde{C}_{\rm match}(m,\mu) \times \Phi(\mu)$$

- Chose a "convenient" scale: $\mu = m_\star = \overline{m}(m_\star)\,,\; g_\star = \bar{g}(m_\star)$
- may set more generally

$$\mu = s^{-1}\, m_\star = \overline{m}(m_\star)\,,\quad g_\star = \bar{g}(m_\star)$$

- note: in the effective theory

  – one does NOT INTEGRATE OUT DOFs ABOVE $\mu = m_\star$

  – one matches the physics BELOW

  $\longrightarrow$ expect $s > 1$ is better

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  $\rightarrow$ expect $s > 1$ is better
- the result is simply ($\hat{g} = \bar{g}(s^{-1} m_\star)$))

$$\begin{aligned}
\gamma_{\rm match}(g_\star) &= \hat{\gamma}_{\rm match}(\hat{g}) = -\hat{\gamma}_0 \alpha - \hat{\gamma}_1 \alpha^2 + \ldots \\
&\quad \hat{\gamma}_0 = \gamma_0\,,\; \hat{\gamma}_1 = \gamma_1 + 2 b_0 \gamma_0 \ldots \\
C_{\rm PS}(M/\Lambda) &= \exp\left\{ \int^{\hat{g}} {\rm d}x \frac{\hat{\gamma}_{\rm match}(x)}{\beta(x)} \right\}.
\end{aligned}$$

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					s
$A_0,\ \hat\gamma_i$	-0.31831	-0.57010	-0.94645		1
	-0.31831	0	0.39720		3.4916
$V_0,\ \hat\gamma_i$	-0.31831	-0.87406	-3.12585		1
	-0.31831	0	-0.231121		6.8007
...					
$A_0/V,\ \hat\gamma_i$	0	0.30396	2.17939	14.803	1
	0	0.30396	0.972221	4.733	4
	0	0.30396	-0.05414	1.82678	13
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[Very similar for $C_{\rm match}(m_{\rm Q},\mu)$ with $\mu = s^{-1}m_{\rm Q}$ expanded in $\alpha(\mu)$]

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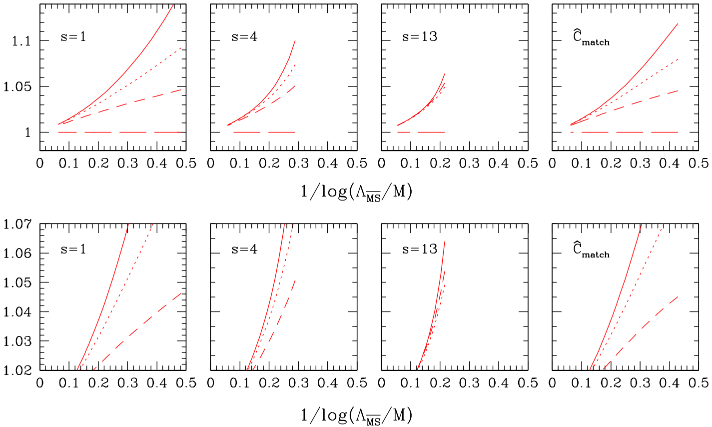
The behavior can be improved significantly but $s \gtrsim 4$ is required

$\alpha(m_{\rm b}/4)$ is not small!

[Very similar for $C_{\rm match}(m_{\rm Q},\mu)$ with $\mu = s^{-1}m_{\rm Q}$ expanded in $\alpha(\mu)$]

Conversion functions with and without scale optimization



The ratio $C_{\rm PS}/C_{\rm V}$, evaluated in the first column as described here. In columns two and three the expansion in $g_\star$ is generalized to an expansion in $\bar{g}(m_\star/s)$. The last column contains the conventionally used $\hat{C}^{\rm PS}_{\rm match}(m_{\rm Q},m_{\rm Q},m_{\rm Q})/\hat{C}^{\rm V}_{\rm match}(m_{\rm Q},m_{\rm Q},m_{\rm Q})$. For B-physics we have $\Lambda_{\overline{\rm MS}}/M_{\rm b}\approx 0.04$ and $-1/\ln(\Lambda_{\overline{\rm MS}}/M_{\rm b})\approx 0.3$. The loop order changes from one-loop (long-dashes) up to 4-loop (full line) anomalous dimension.

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- other ideas?
- in any case perturbative matching is only theoretically consistent at leading order in $1/m_{\rm b}$

$$\alpha^k(m) \sim \left[\frac{1}{2b_0 \log(m/\Lambda_{\rm QCD})}\right]^k \overset{m \gg \Lambda_{\rm QCD}}{\gg} 1/m$$

With $N_f = 2$ dynamical quarks and NP matching

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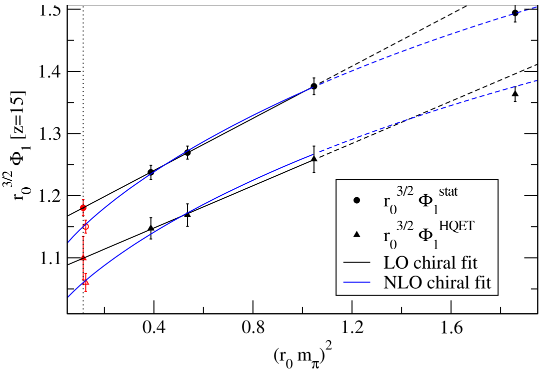


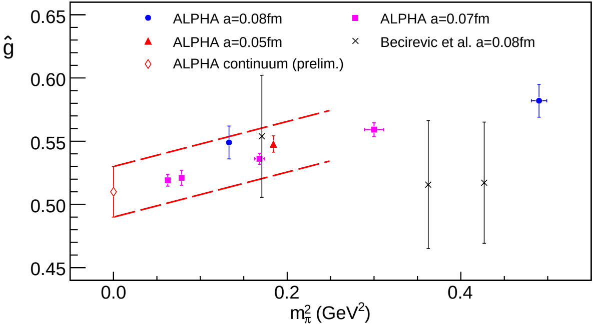
$64 \times 32^3 \ldots 96 \times 48^3$ lattices simulated on JUROPA, JUGENE

$$F_B = F_B|_{m_\pi^2=0} \times \left(1 - \frac{3}{4}\frac{1+3g_{B^*B\pi}^2}{16\pi^2 F_\pi^2} m_\pi^2 \log(m_\pi^2/F_\pi^2) + b\, m_\pi^2\right)$$

Low energy coupling $g_{B^*B\pi}$ is needed (for static quarks).

Low energy coupling $\hat{g} = g_{B^* B \pi}$ with unprecedented precision

$$\hat{g} = \text{const.} \times \langle B_k^{*-} | \hat{A}_k(0) | B^0 \rangle \,, \quad A_k = \bar{d}\gamma_k\gamma_5 u$$

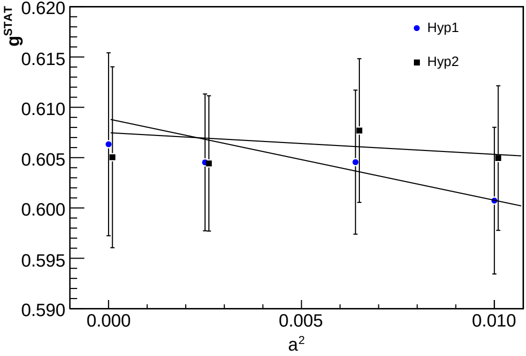


precision due to improved techniques

Low energy coupling $g_{B^* B \pi}$ with unprecedented precision

[Bulava, Donnellan, S.]

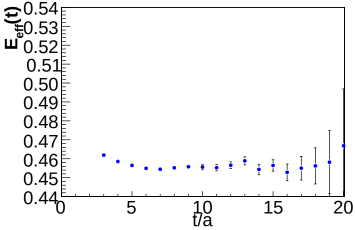
quenched



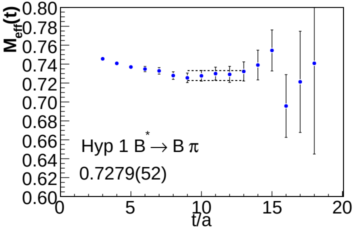
Determination of $\hat{g}$: plateaux

$N = 3$ basis of different size Gaussian wavefunctions

Energy level



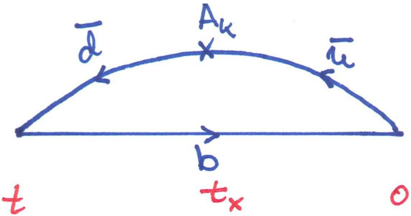
Matrix element $\hat{g}$



$$\hat{g}_{\rm eff}(t) = \hat{g} + {\rm O}(t\Delta_{N+1,1}\, {\rm e}^{-t\Delta_{N+1,1}}) \qquad \Delta_{N+1,1} \approx 6/{\rm fm}$$

Determination of $\hat{g}$: standard ratio

$$\hat{g}_{\rm eff}(t, t_x) \equiv \frac{C_3(t, t_x)}{C_2(t)}$$



$$C_3(t, t_x) =$$

$$C_3(t, t_x) = \langle 0|\hat{B}_k^* \sum_n |n\rangle {\rm e}^{-(t-t_x)E_n} \langle n|\, \hat{A}_k(0) \sum_m |m\rangle {\rm e}^{-t_x E_m} \langle m|\hat{B}^\dagger\, |0\rangle$$

$$= {\rm const.} \times {\rm e}^{-t m_B} \langle B_k^{*-}|\hat{A}_k(0)|B^0\rangle \times (1 + {\rm O}({\rm e}^{-(t-t_x)\Delta_{2,1}}, {\rm e}^{-t_x\Delta_{2,1}}))$$

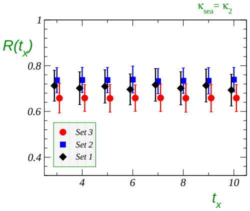
and

$$\hat{g}_{\rm eff}(t, t_x) = \hat{g} + {\rm O}({\rm e}^{-(t-t_x)\Delta_{2,1}}, {\rm e}^{-t_x\Delta_{2,1}})$$

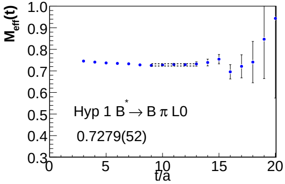
$$\Delta_{2,1} = E_{2,B} - E_{1,B} = E_{2,B} - m_B \approx 2/{\rm fm}$$

Determination of $\hat{g}$, comparison

previous[Becirevic et al, 2009]



New matrix element $\hat{g}$



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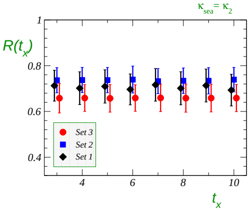

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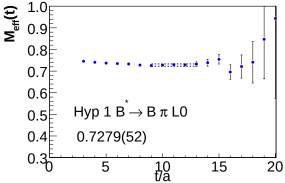
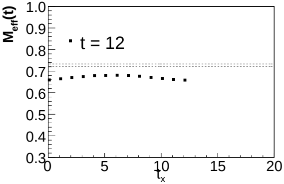


old method with improved statistical accuracy (all-to-all)



Improved techniques: the GEVP

matrix of correlation functions on an infinite time lattice

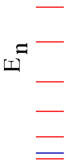
$$C_{ij}(t) \;=\; \langle O_i(0) O_j(t) \rangle = \sum_{n=1}^{\infty} \mathrm{e}^{-E_n t} \psi_{ni} \psi_{nj} \,, \quad i,j = 1,\ldots,N$$

$$\psi_{ni} \equiv (\psi_n)_i = \langle n | \hat{O}_i | 0 \rangle = \psi_{ni}^* \quad E_n \le E_{n+1}$$

the GEVP is

$$C(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0)\,, \quad n = 1,\ldots,N \quad t > t_0,$$

Lüscher & Wolff [1990] showed that



$$E_n^{\mathrm{eff}} \;=\; \frac{1}{a} \log \frac{\lambda_n(t,t_0)}{\lambda_n(t+a,t_0)} = E_n + \varepsilon_n(t,t_0)$$

$$\varepsilon_n(t,t_0) \;=\; \mathrm{O}(\mathrm{e}^{-\Delta E_n (t-t_0)})\,, \Delta E_n = |\min_{m \neq n} E_m - E_n|\,.$$

The GEVP method

[1990] correction term

$$\varepsilon_n(t,t_0) \;=\; \mathrm{O}(\mathrm{e}^{-\Delta E_n\,(t-t_0)})\,,\; \Delta E_n = |\min_{m\neq n} E_m - E_n|\,.$$

[$\sim$ 2000] F.Niedermayer, P.Weisz: private notes on GEVP, including perturbation theory in $n > N$ levels

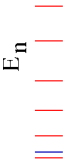
[2009] we could prove that [B. Blossier, M. Della Morte, G. von Hippel, T. Mendes, R.S.]

$$\varepsilon_n(t,t_0) = \mathrm{e}^{-\Delta_{N+1,n}\,t} \text{ if } t_0 \geq t/2\,, \qquad \Delta_{N+1,n} = E_{N+1} - E_n$$

[2009] similar formula for a decay constant: **excited states as well**

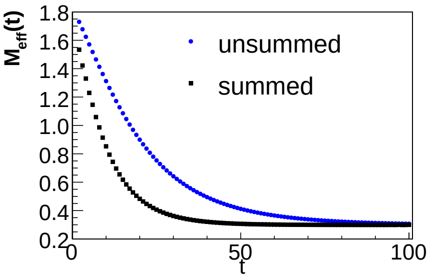
[2011] and now also a formula [J. Bulava, M. Donnellan, R.S.]
($t_0 \geq t/2$)

$$\langle f|\mathcal{O}|i\rangle = \mathcal{M}_{\rm eff}(t,t_0) + \mathrm{O}(t\Delta_{N+1,n}\mathrm{e}^{-t\,\Delta_{N+1,n}})$$



Demonstration in a toy model

- 5 states
- matrix elements between $1/5$ and 1
- matrix element $\langle 3|h_w|3\rangle$



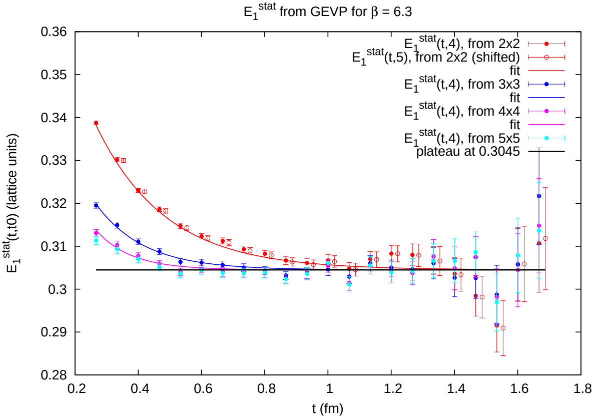
Demonstration in HQET: energies

$a = 0.07\,\mathrm{fm}$

$aE_1^{\mathrm{eff,stat}}(t, t_0)$

curve:

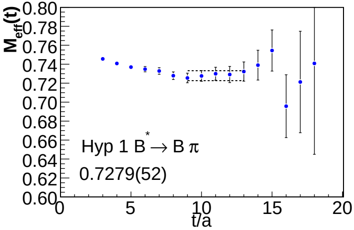
$E_1 + \alpha_N\, \mathrm{e}^{-\Delta_{N+1,1}\, t}$



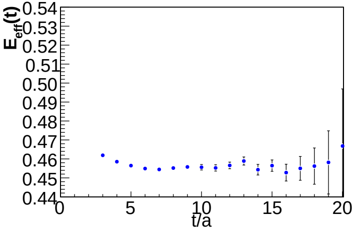
$\Delta_{N+1,1}$ agree with plateaux of $E_{N+1}^{\mathrm{eff,stat}}(t, t_0) - E_1^{\mathrm{eff,stat}}(t, t_0)$ for large N' and t.

Demonstration in HQET: $\hat{g}$

GEVP



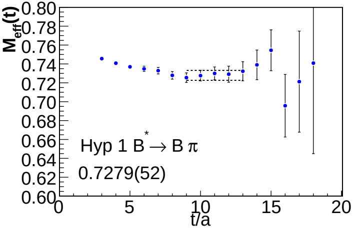
Energies

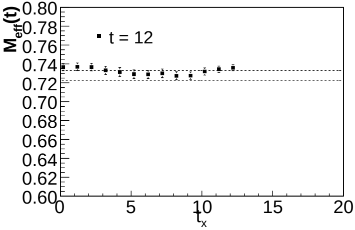


Demonstration in HQET: $\hat{g}$

GEVP

best wavefunction (one may be lucky)





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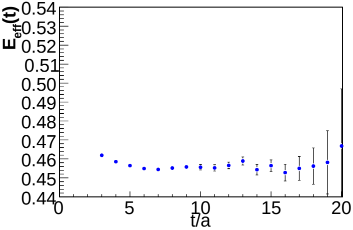

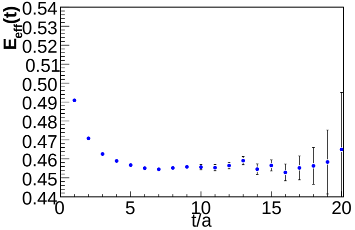

f_b with $N_\mathrm{f} = 2$ dynamical quarks

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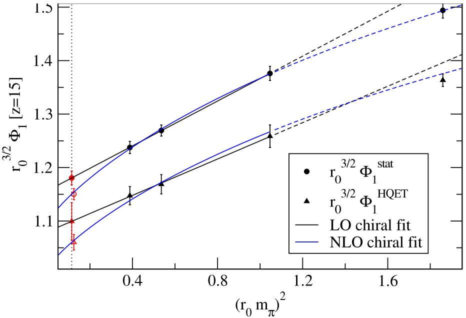

$64 \times 32^3 \ldots 96 \times 48^3$ lattices simulated on JUROPA, JUGENE

$$F_\mathrm{B} = F_\mathrm{B}|_{m_\pi^2=0} \times \left(1 - \frac{3}{4}\frac{1+3g_{B^*\,B\,\pi}^2}{16\pi^2 F_\pi^2} m_\pi^2 \log(m_\pi^2/F_\pi^2) + b\, m_\pi^2\right)$$

To be done:

- Chiral extrapolation with known $g_{B^*\,B\,\pi}$
- continuum extrapolation (from $a = 0.08\mathrm{fm} \ldots 0.045\mathrm{fm}$)

Summary

- B-deacys are an important piece in
  the validation of the SM of particle physics
  the search for new physics
- On the lattice an effective theory is needed
- NP HQET is well on its way for $N_\mathrm{f} > 0$
- Precision chiral extrapolations require a determination of $\hat{g}$
- High precision determination of $\hat{g}$ is done using new methods which are applicable more generally
- With our preliminary number for $B \to \tau\nu$ (f_B), the V_{ub} puzzle remains.
  A precise number will come soon ($N_\mathrm{f} = 2$).
- $B_s \to \mu^+\mu^-$ immediately after (f_{B_s}, LHCb).
- $B \to \pi l \nu$ is the next step.