The shape of the static potential with dynamical fermions

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Definitions

Continuum

- $V_n(r)$: energy levels of a static quark and anti-quark pair at distance r
  $V_0 \equiv V$ ground state, V_1 first excitation . . .
- static force $F(r) = V'(r)$ is a renormalized quantity
- scale r_0 [Sommer, 1993]

$$r^2\, F(r)\Big|_{r=r_0} \quad = \quad 1.65$$

  $r_0 \approx (0.45 \ldots 0.5)\,\mathrm{fm}$ through phenomenological potential models
- slope

$$c(r) \quad = \quad \frac{1}{2}\, r^3\, F'(r)$$

  defines a running coupling

$$\bar{g}_{\mathrm{c}}^2(\mu) \quad = \quad -\frac{4\pi}{C_{\mathrm{F}}} c(r)\,, \;\; \mu = 1/r\,, \;\; C_{\mathrm{F}} = 4/3$$

Definitions

Lattice

- energy levels $V_n(r)$ can be extracted from expectation values of Wilson loops $W(r,T)$, defined as traces of product of gauge links along rectangular paths $r \times T$ (here only on-axis)
- improved definition of the force [Sommer, 1993]

$$F(r_{\mathrm{I}}) \quad = \quad [V(r) - V(r-a)]/a$$

  $r_{\mathrm{I}} = r - a/2 + \mathcal{O}(a^2)$ is chosen such that at tree level $F_{\mathrm{tree}}(r_{\mathrm{I}}) = C_{\mathrm{F}}\, g_0^2/(4\pi r_{\mathrm{I}}^2)$
- improved definition of the slope $c(r)$ [Lüscher and Weisz, 2002]

$$c(\tilde{r}) \quad = \quad \frac{1}{2}\tilde{r}^3[V(r+a) + V(r-a) - 2V(r)]/a^2$$

  $\tilde{r} = r + \mathcal{O}(a^2)$ is chosen such that at tree level $c_{\mathrm{tree}}(\tilde{r}) = -C_{\mathrm{F}}\, g_0^2/(4\pi)$

Definitions

Lattice - continued

- due to confinement

$$\langle W(r,T)\rangle \text{``} = \langle \pm \rangle \text{``} \quad \approx \quad \exp(-\sigma r T)$$

  but

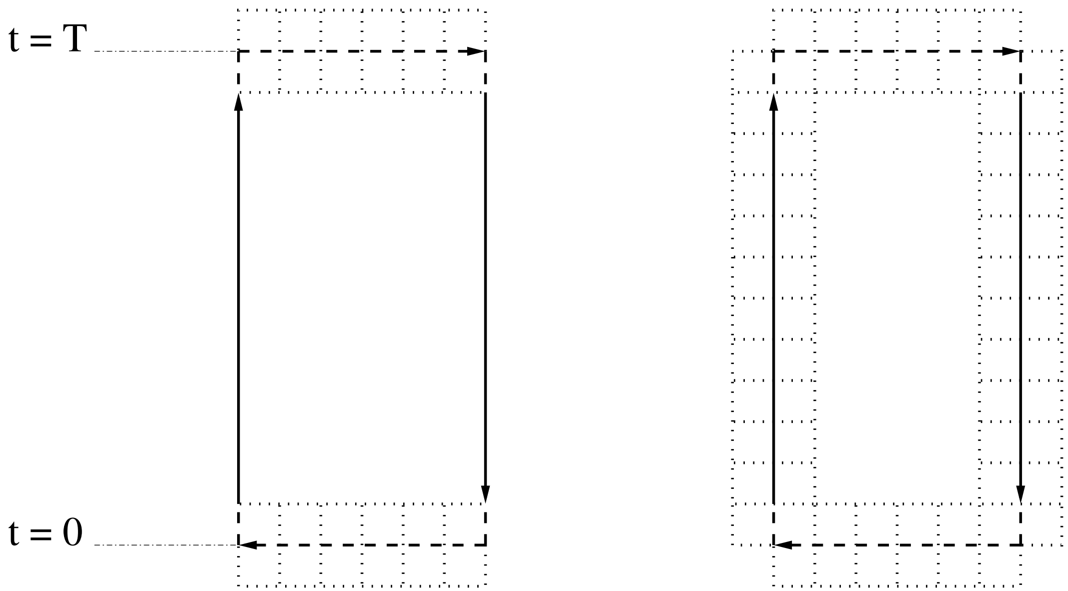
$$\left\langle W(r,T)^2 \right\rangle \text{``} = \langle + \rangle \text{``} \quad \approx \quad \text{const}$$

  $\Rightarrow$ noise-to-signal ratio grows exponentially with the area of the loop

- in pure gauge theory there is a cure: exponential suppression of the statistical noise through multi-hit method [Parisi, Petronzio and Rapuano, 1983] and more efficient multilevel algorithm [Lüscher and Weisz, 2001]. But these methods are not applicable . . .
- with dynamical fermions. Here we use the method of HYP smearing the links in the Wilson loops [A. Hasenfratz and FK, 2001; A. Hasenfratz, R. Hoffmann and FK, 2002; ALPHA, Della Morte et al., 2004]

Techniques

[Donnellan, FK, Leder and Sommer, 2011]



- (left figure) HYP smearing of space-like links corresponds (in the Hamiltonian formalism) to an operator $\hat{O}^\dagger$ that creates a $|Q\overline{Q}(r)\rangle$ state
- (right figure) HYP smearing of the time-like links corresponds to the choice of a static quark action (and a modification of the operator $\hat{O}$)

Techniques

- choice of the HYP-smearing parameters

$$\text{HYP2:} \qquad \alpha_1 = 1.0\,, \quad \alpha_2 = 1.0\,, \quad \alpha_3 = 0.5$$

- using the transfer matrix on a lattice with $N_t \longrightarrow \infty$ time-slices

$$\langle W(r,T)\rangle \quad \sim \quad \sum_n c_n c_n^* \mathrm{e}^{-V_n(r)(T-2a)}$$

  c_n depends on $\hat{O}$ and $V_n(r)$ on the static quark action

- binding energy of a meson made of a static and a light dynamical quark

$$E_{\mathrm{stat}} \quad \sim \quad \frac{1}{a} e^{(1)} g_0^2 + \ldots$$

  HYP2 smearing minimizes $e^{(1)}$ [Della Morte, Shindler and Sommer, 2005]

- with $\mathcal{O}(a)$ improved dynamical fermions (cf. [Necco and Sommer, 2002])

$$V_n^{\mathrm{HYP2}}(r) - 2E_{\mathrm{stat}}^{\mathrm{HYP2}} = V_n^{\mathrm{continuum}}(r) - 2E_{\mathrm{stat}}^{\mathrm{continuum}} + \mathcal{O}(a^2)$$

Techniques

- variational basis: space-like links ($\leftrightarrow$ operator $\hat{O}$) are smeared using n_l iterations of *spatial* HYP smearing

$$\langle W(r,t)\rangle \quad \longrightarrow \quad C_{lm}(r,T)$$

we take a basis with $M=3$ levels ($n_{2,3,5}=8,12,20$ at $\beta=5.3$)

- generalized eigenvalue method to extract V_n

$$\begin{aligned} C(t)\,\psi_\alpha &= \lambda_\alpha(t,t_0)\,C(t_0)\,\psi_\alpha\,, \qquad \alpha=0,1,\ldots,M-1 \\ E_\alpha(t+\frac{a}{2},t_0) &= \ln\left(\lambda_\alpha(t,t_0)/\lambda_\alpha(t+a,t_0)\right) \end{aligned}$$

if $t_0+a\le t\le 2t_0$ [Blossier et al., 2009]

$$E_\alpha(t+\frac{a}{2},t_0) \quad = \quad E_\alpha+\beta_\alpha \mathrm{e}^{-(E_M-E_\alpha)(t+\frac{a}{2})}$$

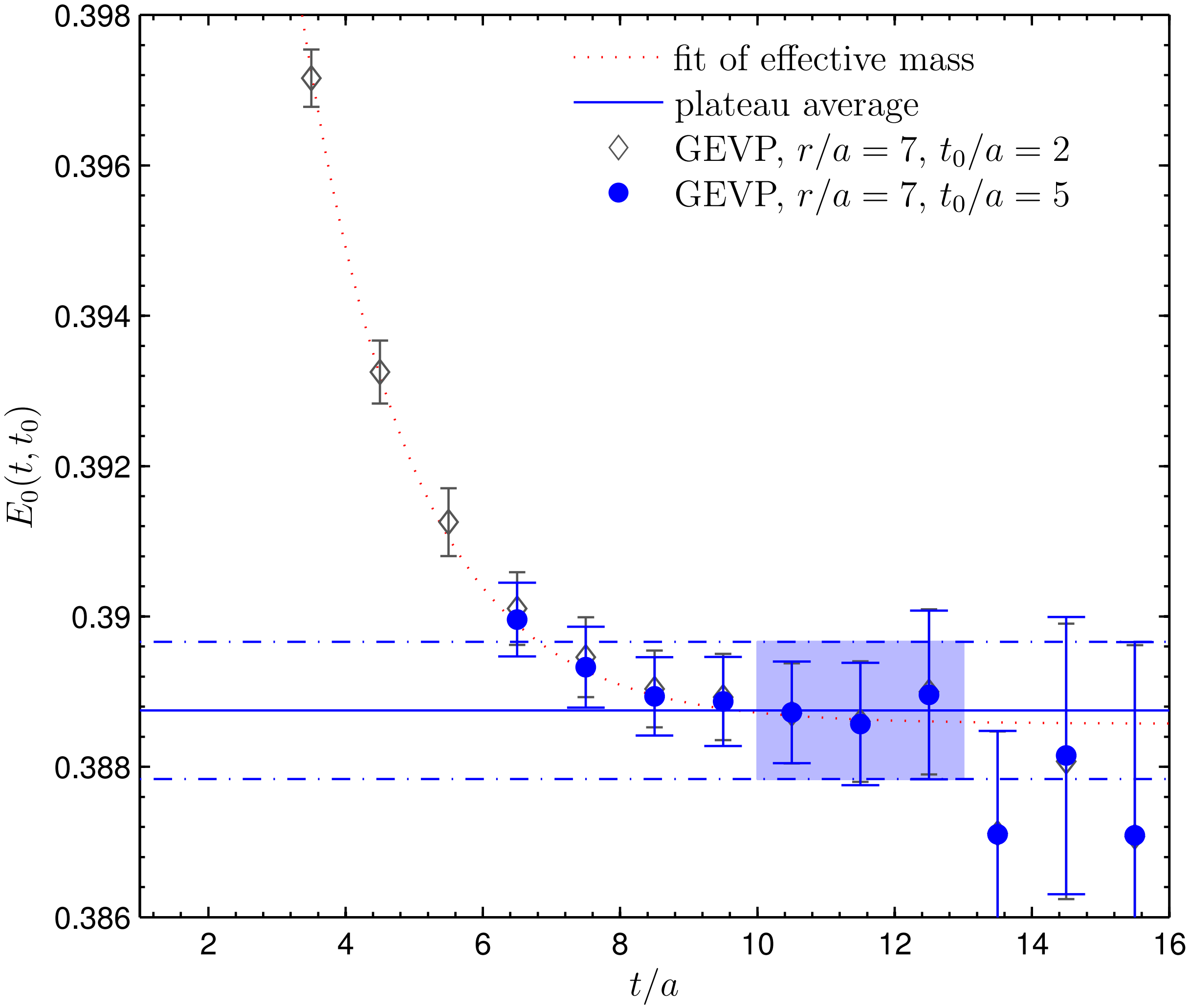
we fit this expression to determine the systematic error and where to start the plateau average ($t=2t_0+\frac{a}{2}$)

Potential

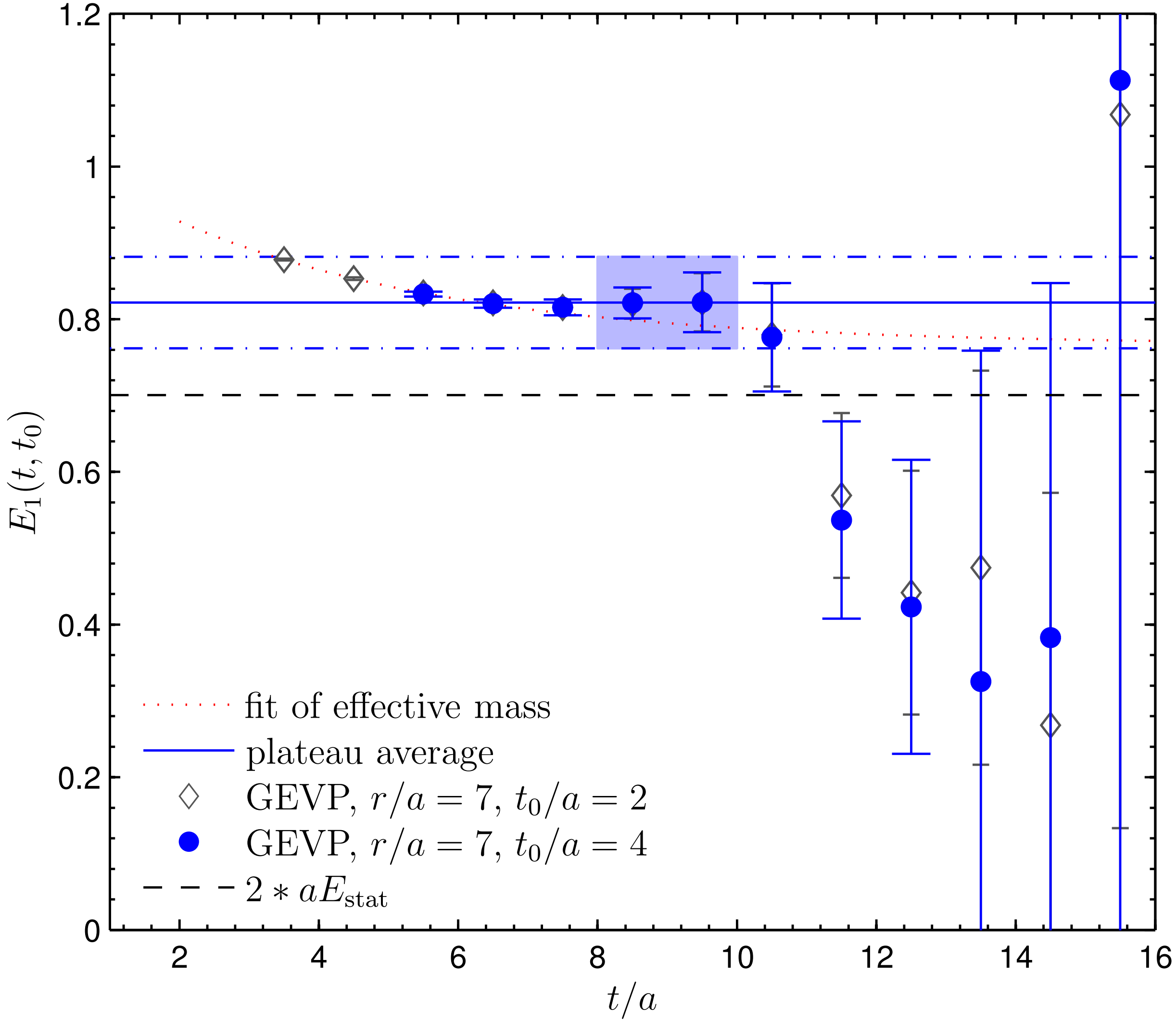
CLS ensemble E5g (`https://twiki.cern.ch/twiki/bin/view/CLS/WebHome`)

- Wilson gauge action and $N_\mathrm{f} = 2$ flavors of $\mathcal{O}(a)$ improved Wilson quarks with periodic boundary conditions for all fields apart from anti-periodic boundary conditions for the fermions in time
- $\beta = 5.3$, $\kappa = 0.13625$, 64×32^3
- $r_0/a(\mathrm{E5g}) = 6.75(6)$, $r_0(m_q = 0)m_\mathrm{PS} = 1.02$ using chiral extrapolation $r_0/a(m_q = 0) = 7.05$ from [Leder and FK, 2010]
- deflation accelerated DD-HMC algorithm [Lüscher 2005; 2007] with trajectory length $\tau = 4$
- 1000 Wilson loops measurements separated by approximately 6 molecular dynamics units

Potential

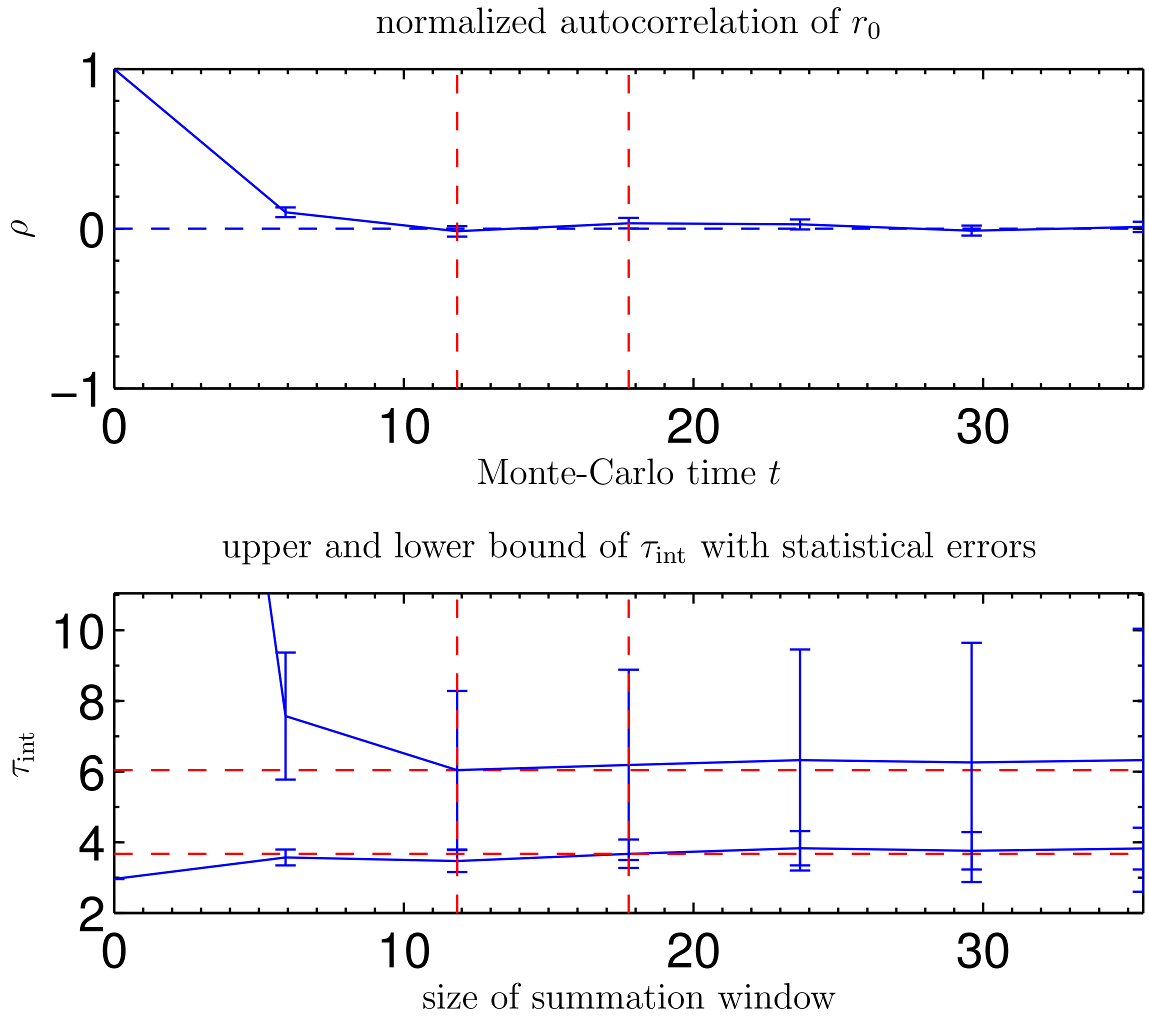

fit of effective mass
plateau average
GEVP, $r/a = 7$, $t_0/a = 2$
GEVP, $r/a = 7$, $t_0/a = 5$
$E_0(t, t_0)$
t/a
0.398
0.396
0.394
0.392
0.39
0.388
0.386
2
4
6
8
10
12
14
16

Potential



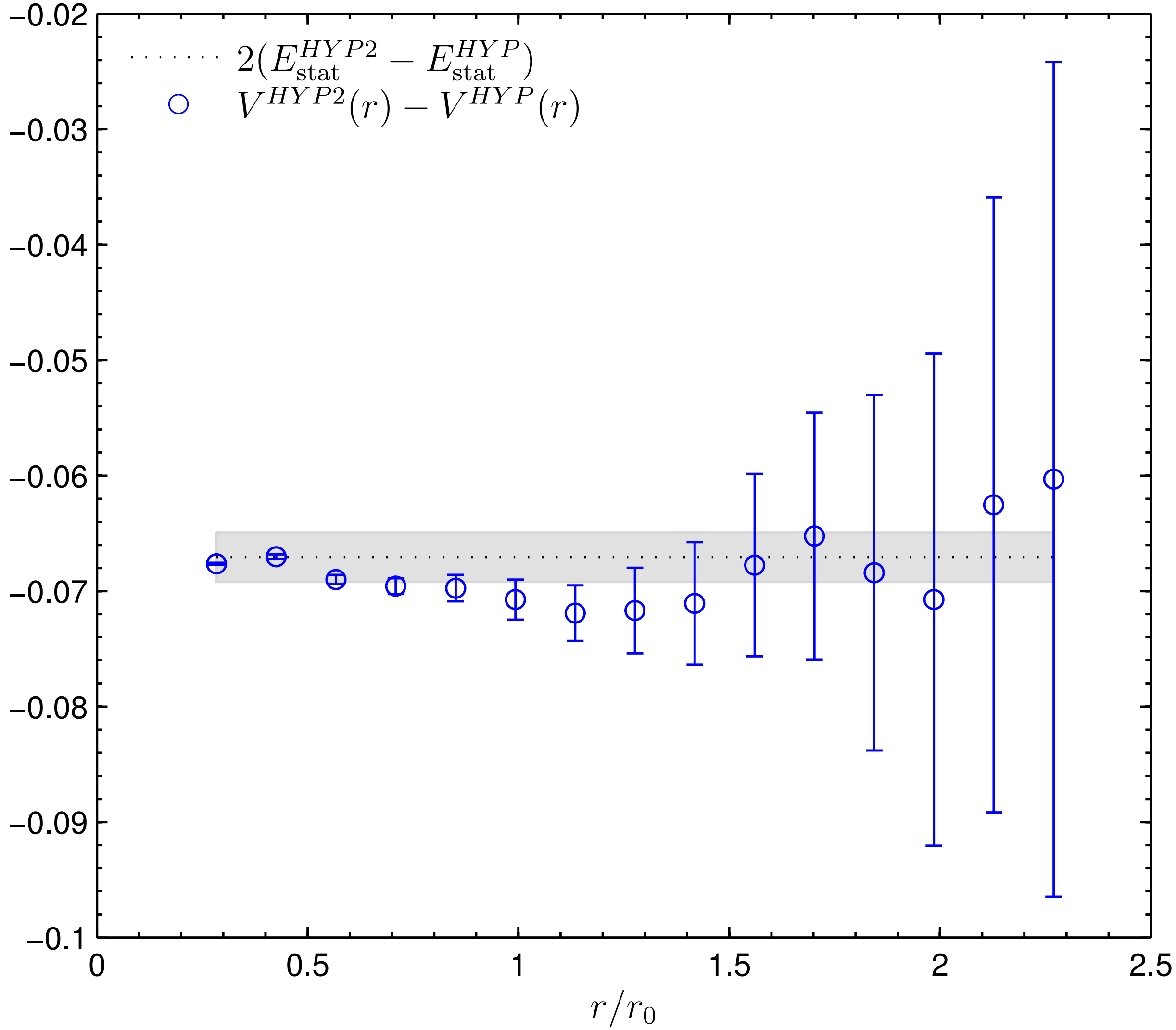
1.2
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14
16
$E_1(t, t_0)$
t/a
fit of effective mass
plateau average
GEVP, $r/a = 7$, $t_0/a = 2$
GEVP, $r/a = 7$, $t_0/a = 4$
$2 * aE_{\rm stat}$

Potential



$\tau_{\rm exp} = 39$ from [Schäfer, Sommer and Virotta, 2010]

Potential



−0.02
−0.03
−0.04
−0.05
−0.06
−0.07
−0.08
−0.09
−0.1
0
0.5
1
1.5
2
2.5
$2(E_{\rm stat}^{HYP2} - E_{\rm stat}^{HYP})$
$V^{HYP2}(r) - V^{HYP}(r)$
r/r_0

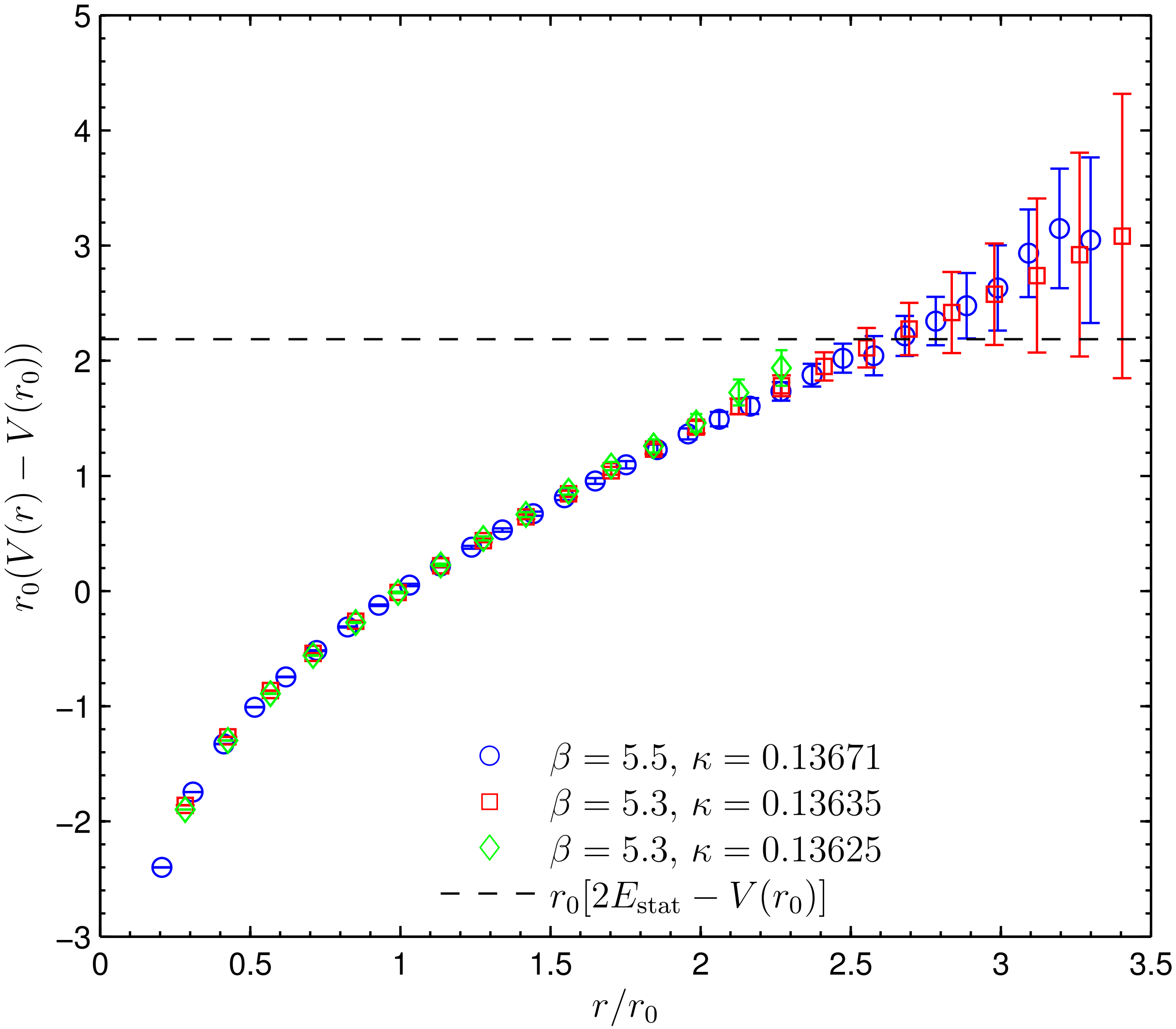
Potential

Comparison with two other ensembles (*preliminary analysis*):

- E5g at $\beta = 5.3$, $\kappa = 0.13625$, 64×32^3:
  $r_0/a(\mathrm{E5g}) = 6.75(6)$, $r_0(m_q = 0)m_{\mathrm{PS}} = 1.02$ using $r_0/a(m_q = 0) = 7.05$
- F6 at $\beta = 5.3$, $\kappa = 0.13635$, 96×48^3:
  $r_0/a(\mathrm{F6}) = 6.983(38)$, $r_0(m_q = 0)m_{\mathrm{PS}} = 0.73$
- O7 at $\beta = 5.5$, $\kappa = 0.13671$, 128×64^3:
  $r_0/a(\mathrm{O7}) = 9.57(6)$, $r_0(m_q = 0)m_{\mathrm{PS}} = 0.64$ using $r_0/a(m_q = 0) = 9.72$

Potential

preliminary



Shape of the potential

The slope

$$c(r) \quad = \quad \frac{1}{2}\, r^3\, F'(r)$$

defines the coupling ($C_{\rm F} = 4/3$)

$$\bar{g}_{\rm c}^2(\mu) \quad = \quad -\frac{4\pi}{C_{\rm F}} c(r)\,,\;\; \mu = 1/r\,.$$

It obeys the renormalization group equation

$$\mu \frac{\rm d}{{\rm d}\mu} \bar{g}_{\rm c}(\mu) \quad = \quad \beta_{\rm c}(\bar{g}_{\rm c}(\mu))\,,$$

which is solved by

$$\frac{\Lambda_{\rm c}}{\mu} \quad = \quad \left(b_0 \bar{g}_{\rm c}^2\right)^{-b_1/(2b_0^2)} {\rm e}^{-1/(2b_0\bar{g}_{\rm c}^2)} \exp\left\{-\int_0^{\bar{g}_{\rm c}} {\rm d}x \left[\frac{1}{\beta_{\rm c}(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}\,.$$

Shape of the potential

In perturbation theory, β_{c} is known up to 4 loop [Brambilla, Pineda, Soto and Vairo, 1999 and 2000; Smirnov, Smirnov and Steinhauser, 2010; Anzai, Kiyo and Sumino, 2010] ($C_{\mathrm{A}} = 3$)

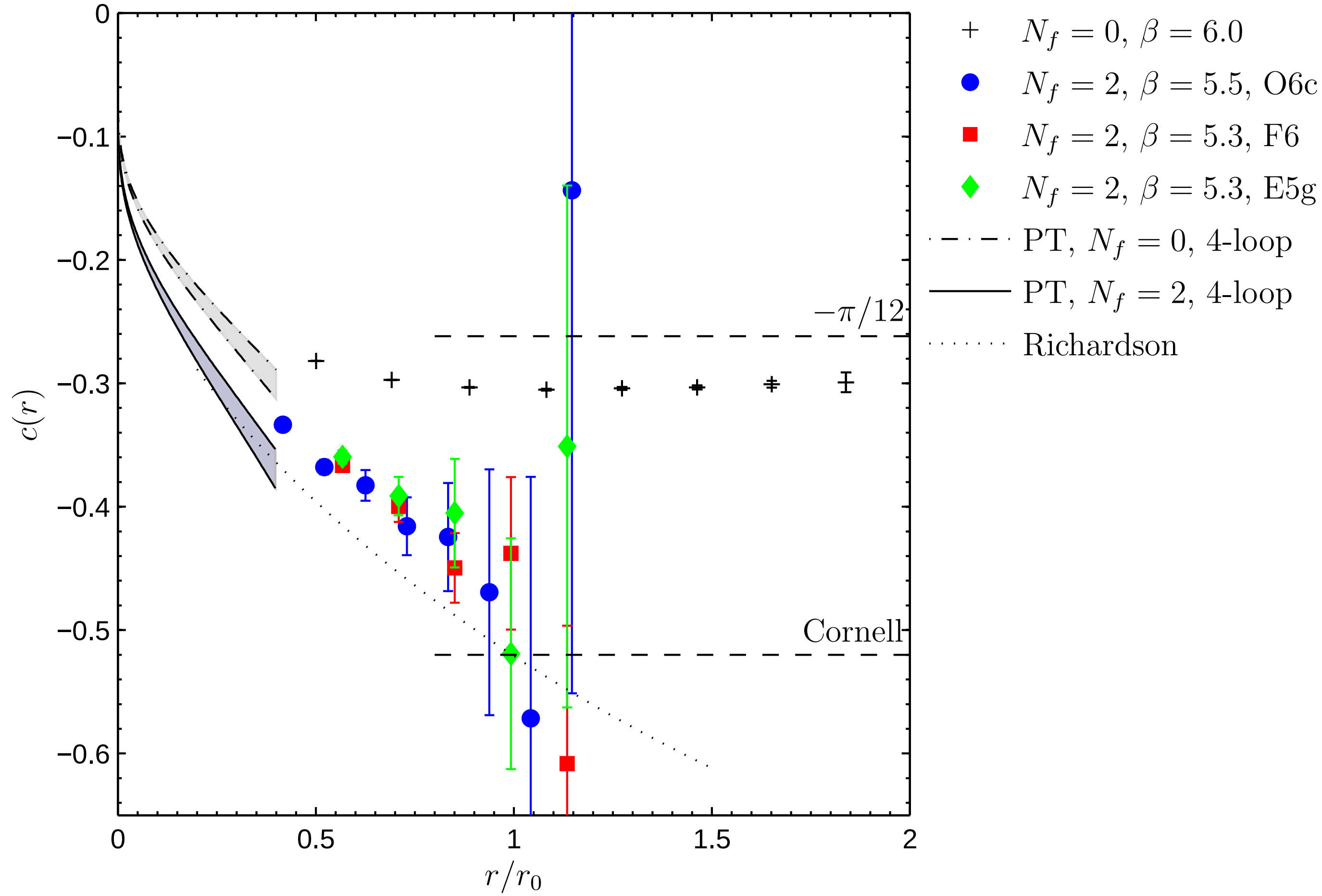
$$\beta_{\mathrm{c}}(\bar{g}_{\mathrm{c}}) = -\bar{g}_{\mathrm{c}}^3 [\sum_{n=0}^{3} b_n^{(\mathrm{c})} \bar{g}_{\mathrm{c}}^{2n} + b_{3,l}^{(\mathrm{c})} \bar{g}_{\mathrm{c}}^6 \log(C_{\mathrm{A}} \bar{g}_{\mathrm{c}}^2/(8\pi)) + \mathrm{O}(\bar{g}_{\mathrm{c}}^8)]$$

Non-perturbatively

- in the $N_{\mathrm{f}} = 0$ theory $c(r)$ approaches the asymptotic value $c(\infty) = -\pi/12$ with corrections $\mathcal{O}(1/r^2)$ predicted from the bosonic effective theory [Lüscher, Symanzik and Weisz, 1980; Lüscher 1980; Lüscher and Weisz, 2002 and 2004]
- comparison with phenomenological potential models: Cornell [Eichten et al., 1980] and Richardson [Richardson, 1979]
- interesting for holographic QCD models [Giataganas and Irges, 2011]

Shape of the potential

preliminary



$$r_0\Lambda_{\overline{\rm MS}}^{N_{\rm f}=0} = 0.60(5)\ [\texttt{hep-lat/9810063}]\ ,\ r_0\Lambda_{\overline{\rm MS}}^{N_{\rm f}=2} = 0.73(3)(5)\ [\texttt{arXiv:1012.1141}]$$

Conclusions and Outlook

- We determine the static potential using the HYP2 static action. Cut-off effects appear to be small
- We determine the scale r_0/a with a precision better than 1%
- We observe large effects from dynamical fermions in the slope $c(r)$
- The statistical precision is worse than in the pure gauge case. Improvement due to the inclusion of fermionic correlators?
- We plan to study string breaking