LIGHT QUARK MASSES FROM RBC & UKQCD COLLABORATIONS

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Light Quark Mass Results

* arXiv/1011.0892: 2+1 flavor dynamical domain-wall fermions with continuum limit

$$m_{ud}^{\overline{MS}}(2GeV) = 3.59(13)_{\rm stat}(14)_{\rm syst}(8)_{\rm ren}\ {\rm MeV},$$
$$m_{s}^{\overline{MS}}(2GeV) = 96.2(1.6)_{\rm stat}(0.2)_{\rm syst}(2.1)_{\rm ren}\ {\rm MeV}$$

* PRD78(2008)114509: 2+1 flavor dynamical domain-wall fermions w/o continuum lim

$$m_{ud}^{\overline{MS}}(2GeV) = 3.72(13)_{\rm stat}(18)_{\rm syst}(33)_{\rm ren}\ {\rm MeV},$$
$$m_{s}^{\overline{MS}}(2GeV) = 107.3(4.4)_{\rm stat}(4.9)_{\rm syst}(9.7)_{\rm ren}\ {\rm MeV}$$

* How these improvements are achieved is reviewed....

Plan

* results (now and then)
* quark mass improvement: step by step
  * 2+1 flavor dynamical domain-wall fermion simulation with 2 lattice spacing
  * non-perturbative renormalization with RI/SMOM schemes
* results
* outlook

* related papers
  * PRD78(2008)114509, arXiv/1011.0892 by RBC/UKQCD
  * arXiv/1006.0422 by R. Arthur and P. Boyle

at Year 2008

* PRD78(2008)114509: 2+1 flavor dynamical domain-wall fermions w/o continuum lim
  * Iwasaki gauge + DWF: β=2.13 (a≃0.11 fm), 24^3x64 (L_s=2.7 fm)
  * Pros
    * chiral fermion (simpler handling of chiral extrapolation)
    * RI/MOM scheme non-perturbative renormalization (NPR)
  * Cons
    * single lattice spacing
    * large NPR error
* arXiv/1011.0892: 2+1 flavor dynamical domain-wall fermions with continuum limit

at Year 2010

- arXiv/1011.0892: 2+1 flavor dynamical domain-wall fermions with continuum limit
  - Iwasaki gauge + DWF: β=2.13 (a≃0.11 fm), 24^3x64 (L_s=2.7 fm) and
  - Iwasaki gauge + DWF: β=2.25 (a≃0.9 fm), 32^3x64 (L_s=2.8 fm)
  - What's new:
    - 2 lattice spacings
      - continuum limit
      - more robust chiral extrapolation possible
    - RI/SMOM scheme NPR
      - much reduced systematic error

2+1 flavor dynamical chiral fermions

$m_s a$	$m_l a$	$\tilde{m}_s/\tilde{m}_l$	Δt_{light}	τ(Ref.[1])	τ(MD)	Acceptance	$\langle P \rangle$	$\langle \bar{\psi}\psi(m_l) \rangle$
$V/a = 24^3 \times 64, L_s = 16, \beta = 2.13, a^{-1} = 1.73(3)$ GeV, $m_{res}a = 0.003152(43), \tau/\text{traj} = 1$								
0.04	0.005	5.3	1/6	4460	8980	73%	0.588053(4)	0.001224(2)
	0.01	3.3	1/5	5020	8540	70%	0.588009(5)	0.001738(2)
$V/a = 32^3 \times 64, L_s = 16, \beta = 2.25, a^{-1} = 2.28(3)$ GeV, $m_{res}a = 0.0006664(76), \tau/\text{traj} = 2$								
0.03	0.004	6.6	1/8	—	6856	72%	0.615587(3)	0.000673(1)
	0.006	4.6	1/8	—	7650	76%	0.615585(3)	0.000872(1)
	0.008	3.5	1/7	—	5930	73%	0.615571(4)	0.001066(1)

* Iwasaki gauge and domain-wall fermions
* 2 lattice spacings
* 1 strange mass & 2-3 u. d quark masses
* combined chiral and continuum extrapolation is performed

extrapolation / interpolation

* strange mass:
  * valence: 2: one unitary and one smaller
  * sea: many points by reweighting
  * combining these makes 2nd unitary point
  * 2nd point is chosen so to interpolate
* u, d quark mass:
  * valence: 3-4 points to cover $225\ \mathrm{MeV} \leqq m_\pi \leqq 420\ \mathrm{MeV}$
  * sea: 2-3 points to cover $290\ \mathrm{MeV} \leqq m_\pi \leqq 420\ \mathrm{MeV}$
  * extrapolation to $m_\pi \to 135$ MeV
* lattice spacing: a=0.114 fm & 0.087 fm to take the continuum limit

determining the physical point

- need to determine (a, m_{ud}, m_s): “physical point” for each lattice spacing
- using mass of π, K, Ω
- m_π, m_K
  - statistically most precisely calculated quantity
  - chiral behavior known better than others
- m_Ω: mass of sss baryon
  - reasonably well controlled statistical error
  - Chiral extrapolation is easy (no chiral log)
- matching the continuum-extrapolated lattice results
  - physical point is determined
- now you can predict the other quantities

chiral and continuum global fit

* mass renormalization needed to handle data with multi lattice spacing
* renormalization scale for each lattice depend on the lattice scale determined from the chiral fit
* decoupling these two simplifies the whole calculation framework
* fixed trajectory method

notation

* coarser lattice is referred to as 32 cubed lattice
* finer lattice is referred to as 24 cubed lattice
* m_l: light quark mass for u, d
* m_h: heavy quark mass for s
* m_{ll}: “pion” with (m_l,m_l)
* m_{lh}: “kaon” with (m_l,m_h)
* m_{hhh}: “Ω” with (m_h,m_h,m_h)

primary lattice ensemble and other

* define the lattice scheme with the primary ensemble (lattice spacing)
  * in this work 32 cubed is the primary ensemble
* do matching the other ensembles to primary
  * in this work we match 24 cubed to 32 cubed
* all results are parametrized with the parameter in the primary ensemble
  * all 24 and 32 cubed data are parametrized in 32 cubed parameter
* do chiral fit with a^2 error taken into account
* input mass of π, K, Ω $\Rightarrow$ primary ensemble physical point is determined
* do renormalization of the primary ensemble parameter: to get quark mass
* (we do need renormalization for another ensemble to get rid of a-error)

parameterization

- everything is parametrized in 32 cubed (finer) lattice scheme:
- introduce
  - ratio of lattice spacing $R_a = a_{32}/a_{24}$
  - ratio of mass renormalization constant
    - $Z_l = Z_m^{24}/Z_m^{32}$ for (u, d)
    - $Z_h = Z_m^{24}/Z_m^{32}$ for (s)
    - Z_m to mach to a mass independent scheme
    - difference arrises due to mass dependent lattice artifact $O(a^2)$
    - function of mass, but, in practice, one value has an “allowed range”
- determined iteratively at a simulated mass on the finer lattice
- then 24 cubed (coarser) lattice can be treated in the same ground as 32

fixed trajectory scheme

* renormalized trajectory obtained by matching to a simulated point of the primary ensemble
  * ex: use 32 cubed (m_l,m_h)=(0.006,0.03)

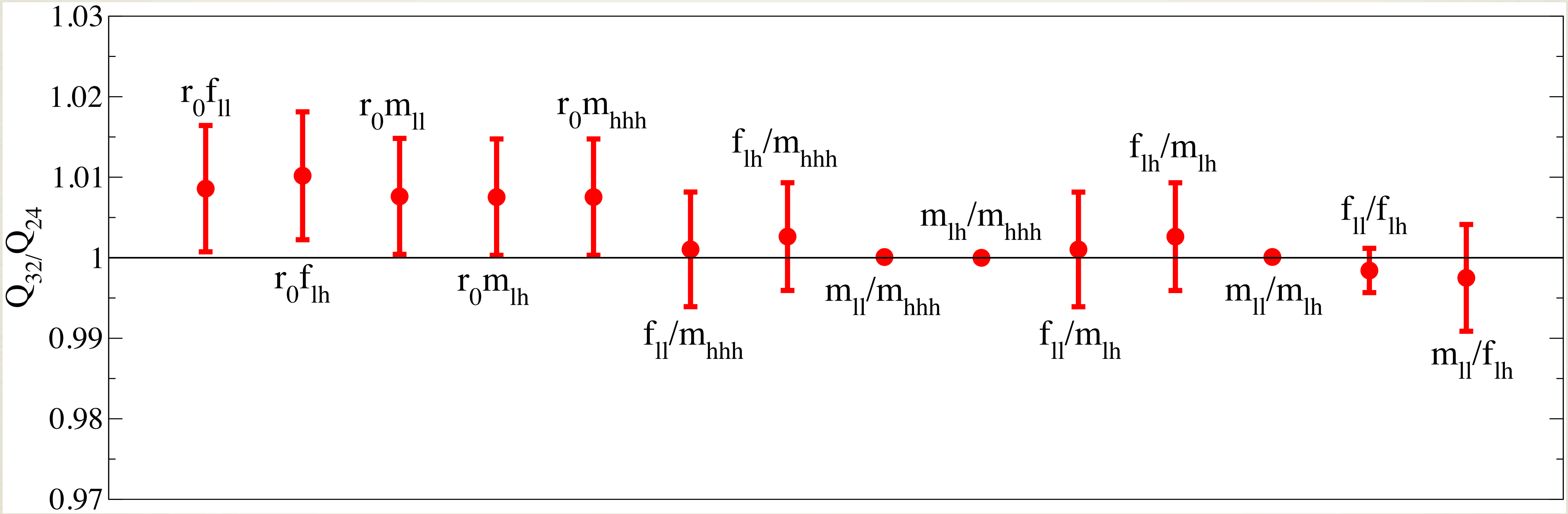
M	$(am_l)^{\mathbf{M}}$	$(am_h)^{\mathbf{M}}$	$(am_l)^{\mathbf{e}}$	$(am_h)^{\mathbf{e}}$	Z_l	Z_h	R_a
32^3	0.004	0.03	0.00313(13)	0.03812(80)	0.980(15)	0.976(11)	0.7617(72)
32^3	0.006	0.03	0.00583(12)	0.03839(51)	0.981(9)	0.974(7)	0.7583(46)
32^3	0.008	0.03	0.00860(19)	0.03869(64)	0.979(10)	0.972(8)	0.7545(58)
24^3	0.005	0.04	0.00545(11)	0.03148(51)	0.985(12)	0.978(9)	0.7620(57)
24^3	0.01	0.04	0.00897(18)	0.03074(57)	0.974(11)	0.968(9)	0.7517(70)

  * stable over the range of simulated quark mass

a scaling test at along the trajectory

* Q_{32}/Q_{24} for Q being a dimension less ratio of observables
* Q_{32}/Q_{24} = 1±0.01 level (1 for perfect scaling)

matched with m_l=0.006, m_h=0.03 for 32 cubed



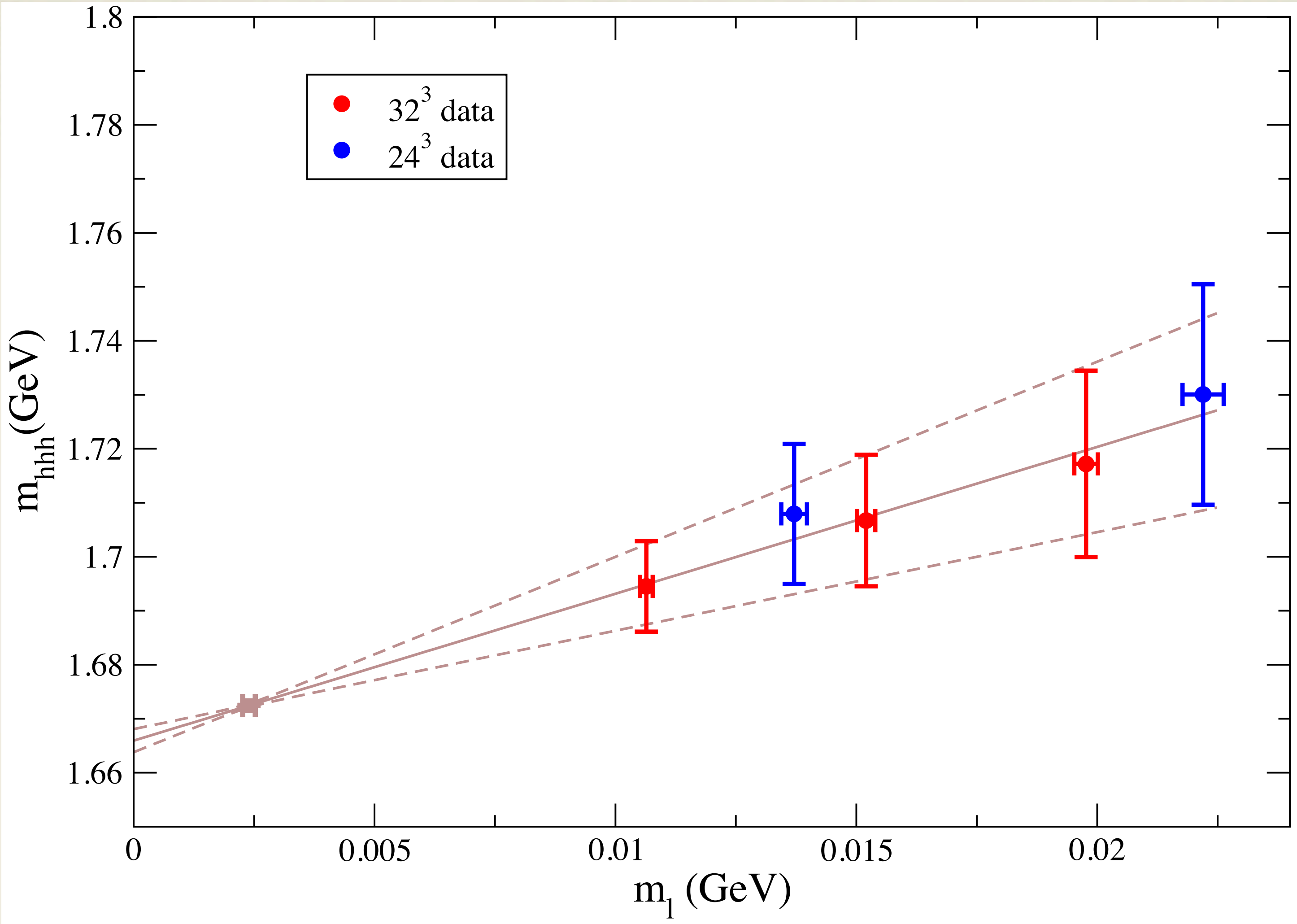
global fit

- Now that we determined the necessary parameters, we can perform the global fit using 24^3 and 32^3
- 2 fit types
  - ChPT　(NLO)
    - good if simulated masses are in the chiral regime
    - with and without finite volume correction to assess finite V error
  - analytic（1st order)
    - good if the chiral regime is below physical point

global fit for non-zero a and m_{res}

* NLO power counting: a^2, $m_\pi{}^2$: same order, neglect higher order.
* m_{res} enters as additive renormalization to bare quark mass
  * $\tilde{m} = m + m_{\rm res}$
  * other m_{res} effects are negligible with this power counting
* SU(2) ChPT (LEC's depend on m_h)
* Analytic fit to first order in m_l (constants depend on m_h)

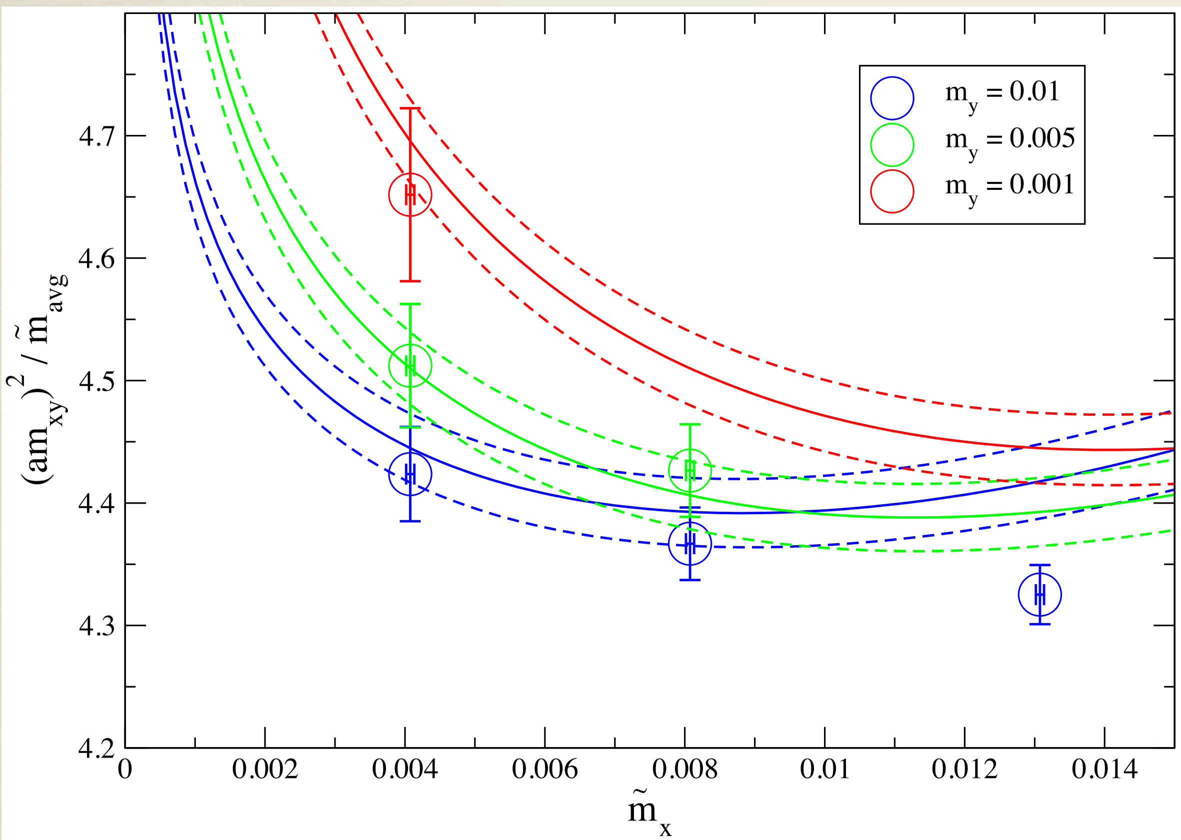
m_Ω



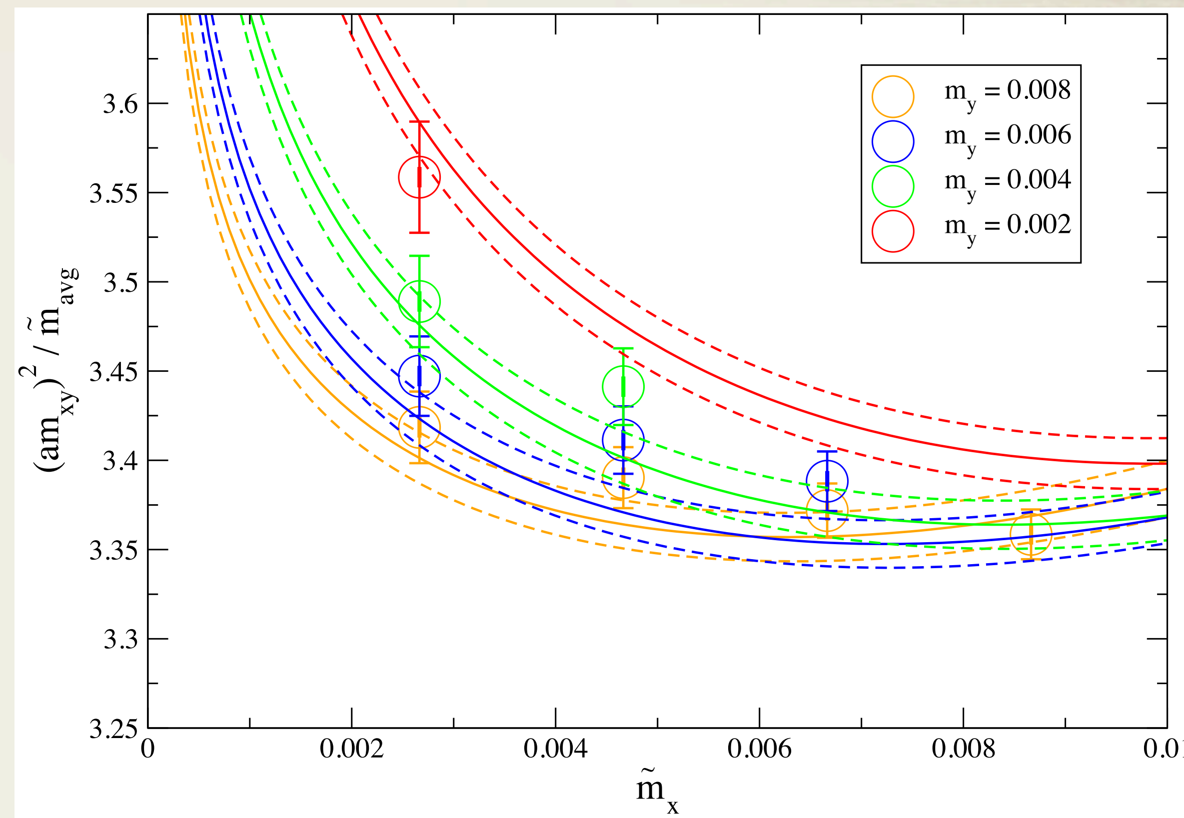
- analytic ansatz
- virtually no difference for ChPT and $ChPT_{FV}$

m_{xx}^2/m_x : partially quenched ChPT$_{FV}$ fit

24^3, m_l=0.005

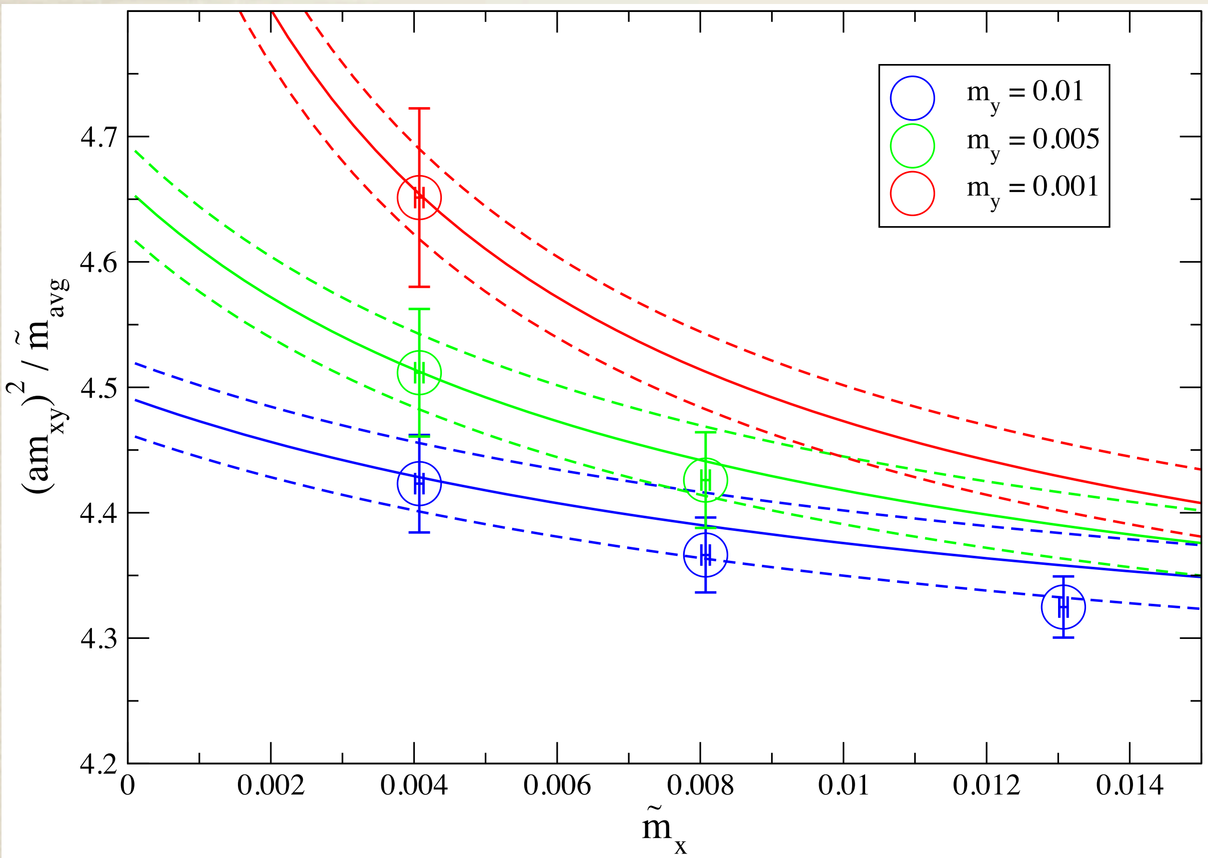


32^3, m_l=0.004

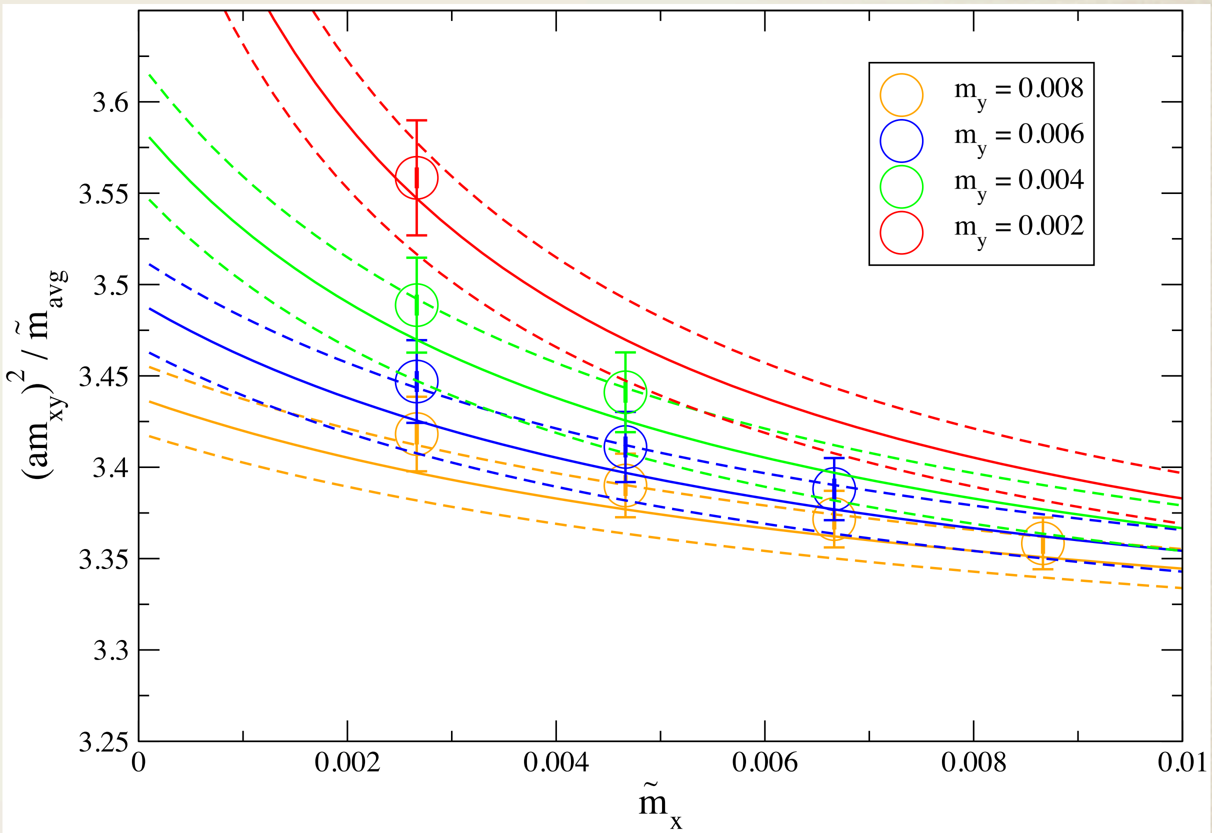


m_{xx}^2/m_x : analytic fit

24^3



32^3



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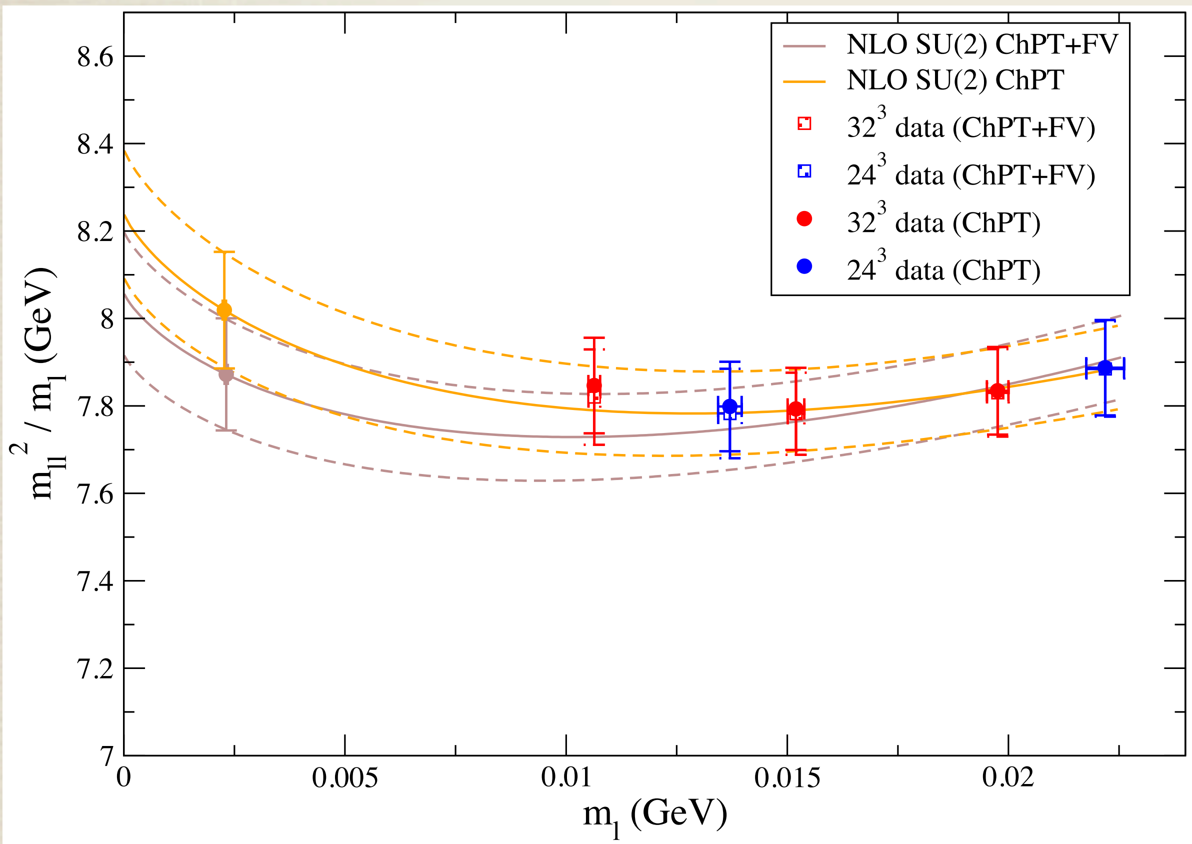
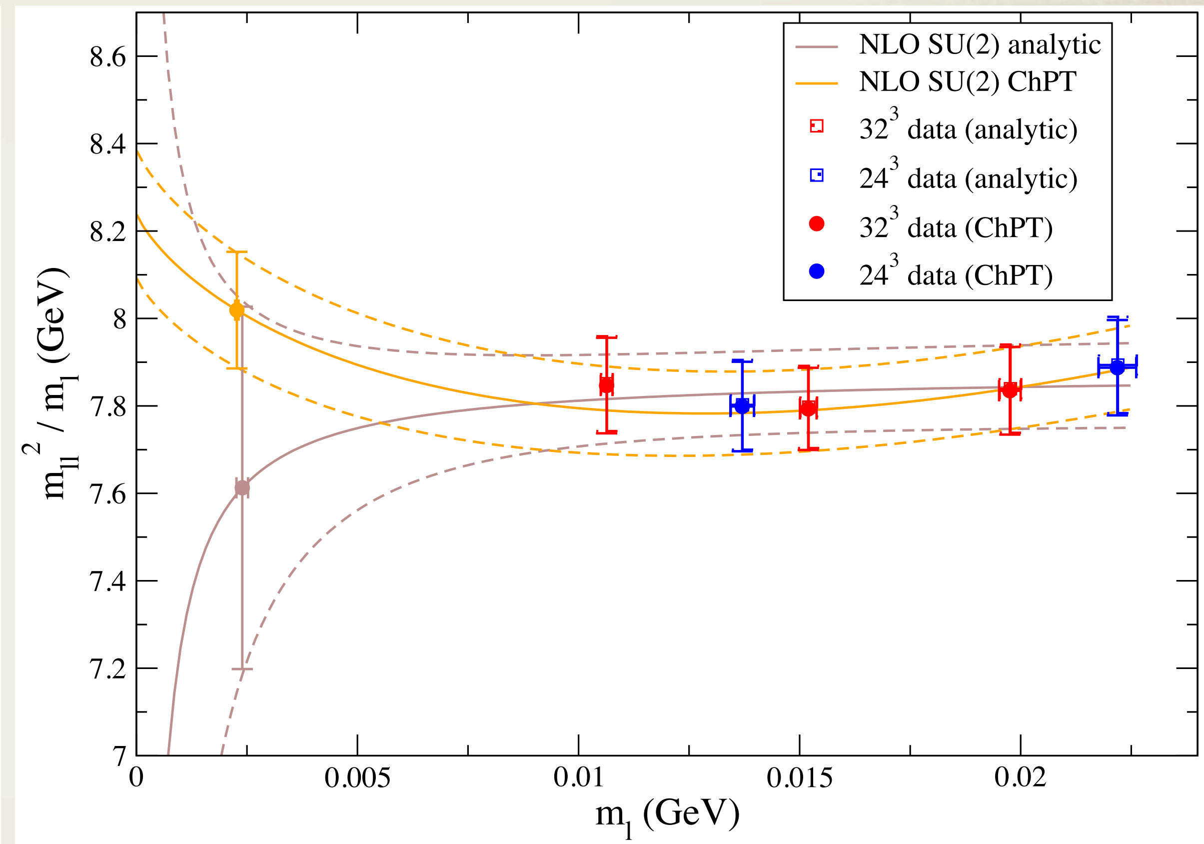
m_{ll}^2/m_l : continuum limit

ChPT(V≠∞ & V→∞) analytic & ChPT(V→∞)

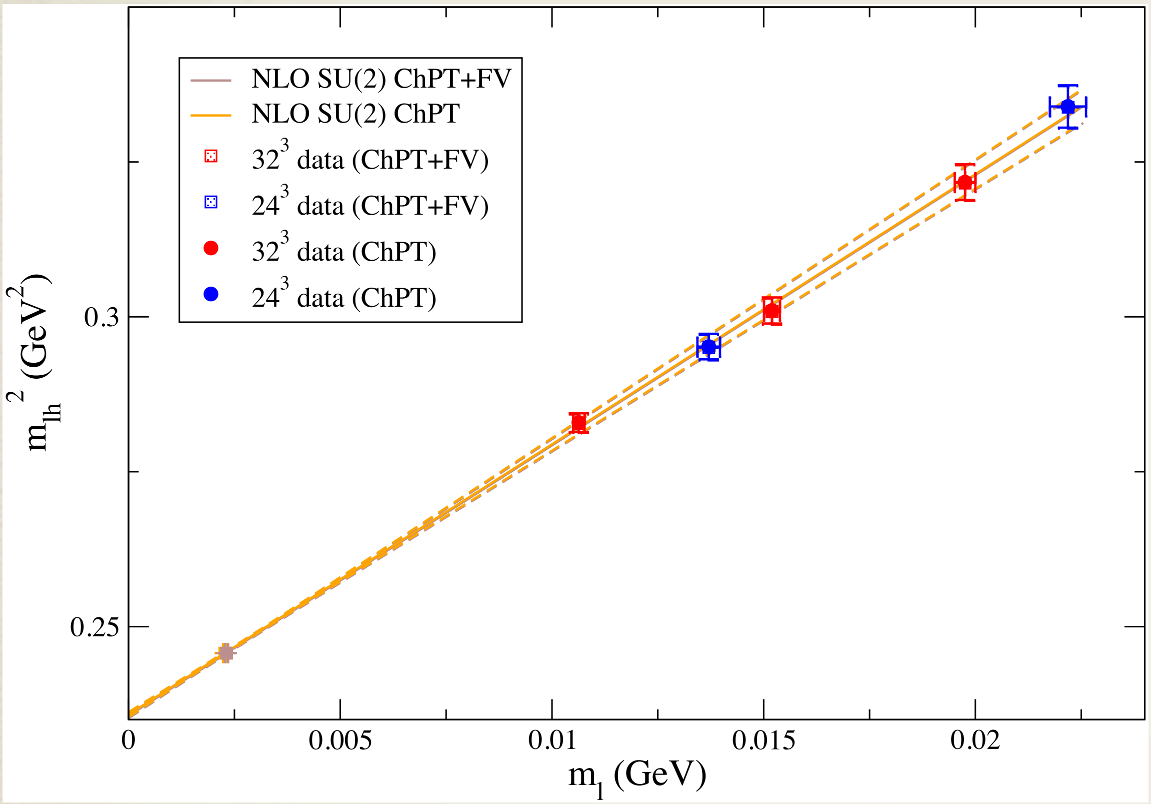




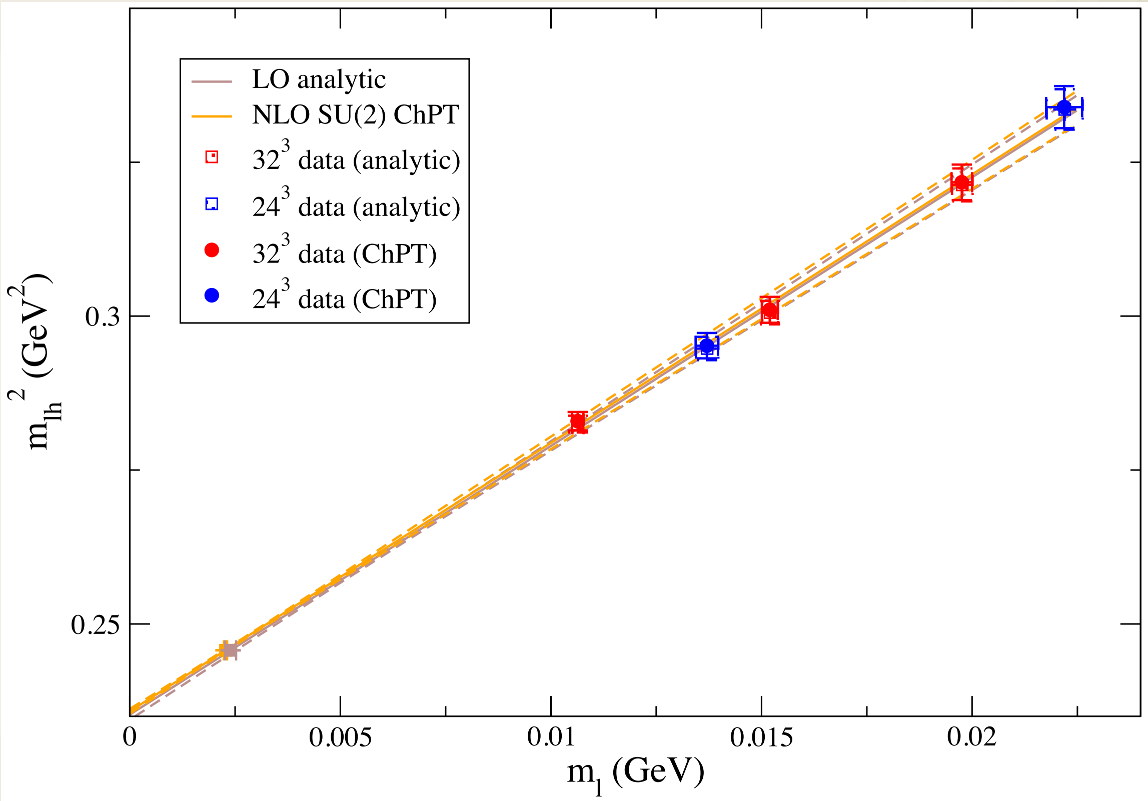
✳ a^2 subtracted from the data from the fit

kaon mass (continuum)

ChPT(V≠∞ & V→∞)



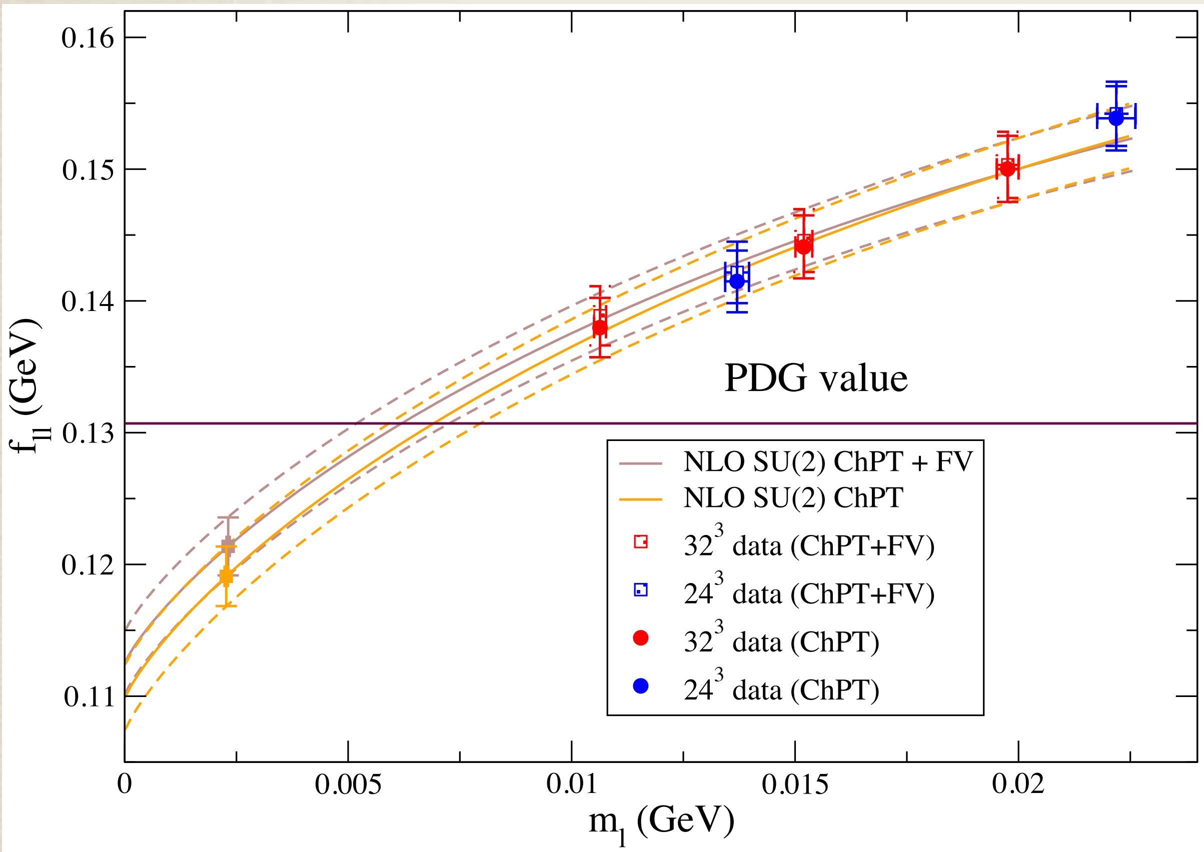
analytic & ChPT(V→∞)

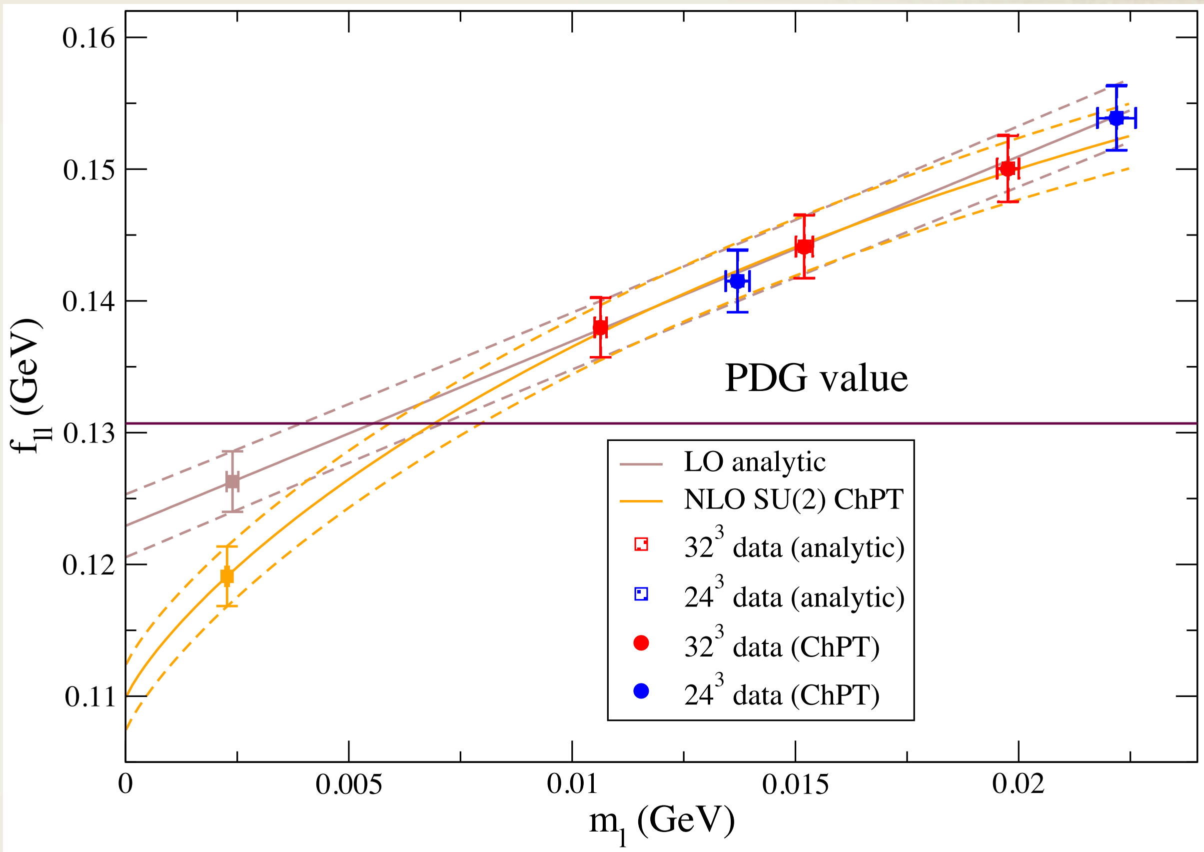


* local axial current renormalized with Z_A calculated from the ratio of conserved and local DWF vector current

pion decay constant (continuum)

ChPT(V≠∞ & V→∞) analytic & ChPT(V→∞)





- local axial current renormalized with Z_A calculated from the ratio of conserved and local DWF vector current

central value and systematic error

* unitary data : surprisingly linear and consistent with analytic ansatz
* but also consistent with ChPT
* no strong indication of fit favoring $ChPT_{FV}$ or analytic
  * take average of them for the central value
  * systematic error of the chiral extrapolation = $|ChPT_{FV} - \text{analytic}|$
  * systematic error due to finite volume = $|ChPT_{FV} - ChPT|$
  * $f_\pi^{\text{continuum}}$=124(2)(5) MeV ↔ 130.4(4)(2) MeV [PDG]
  * consistent at 1 sigma level
* This procedure is followed for all quantities f_K, B_K, quark masses

predictions

$$
\begin{aligned}
f_\pi^{\text{continuum}} &= 124(2)(5)\,\text{MeV} \\
f_K^{\text{continuum}} &= 149(2)(4)\,\text{MeV} \\
(f_K/f_\pi)^{\text{continuum}} &= 1.204(7)(25)\,,
\end{aligned}
$$

$$\tilde{m}_{ud} = 2.35(8)(9)\,\text{MeV} \qquad \text{and} \qquad \tilde{m}_s = 63.7(9)(1)\text{MeV}$$

in 32 cubed scheme. needs to be matched to more convenient schemes with non-perturbative renormalization

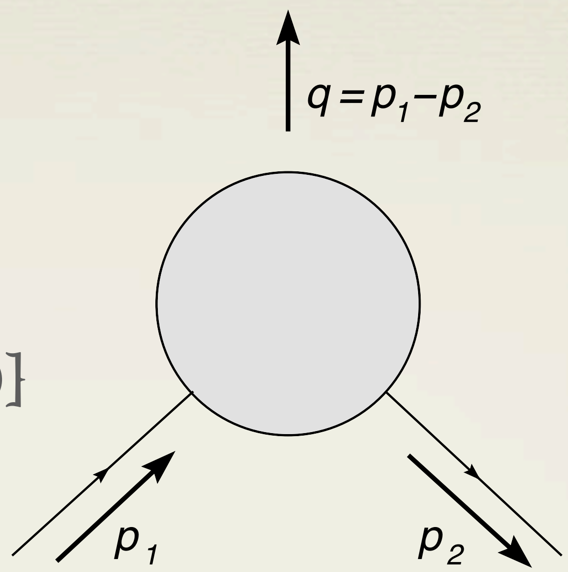
non-perturbative renormalization

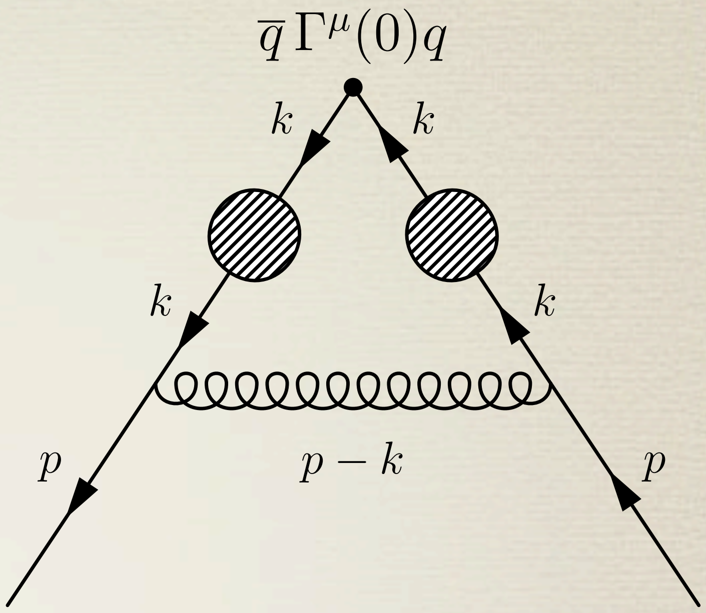
* previous RBC/UKQCD result [PRD78(2008)114509]

$$m_s^{\overline{MS}}(2GeV) = 107.33(4.4)_{\rm stat}(4.9)_{\rm syst}(9.7)_{\rm ren}\ \mathrm{MeV}$$

  * 9% error from RI/MOM renormalization [PRD78(2008)054510]
    * 6% from truncation error of perturbation (NNNLO)
    * 7% from $m_s \neq 0$ (SSB contamination)
      * a promising solution also provided in the same paper
        * momentum kinematics: exceptional -> non-exeptional
* RI/SMOM schemes are constructed utilizing non-exceptional momenta:
  * [C. Stutm, YA, N. Christ, T. Izubuchi, C. Sachrajda, A. Soni, PRD80 (2009)014501]
  * 1 loop matching from SMOM schemes to MSbar provided

SSB contamination





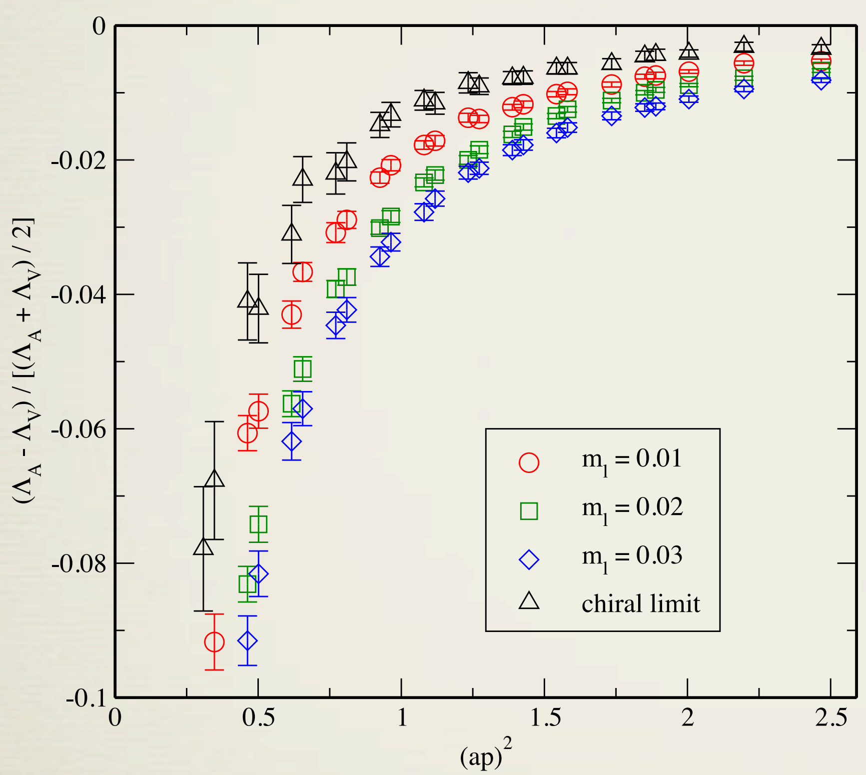
- $q_\mu=0$ [exceptional (∃ partial sum is zero)]
- $1/p^2$ from one gluon exchange
- low momentum flow in the upper part triggers SSB depending on the Γ structure

  → $1/p^2$ contamination (cannot be corrected by PT)
- Backed up from power counting theorem by Weinberg, combined with the group theoretical argument (RBC/UKQCD, PRD 2008)
- avoiding $q_\mu=0$, and going for non-exceptional momenta remove $1/p^2$ contamination

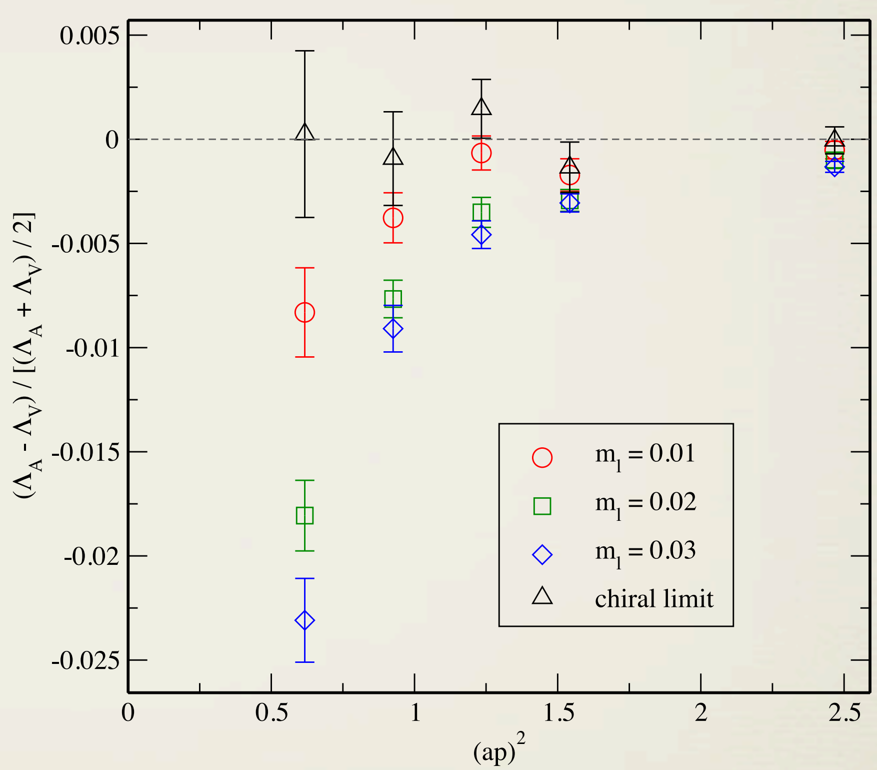
A test of non-exceptional mom

Λ_A-Λ_V: RBC/UKQCD [PRD 2008]

exceptional

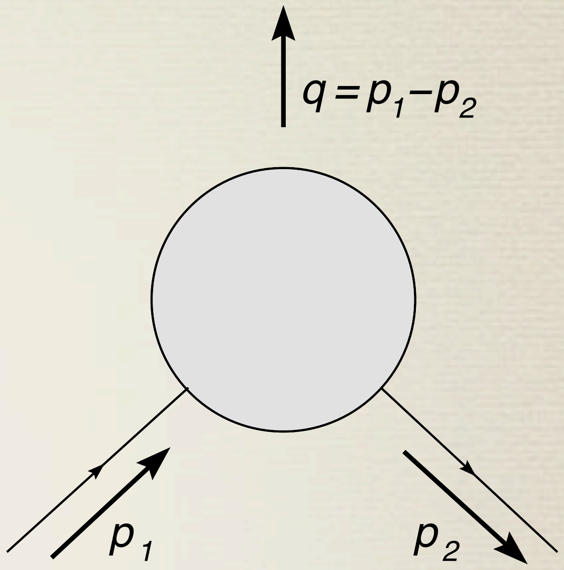


non-exceptional



* The success created a very good motivation to invest in non-exceptional momenta

SMOM scheme



* C. Sturm et al. PRD80 (2009) 014501.
* utilize a non-exceptional momenta for bilinears
* Symmetric $(q^2 = p_1^2 = p_2^2)$ MOM scheme
* SSB contamination is expected to be reduced
* SMOM: $\Lambda_A = \Lambda_V$ exact for chiral fermions
  * $\Lambda_A = \Lambda_V + c/p^2$... for RI/MOM
* projection operator for V: $q_\mu \not{q} / q^2$ gives Z_q of RI'
* 2nd scheme: SMOMγ_μ γ_μ new Z_q

MOM ↔ SMOM comparison: MS matching

* original RI/MOM mass conversion factor
  [Franco & Lubicz, Chetyrkin & Retey, Gracey] $\mu = 2$ GeV

$$C_m(RI/MOM \to \overline{MS}) \quad = \quad 1 + \frac{\alpha_s}{4\pi} C_F\ 4 + \cdots = 1 - 0.123 - 0.070 - 0.048 + \cdots$$

1-loop, 2-loop, 3-loop

$$C_m(RI'/MOM \to \overline{MS}) \quad = \quad 1 + \frac{\alpha_s}{4\pi} C_F\ 4 + \cdots = 1 - 0.123 - 0.065 - 0.044 + \cdots$$

5-6% correction 2→3 loop!

* SMOM: 1 loop [C. Sturm et al. PRD80 (2009) 014501]
* 2 loop [L. Almeida & C. Sturm PRD82 (2010) 054017]

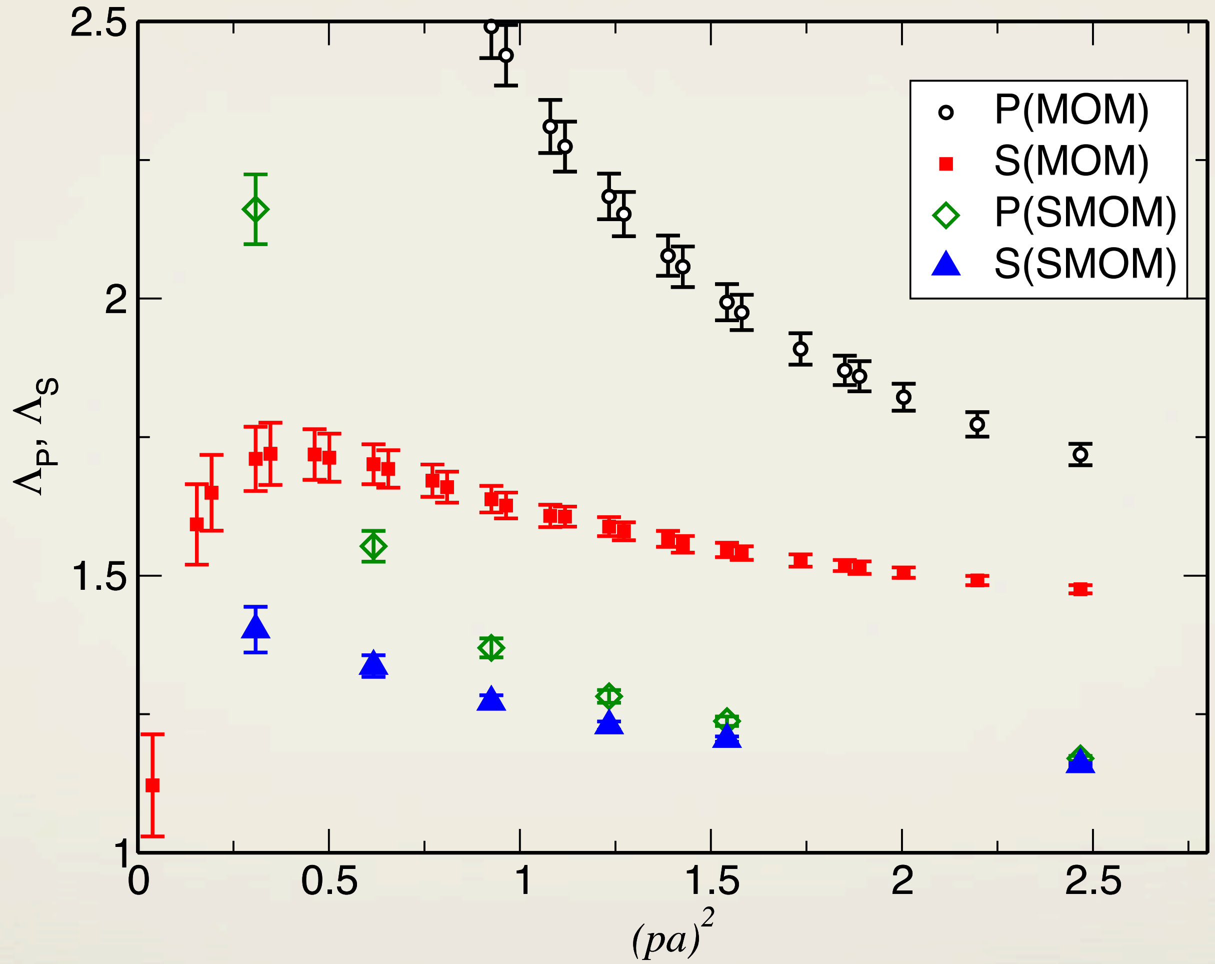
$$C_m(SMOM \to \overline{MS}) \quad = \quad 1 - \frac{\alpha_s}{4\pi} C_F\ 0.484 + \cdots = 1 - 0.015 - 0.006 + \cdots$$

$$C_m(SMOM\gamma_\mu \to \overline{MS}) \quad = \quad 1 - \frac{\alpha_s}{4\pi} C_F\ 1.484 + \cdots = 1 - 0.045 - 0.020 + \cdots$$

* (size of the last term is taken as the systematic error of PT)

results with 2+1f DWF

Λ_P, Λ_S comparison of RI/MOM and SMOM



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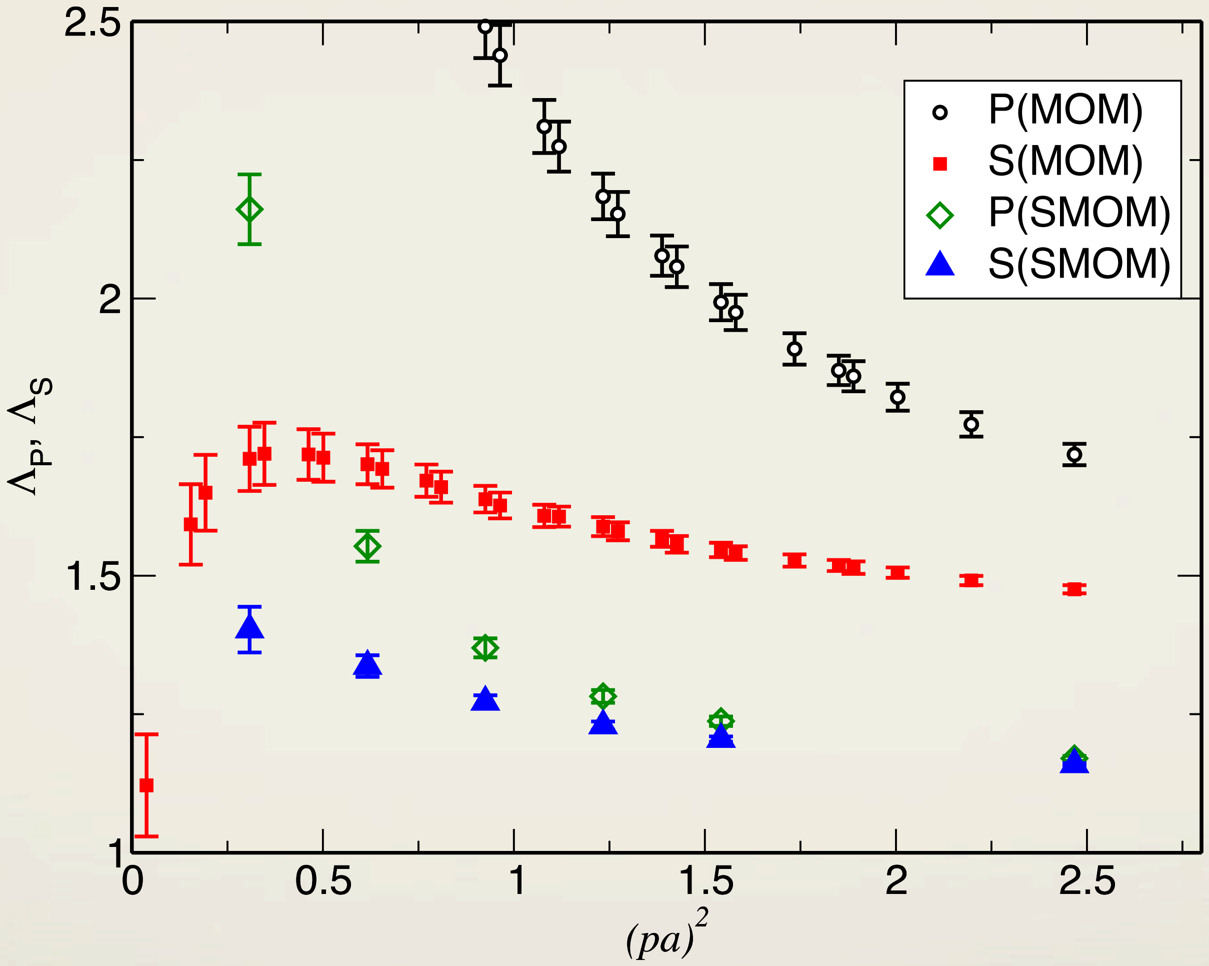
results with 2+1f DWF

Λ_P, Λ_S comparison of RI/MOM and SMOM

* DWF, RBC/UKQCD (YA:Lattice 2008)



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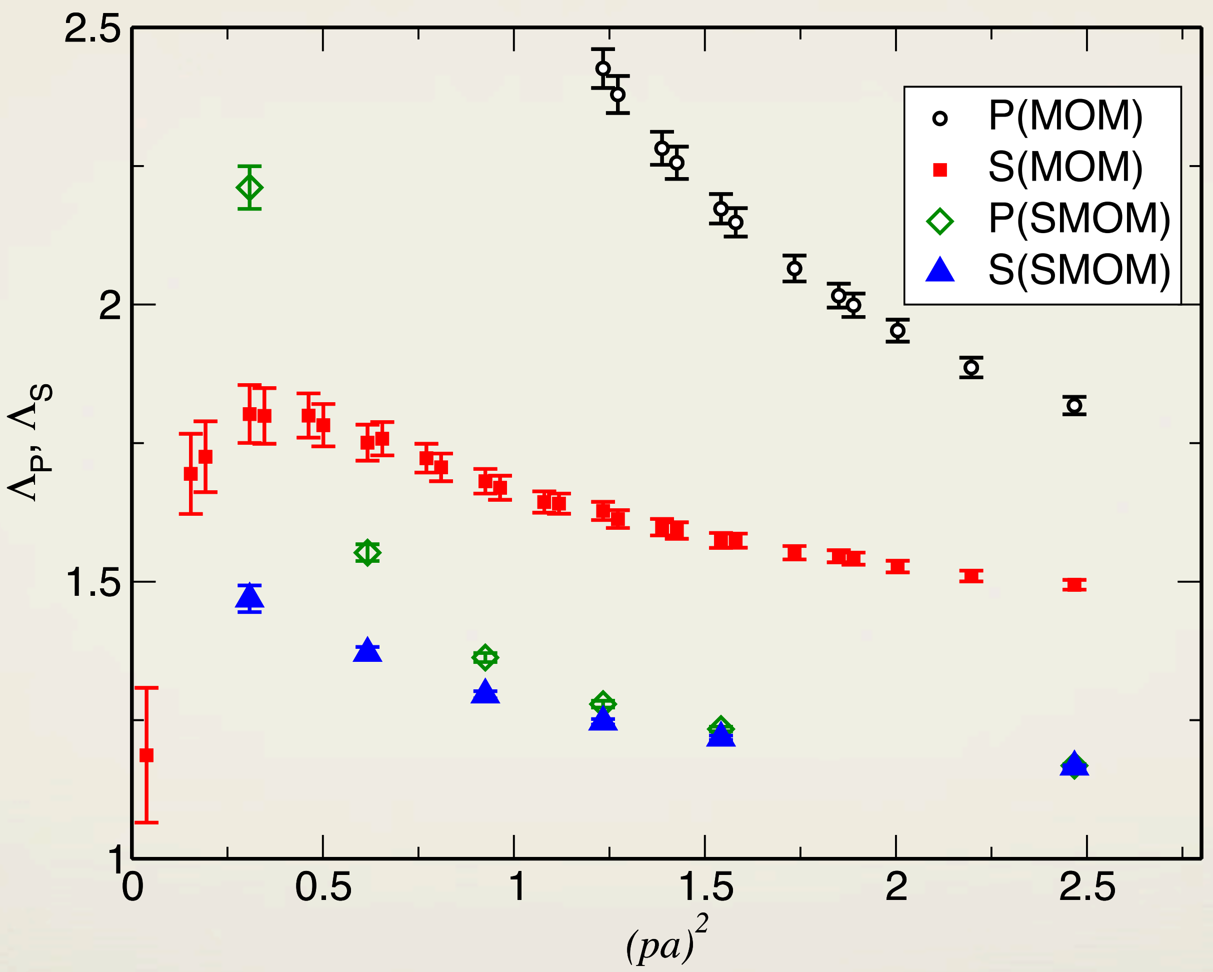

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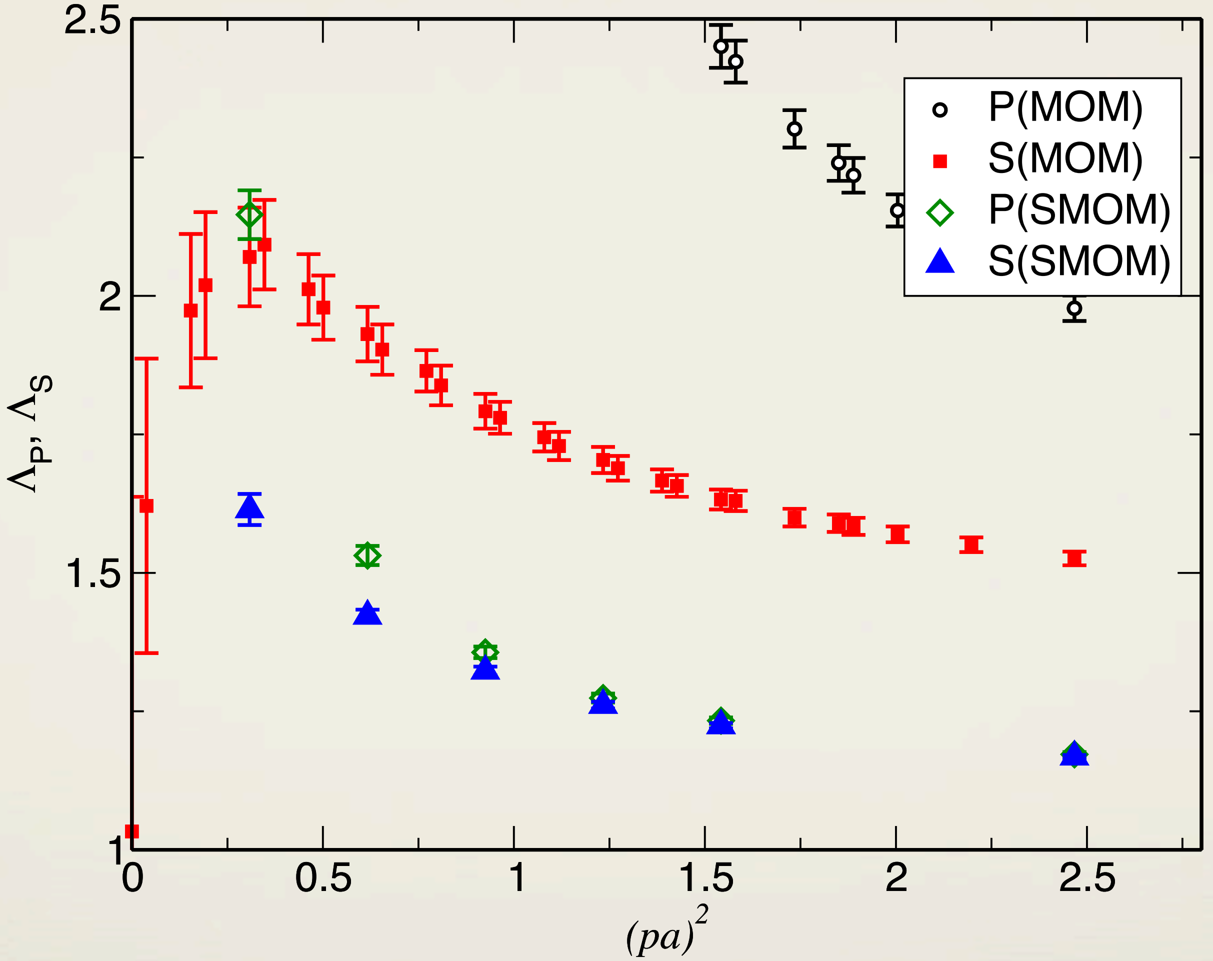

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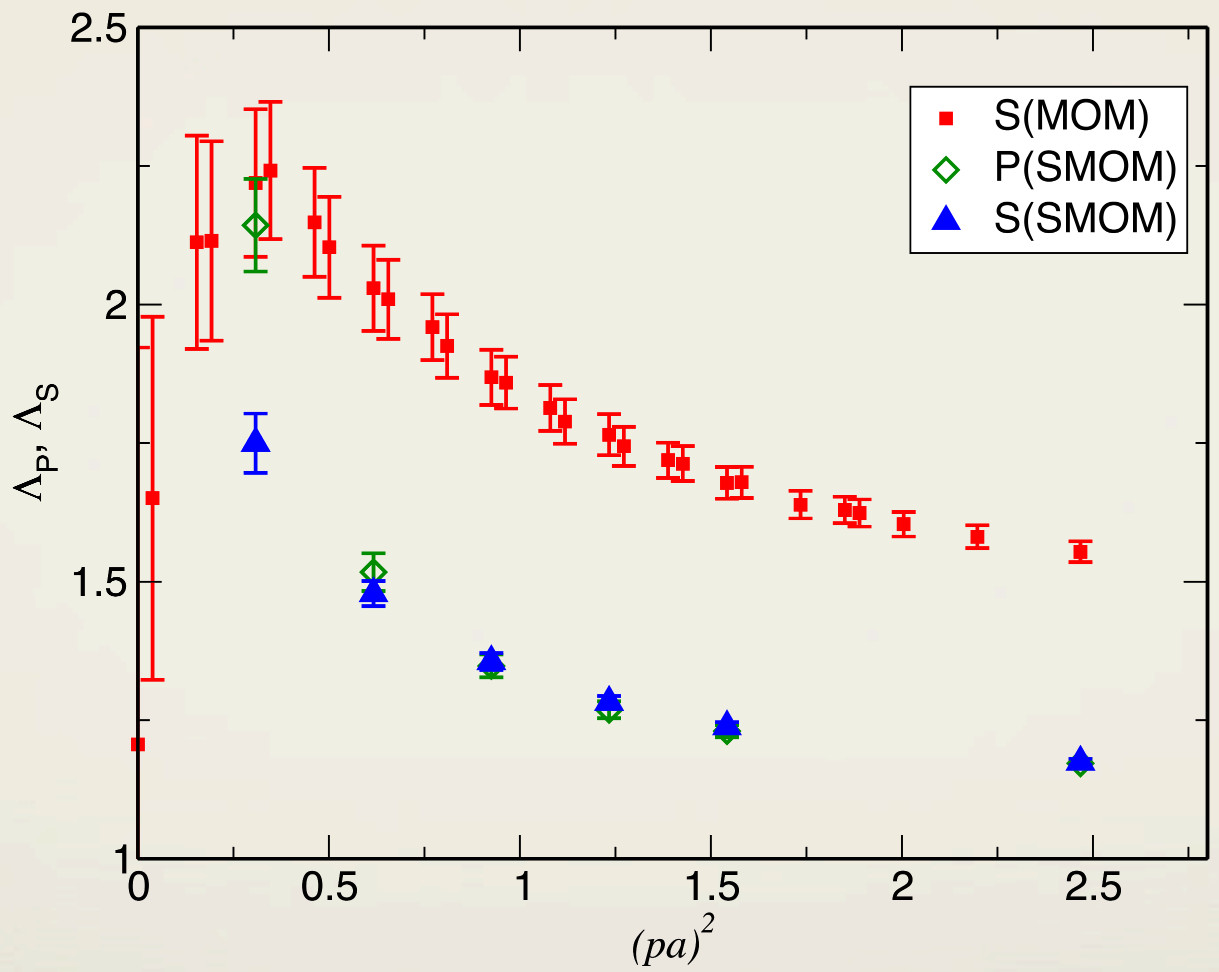
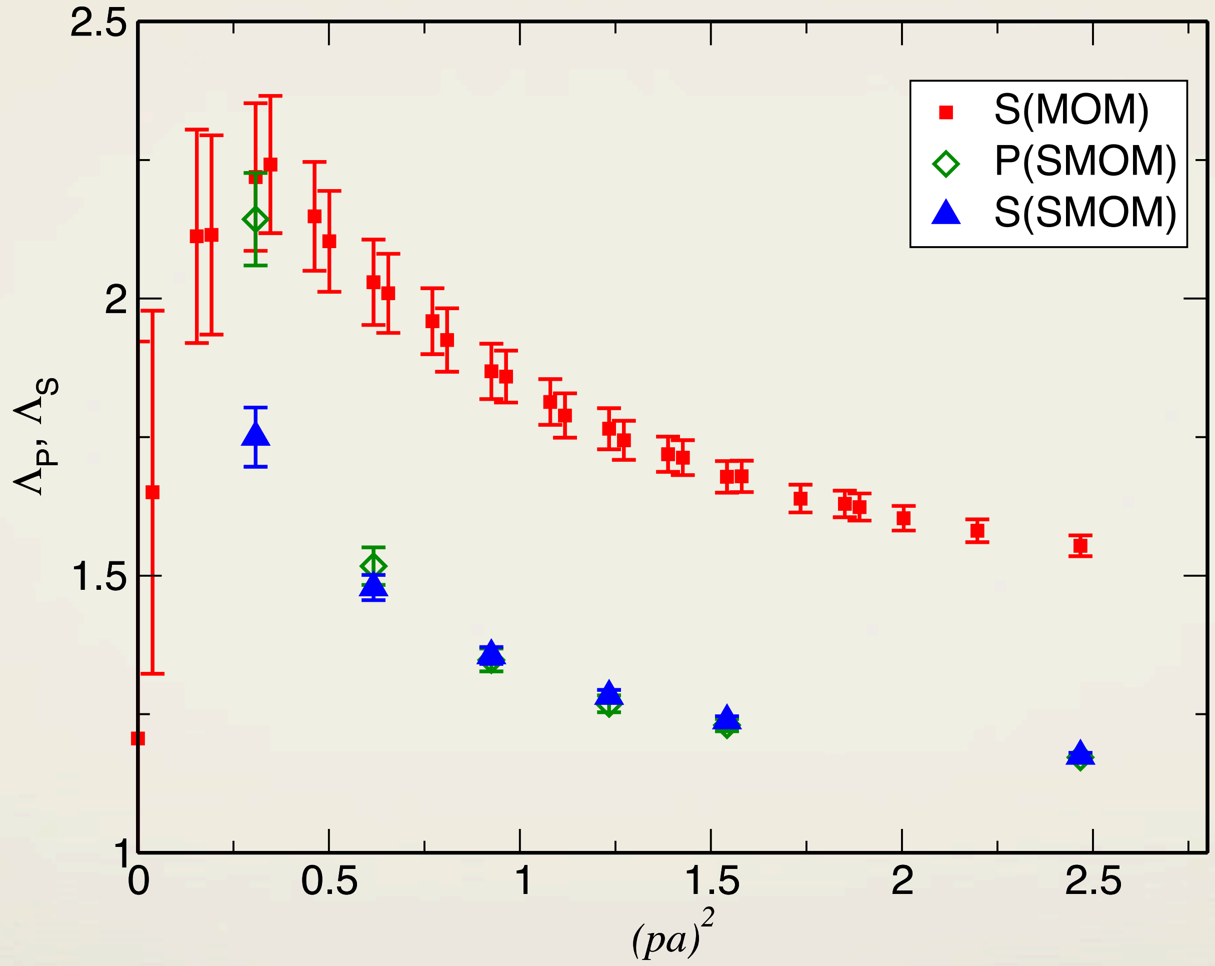


results with 2+1f DWF

Λ_P, Λ_S comparison of RI/MOM and SMOM

- DWF, RBC/UKQCD (YA:Lattice 2008)

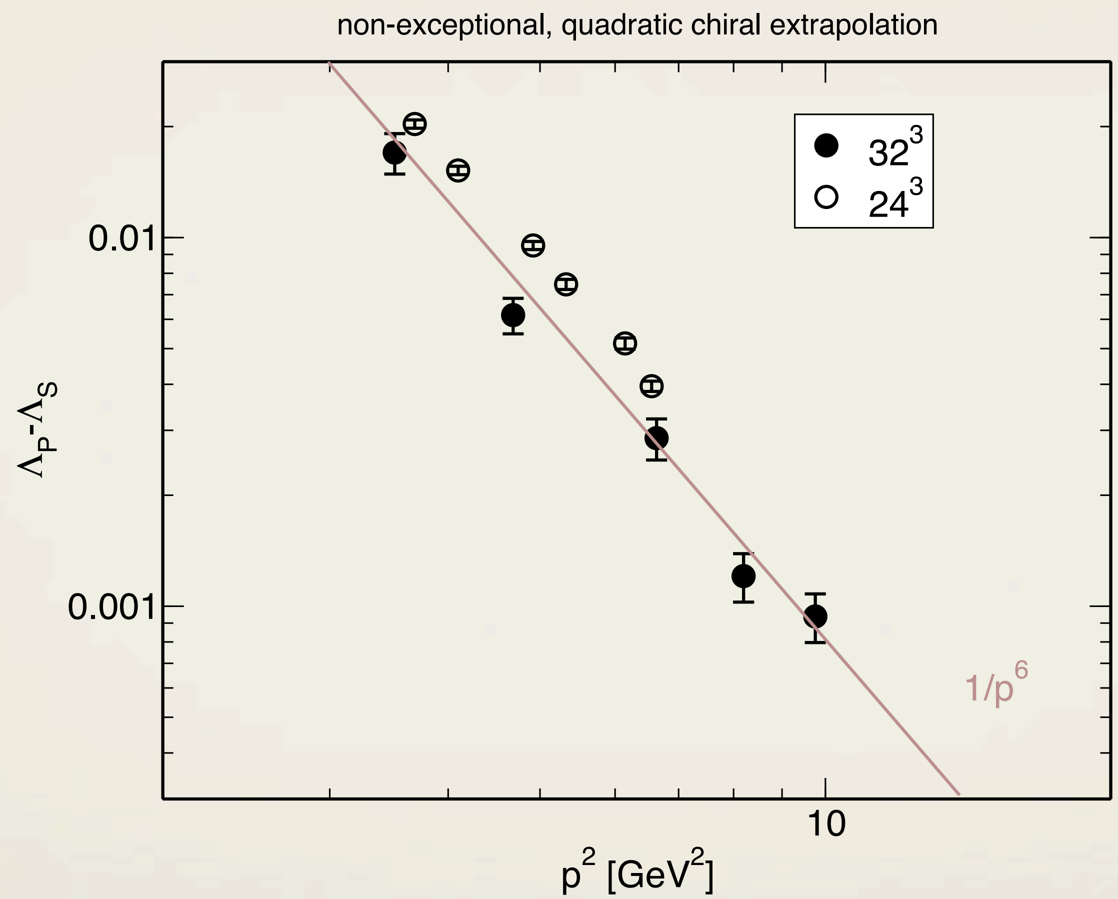


- SMOM: smaller mass dependence, good chiral symm.

SMOM $\Lambda_P - \Lambda_S$

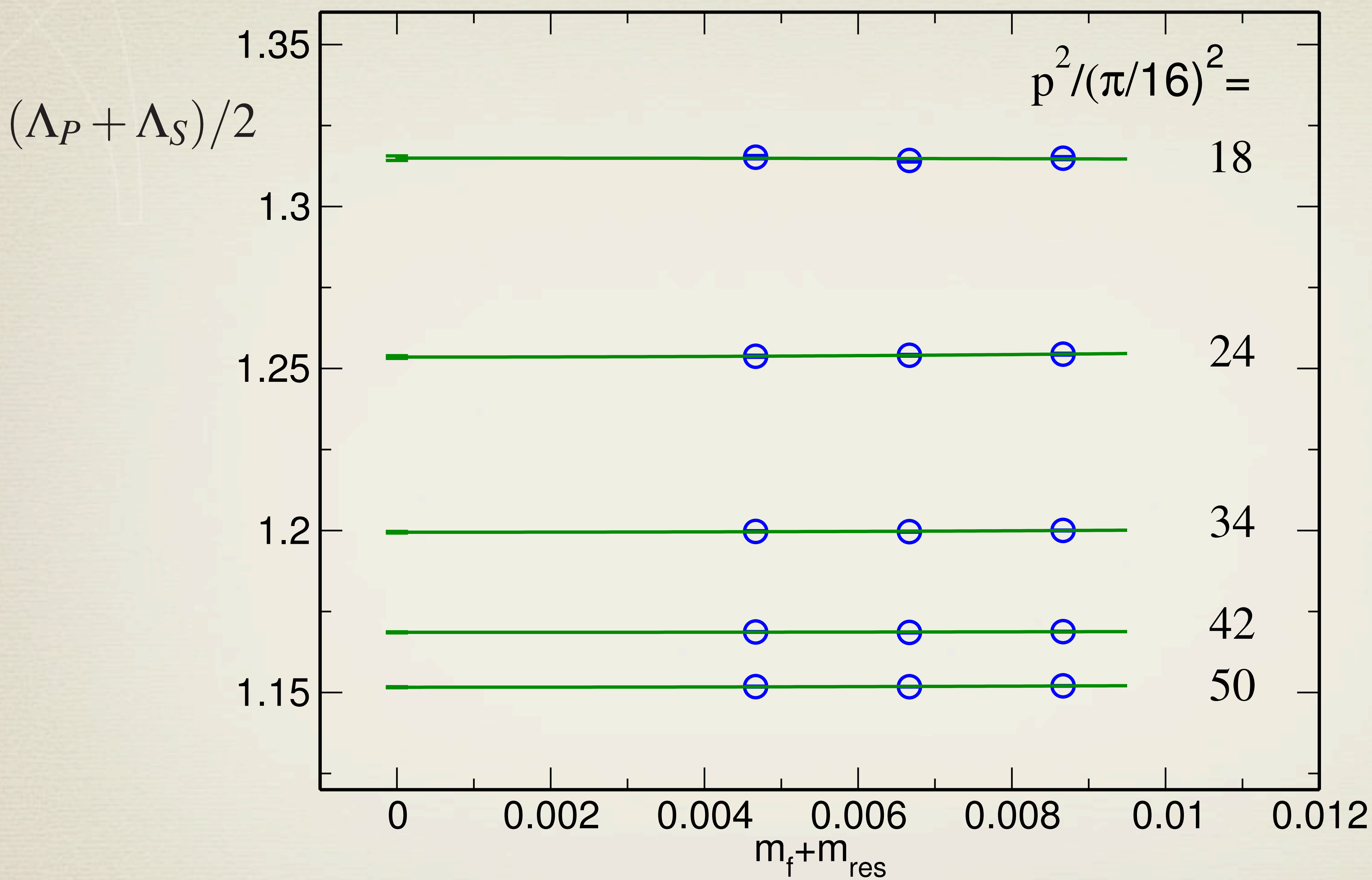
on two lattice spacings, with powerful momentum source

- a^{-1}=1.7GeV (24^3 coarse) & 2.3GeV (32^3 fine), DWF



- $1/p^6$ expected from the Weinberg's theorem

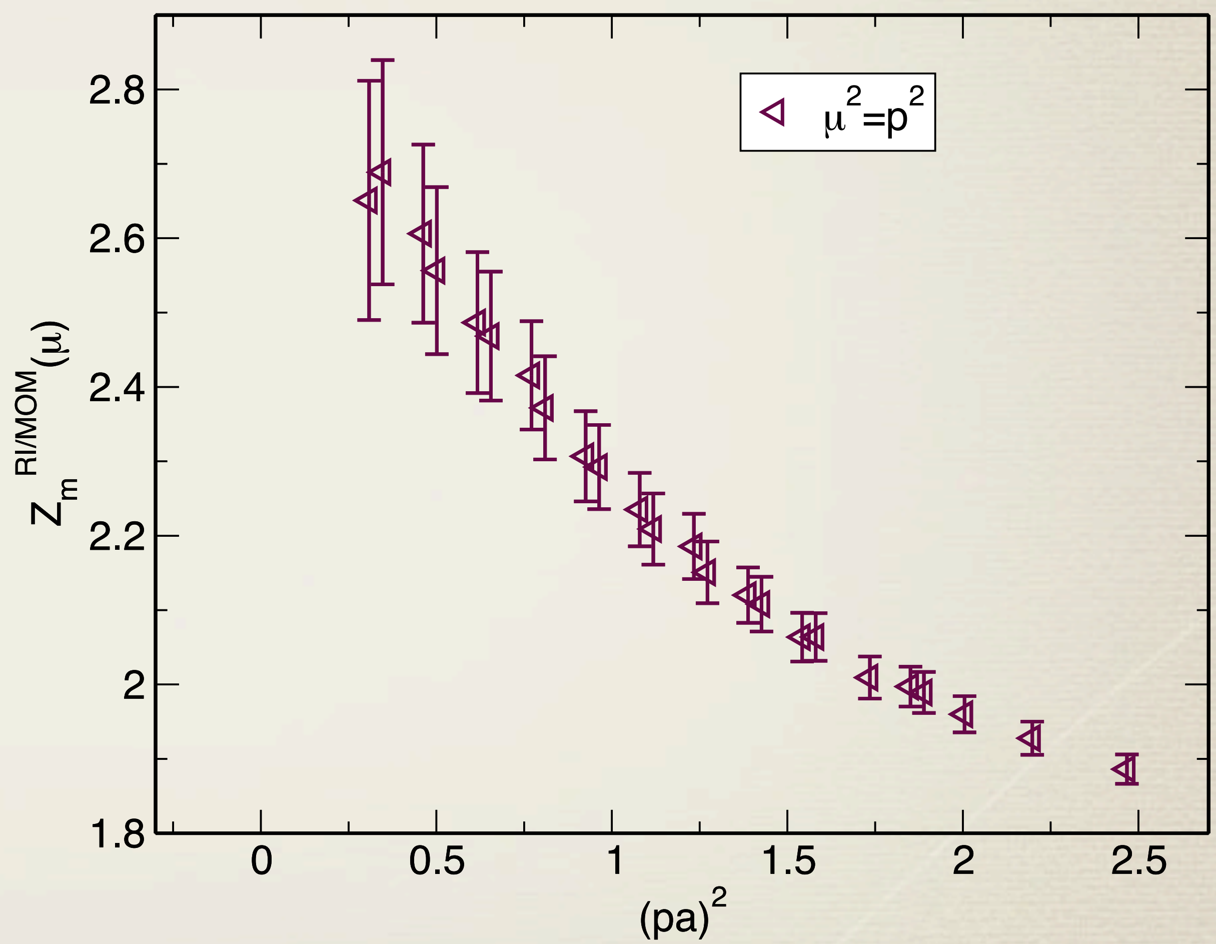
SMOM kinematics: small mass dependence



* $m_s \neq 0$ effect is estimated as 0.1-0.2 % level

RI/MOM procedure

RBC/UKQCD
n_f=2+1 DWF, 1/a≈1.7 GeV



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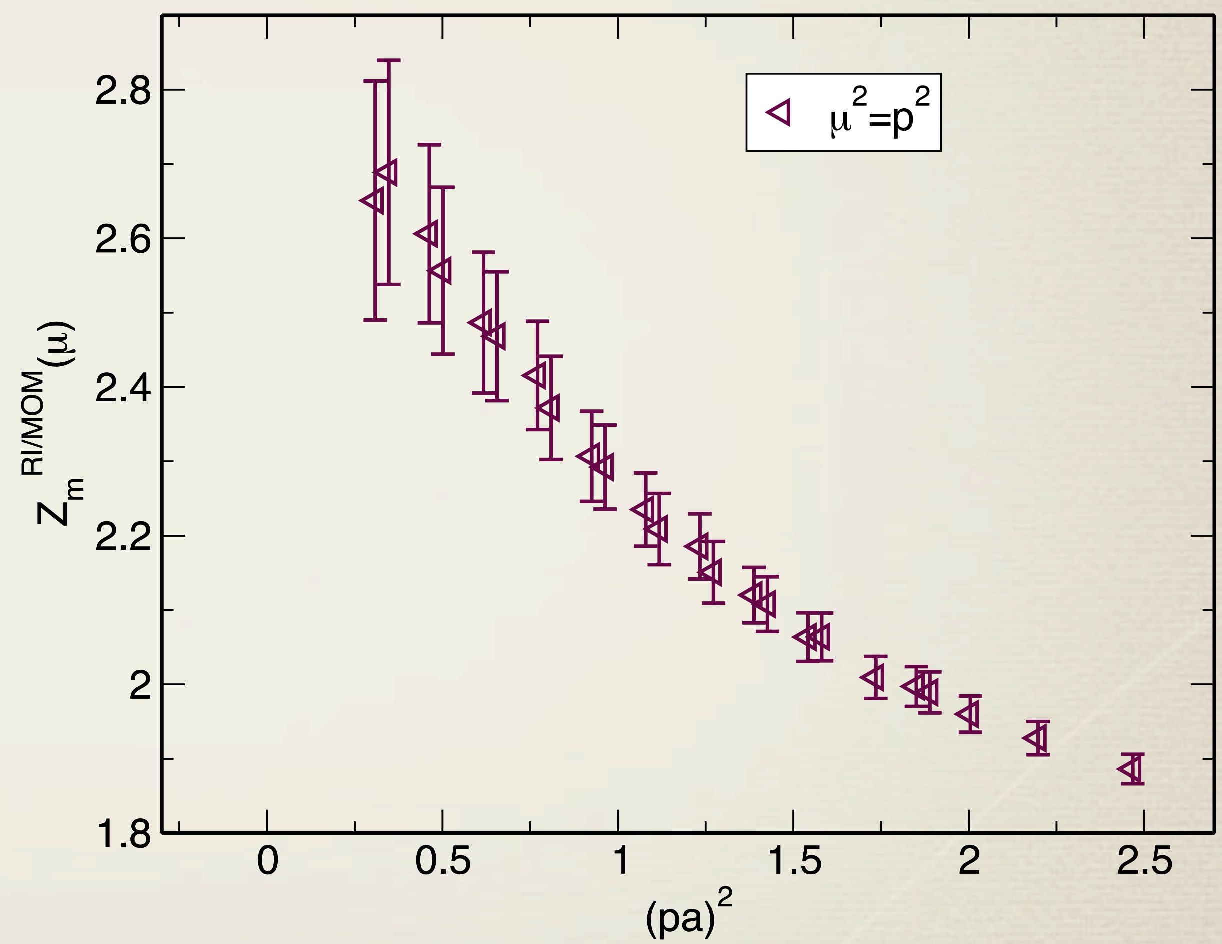
RI/MOM procedure

* $Z_m^{RI}(\mu)$ non-perturbative μ dependence

RBC/UKQCD
n_f=2+1 DWF, 1/a≈1.7 GeV



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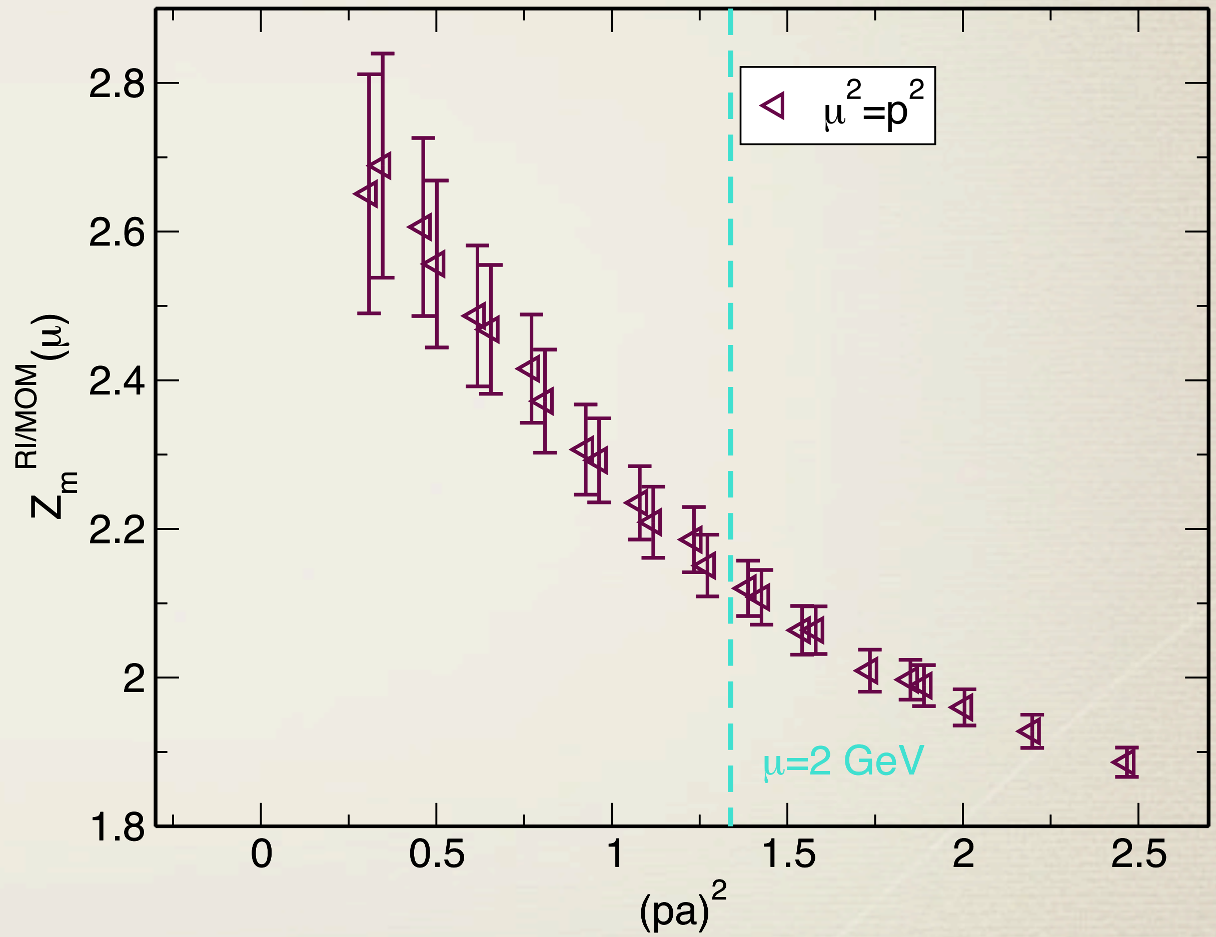
RI/MOM procedure

- $Z_m^{RI}(\mu)$ non-perturbative μ dependence
- $Z_m^{RI}(2\text{ GeV})$ from p=2 GeV intercept

RBC/UKQCD
n_f=2+1 DWF, 1/a≈1.7 GeV



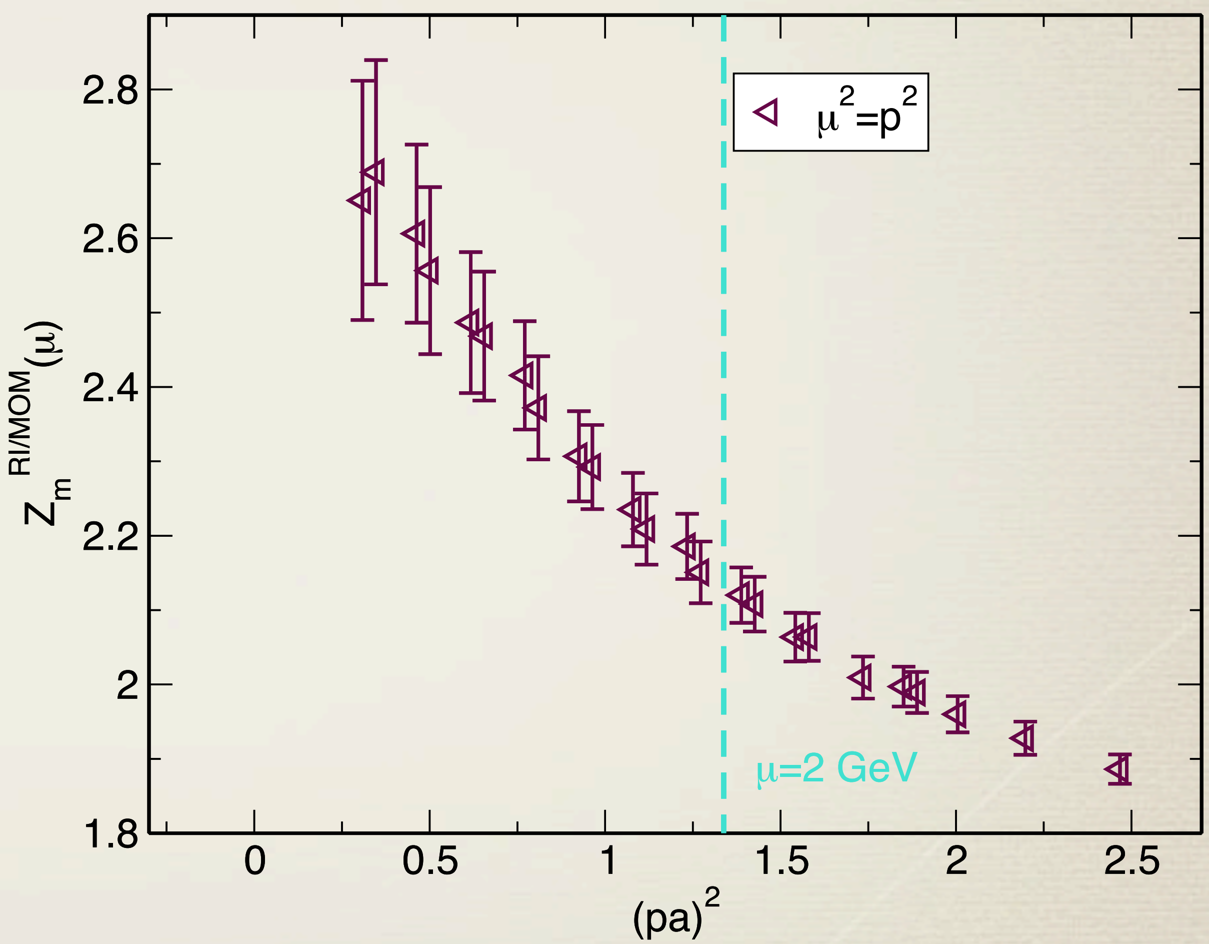
RI/MOM procedure

- ✳ $Z_m^{RI}(\mu)$ non-perturbative μ dependence
- ✳ Z_m^{RI}(2 GeV) from p=2 GeV intercept

$$Z_m^{\overline{MS}}(\mu) = \left(\frac{Z_m^{\overline{MS}}(\mu)}{Z_m^{RI}(\mu)} \right)_{PT} Z_m^{RI,NPR}(\mu)$$

RBC/UKQCD

n_f=2+1 DWF, 1/a≈1.7 GeV



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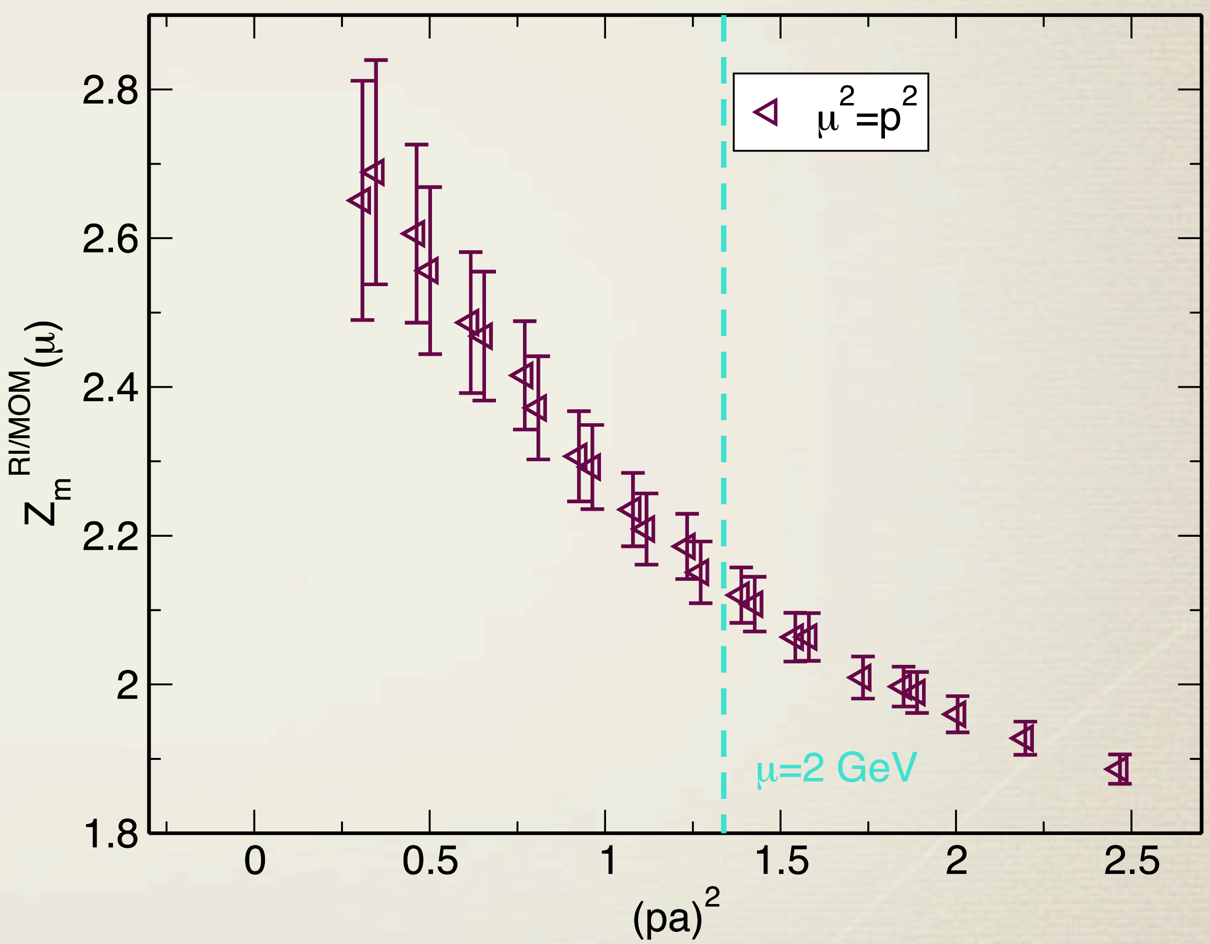
RBC/UKQCD

n_f=2+1 DWF, 1/a≈1.7 GeV



2011年5月2日月曜日

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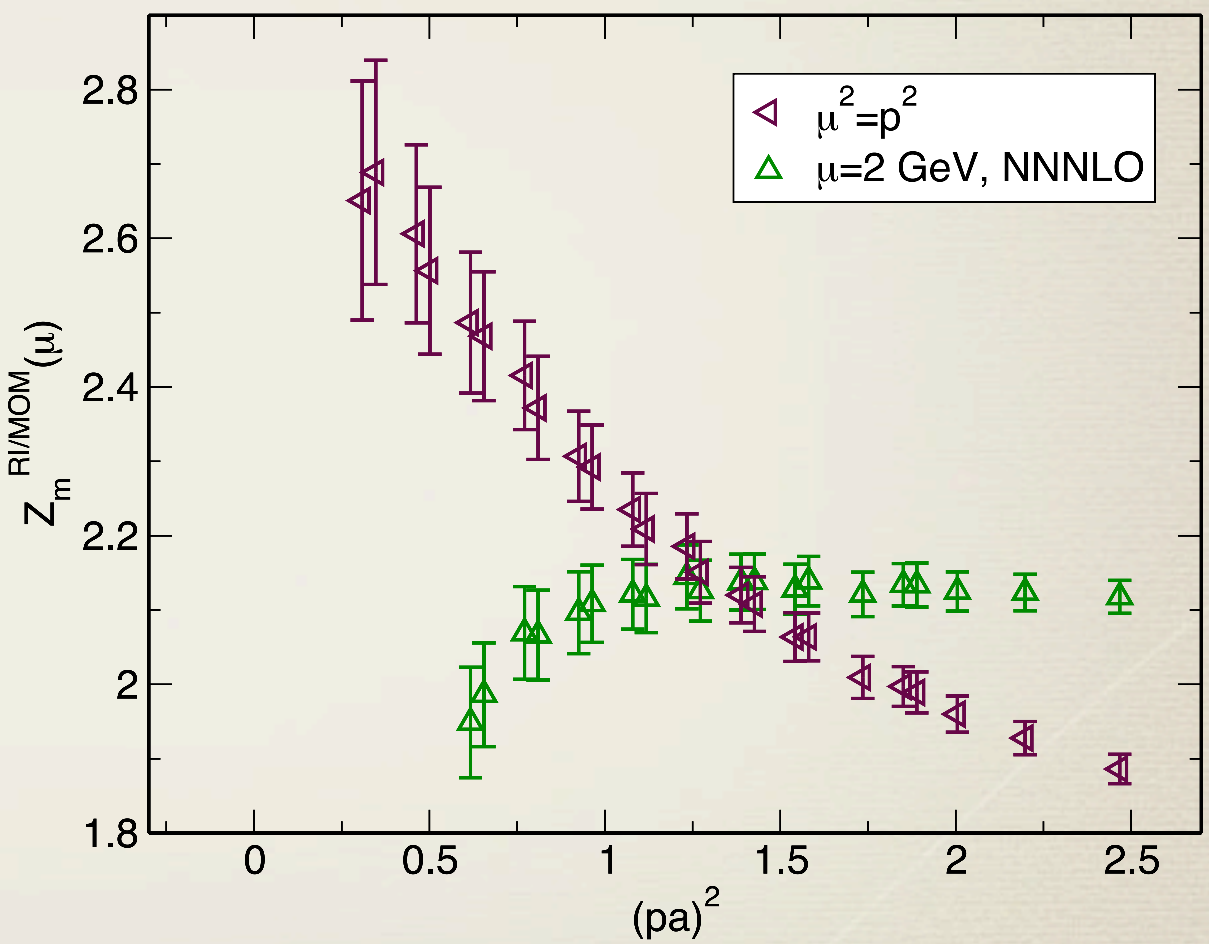

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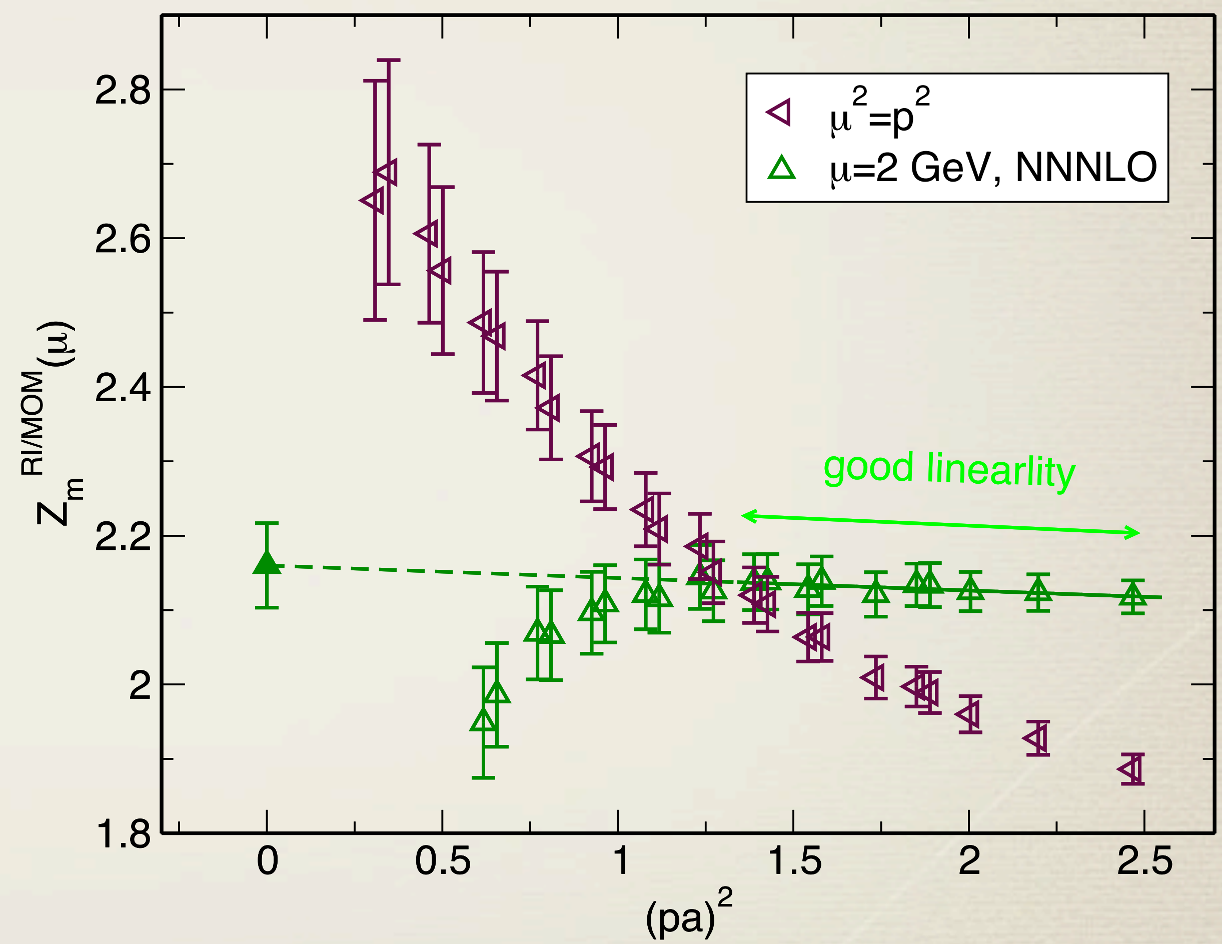

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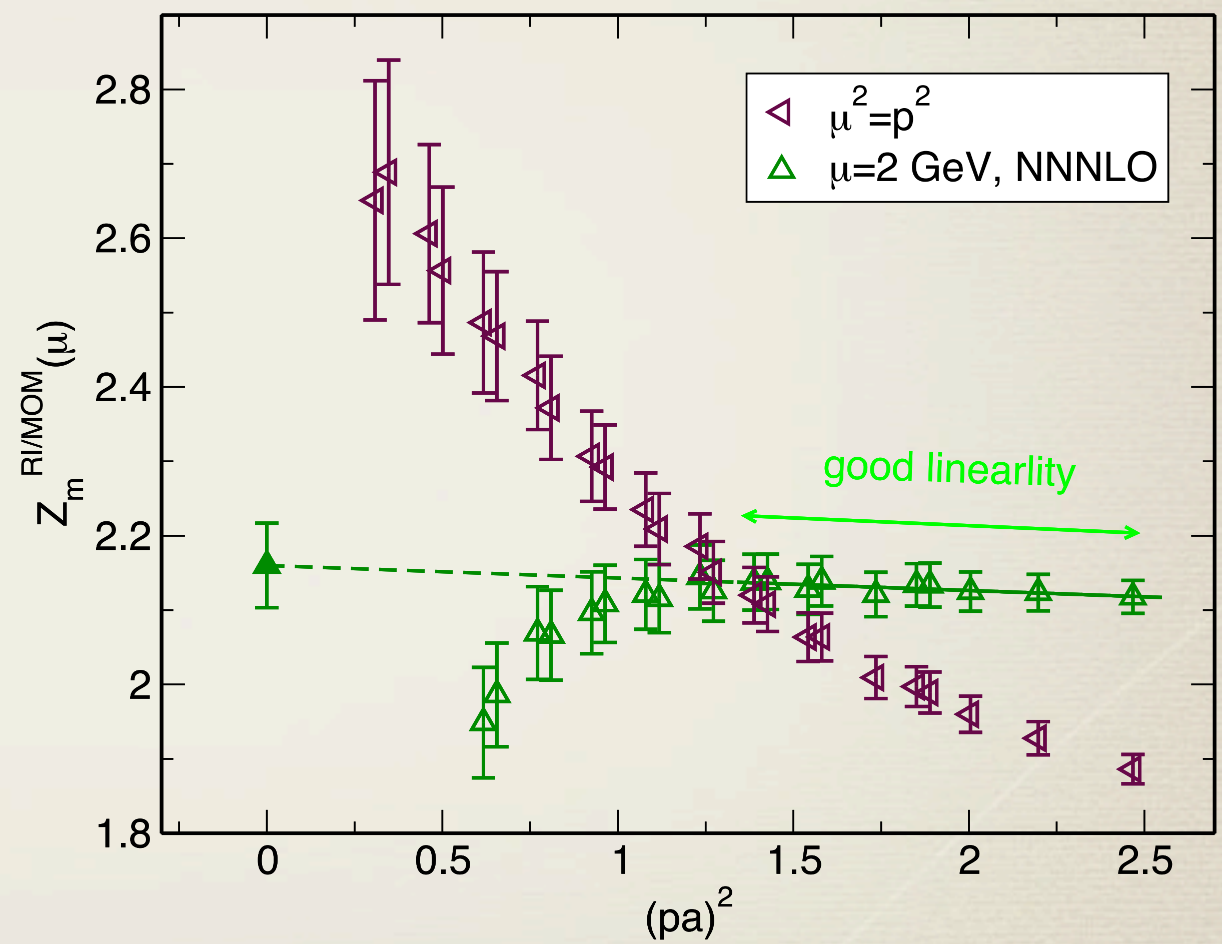

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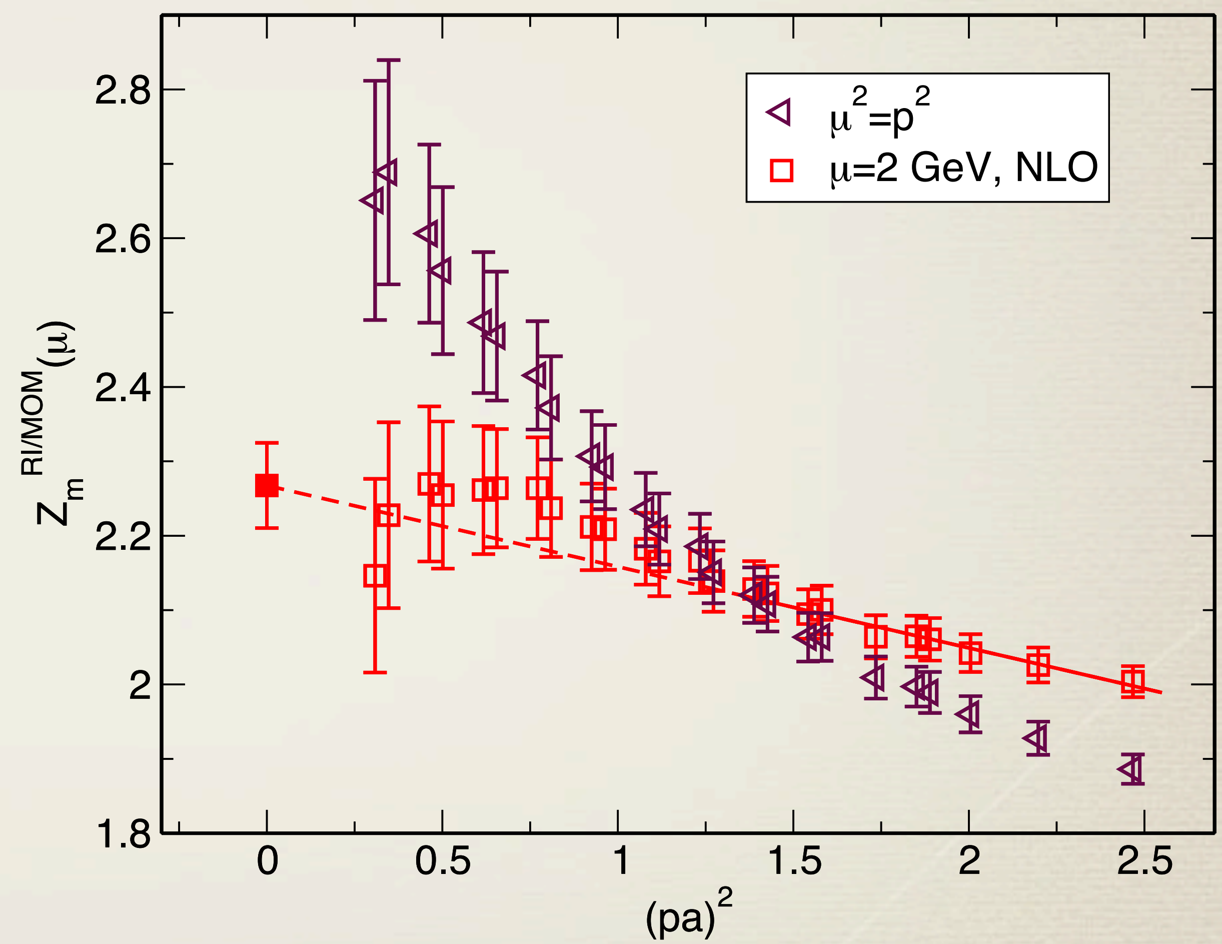

2011年5月2日月曜日

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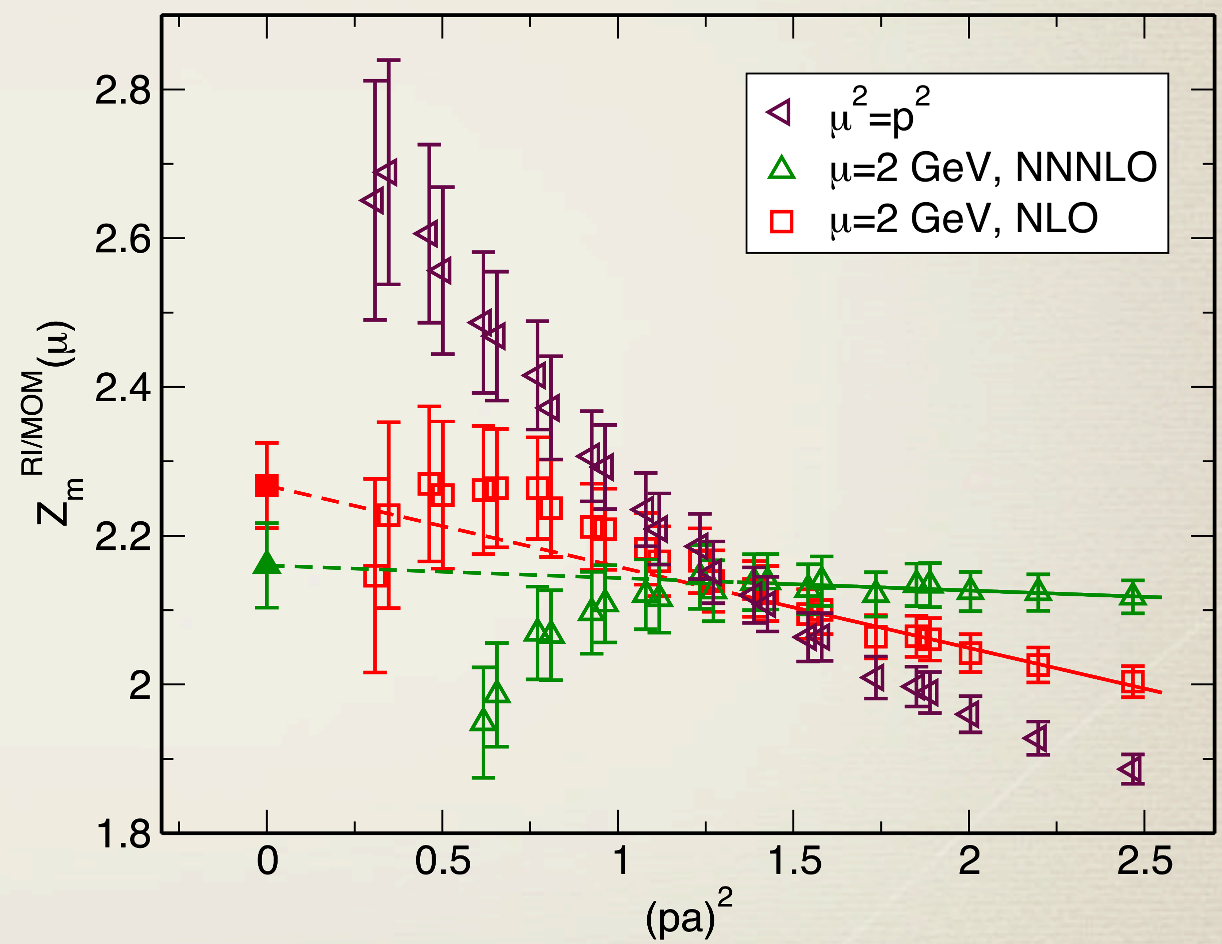

2011年5月2日月曜日

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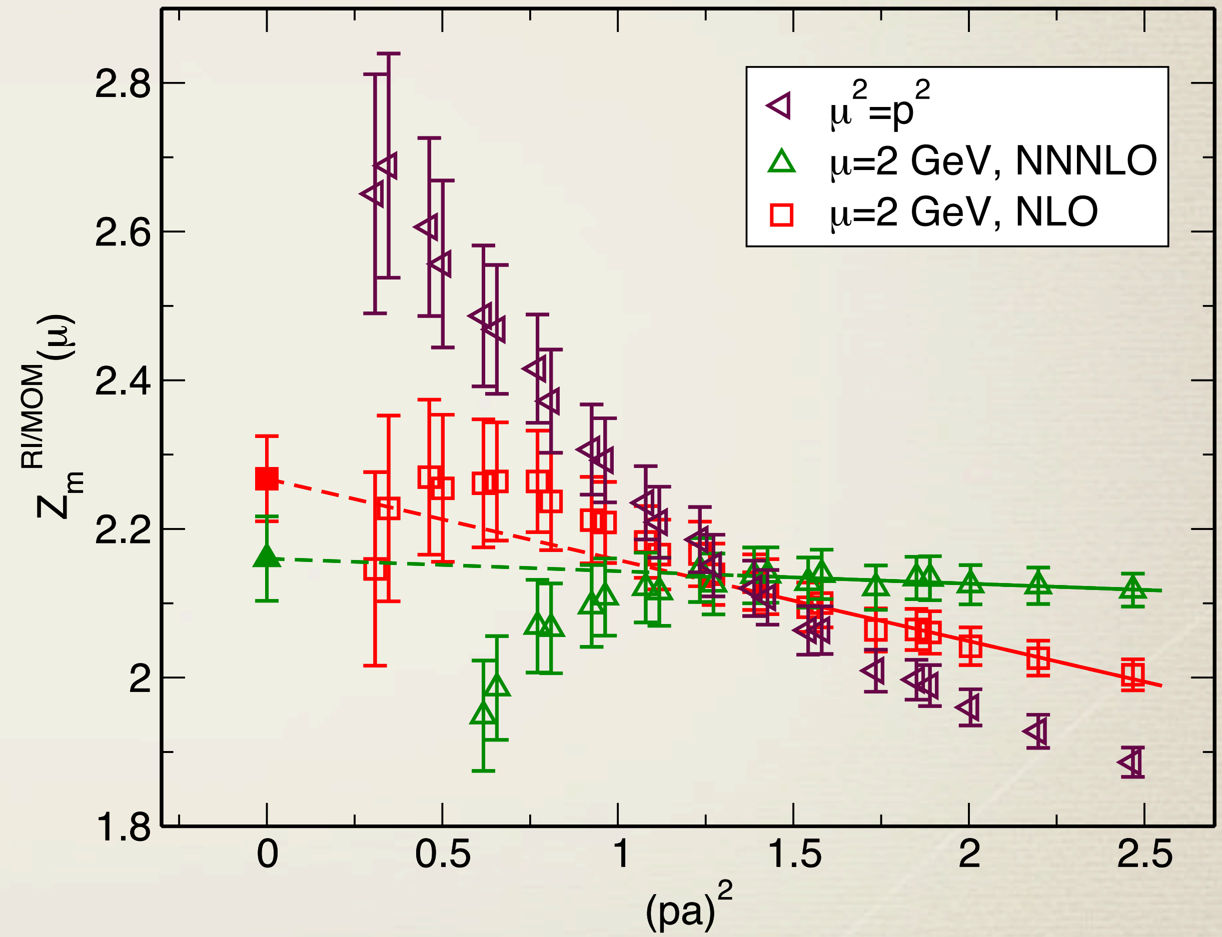

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- ✳ $(pa)^2 \to 0$ using data in the window
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- ✳ What if only known to NLO ? (before Franco and Lubicz 1998)
  - linear extrapolation wrong!
  - Be careful if you observe curvature or large slope
  - a way out: no extrapolation, take p=2 GeV value & add variation 2GeV→0 to systematic error

RBC/UKQCD
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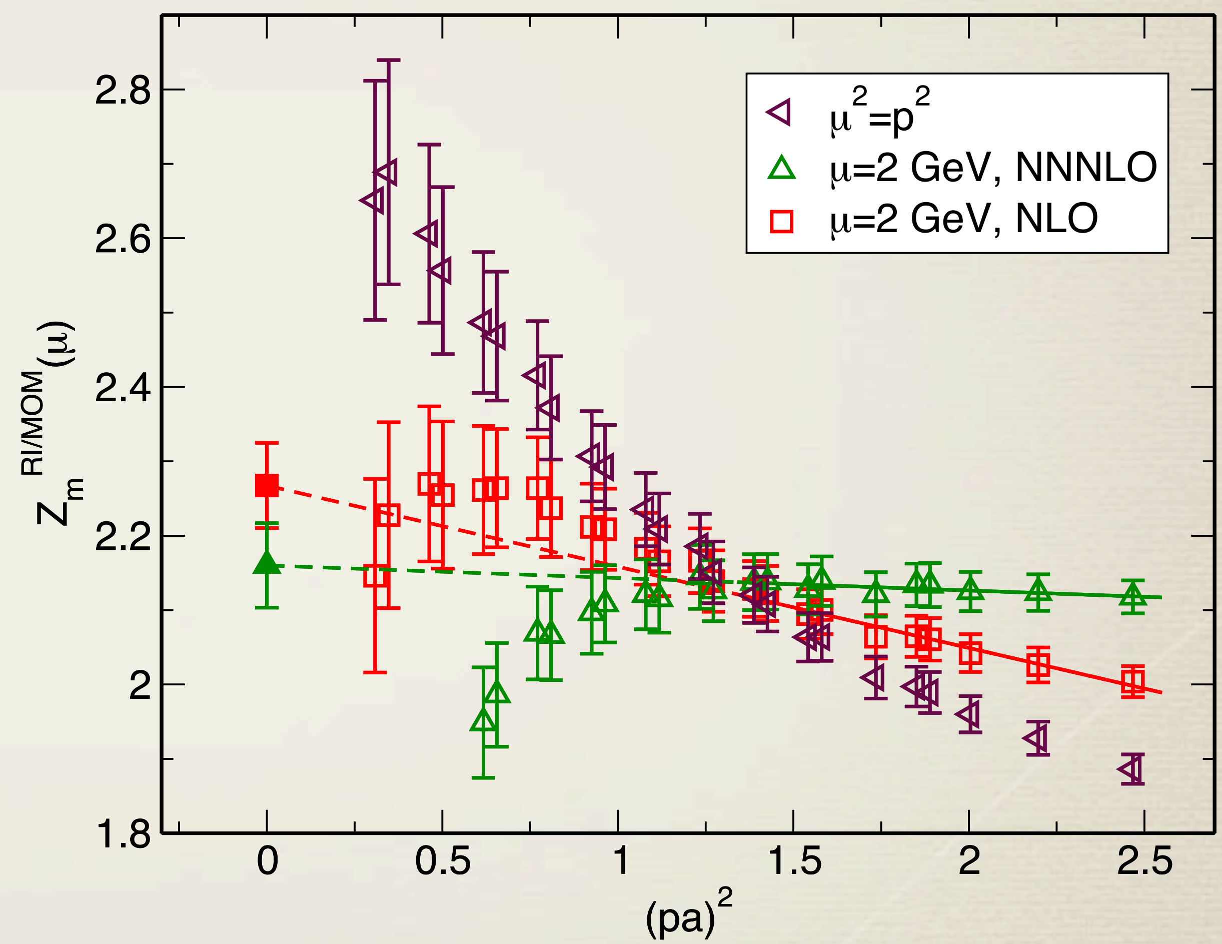

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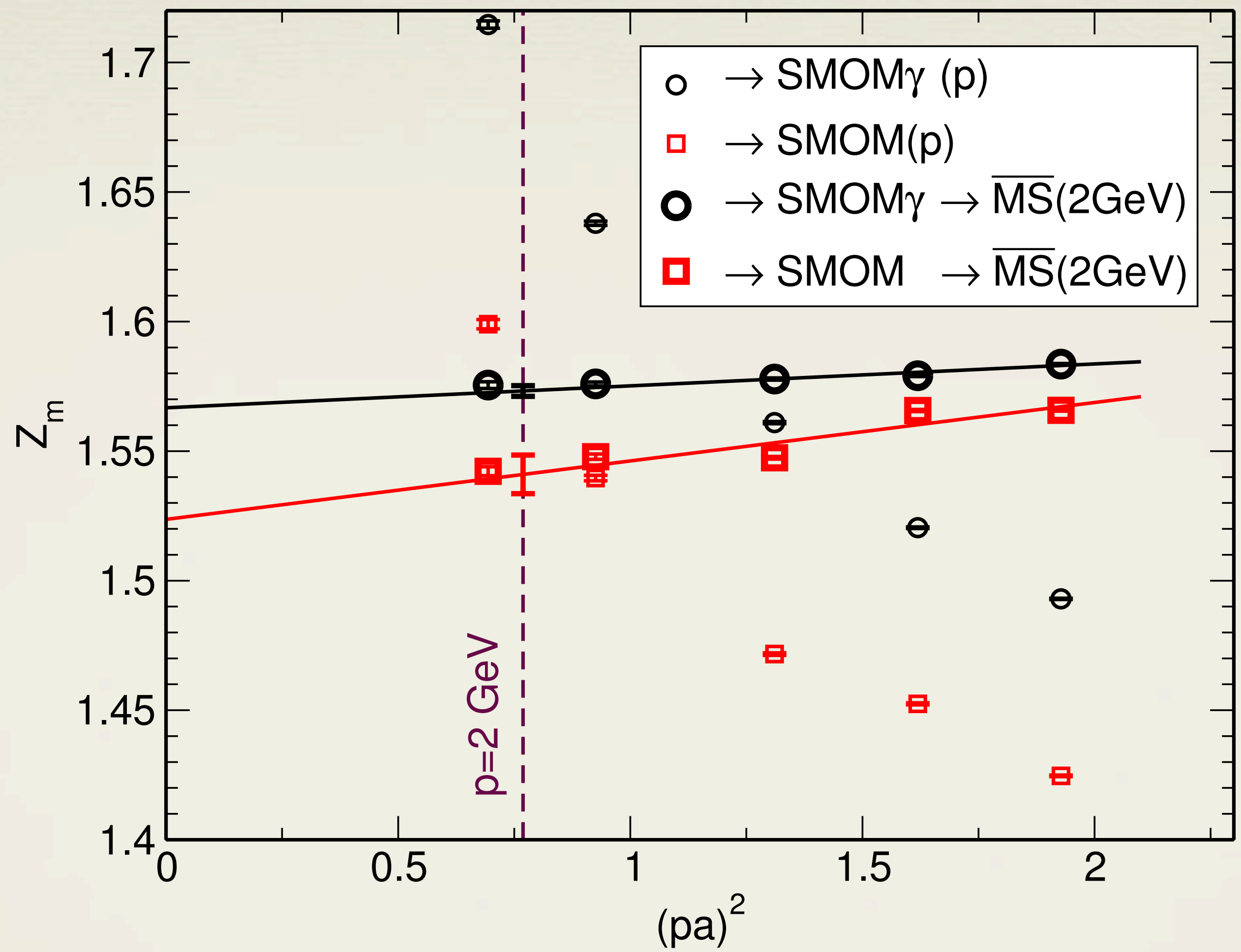
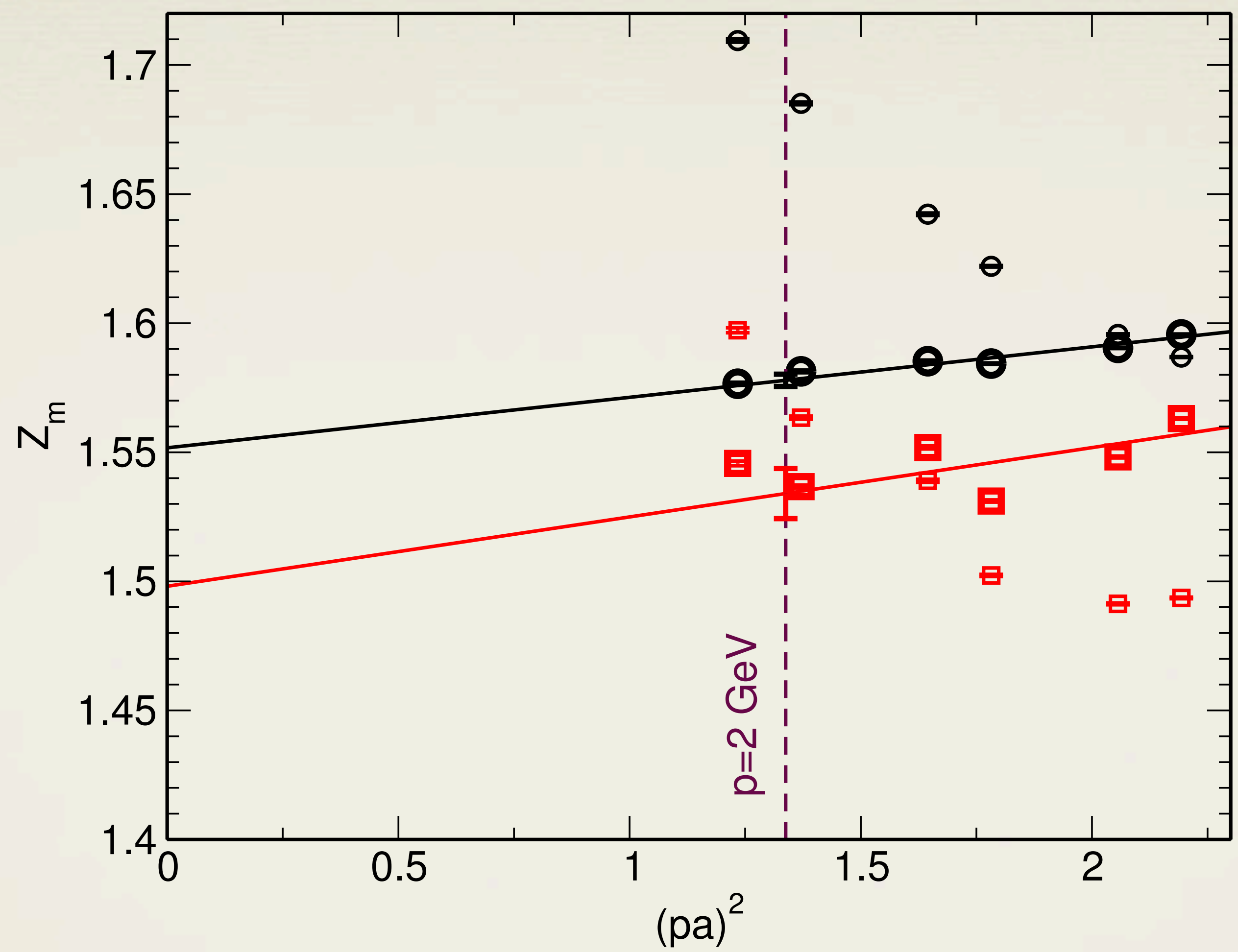
SMOM mass renormalization: 32^3



* larger O(4) breaking observed for SMOM
* take SMOMγ_μ for the intermediate scheme, use SMOM for sys error estimate

SMOM mass renormalization: 24^3



2011年5月2日月曜日

SMOM results

$$Z_m^{\overline{\rm MS}(32)}(\mu = 2\ {\rm GeV}, n_f = 3; {\rm SMOM}_{\gamma_\mu}) = 1.573(2),$$

$$Z_m^{\overline{\rm MS}(24)}(\mu = 2\ {\rm GeV}, n_f = 3; {\rm SMOM}_{\gamma_\mu}) = 1.578(2),$$

- taking the continuum limit of Z(32), Z(24)/Z_l or Z(24)/Z_h (to get rid of $O(p^2a^2)$ error)

$$Z_{mh}^{\overline{\rm MS}(32)c}(\mu = 2\ {\rm GeV}, n_f = 3; {\rm SMOM}_{\gamma_\mu}) = 1.510(6),$$

$$Z_{mh}^{\overline{\rm MS}(32)c}(\mu = 2\ {\rm GeV}, n_f = 3; {\rm SMOM}) = 1.495(22)$$

- sys error due to PT truncation may be estimated from the difference or the size of the highest order (2-loop) term of PT matching

Z_m error budget

ensemble	fine (32^2)	coarse (24^3)	coarse (16^3)[13]
intermediate scheme	RI/SMOM	RI/SMOM	RI/MOM
PT truncation error	2.1%	2.1%	6%
$m_s \neq 0$	0.1%	0.2%	7%
$(\Lambda_P - \Lambda_S)/2$	0.5%	0.6%	N.A. (∞)
$(\Lambda_A - \Lambda_V)/2$	0.0%	0.0%	1%
total	2.2%	2.2%	9%

the light quark mass results

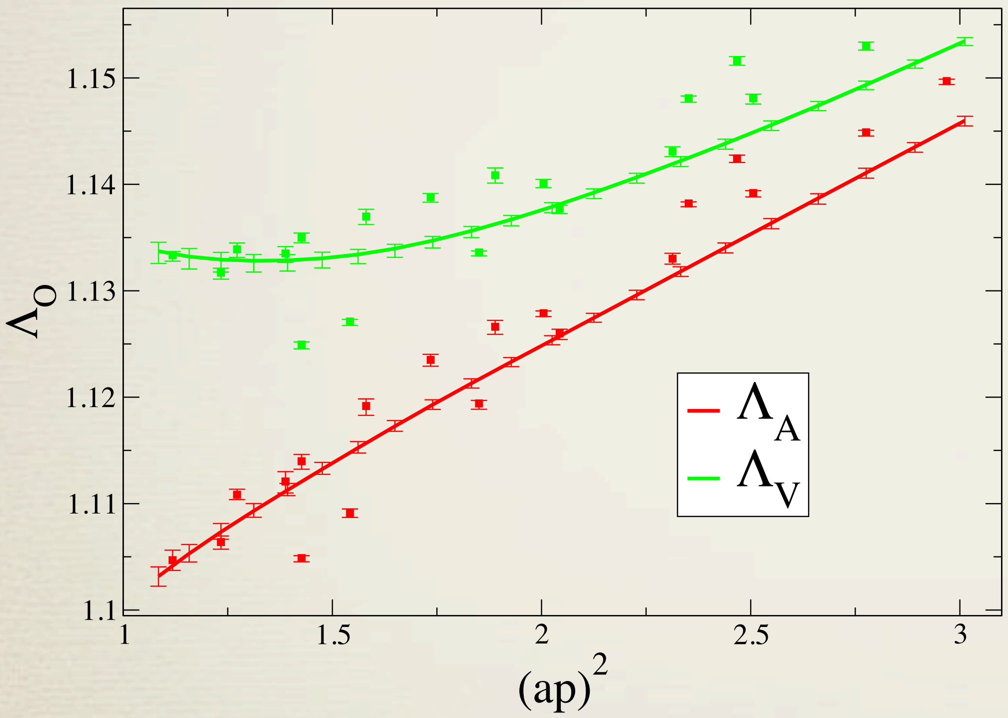
$$
\begin{aligned}
m_{ud}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) &= Z_{ml}^{\overline{\mathrm{MS}}(32)c}(\mu = 2\,\mathrm{GeV}, n_f = 3) \cdot \tilde{m}_{ud}(32^3) \cdot a^{-1}(32^3) \\
&= 3.59(13)_{\mathrm{stat}}(14)_{\mathrm{sys}}(8)_{\mathrm{ren}}\ \mathrm{MeV}, \\
m_{s}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= Z_{mh}^{\overline{\mathrm{MS}}(32)c}(\mu = 2\mathrm{GeV}, n_f = 3) \cdot \tilde{m}_{s}(32^3) \cdot a^{-1}(32^3) \\
&= 96.2(1.6)_{\mathrm{stat}}(0.2)_{\mathrm{sys}}(2.1)_{\mathrm{ren}}\ \mathrm{MeV},
\end{aligned}
$$

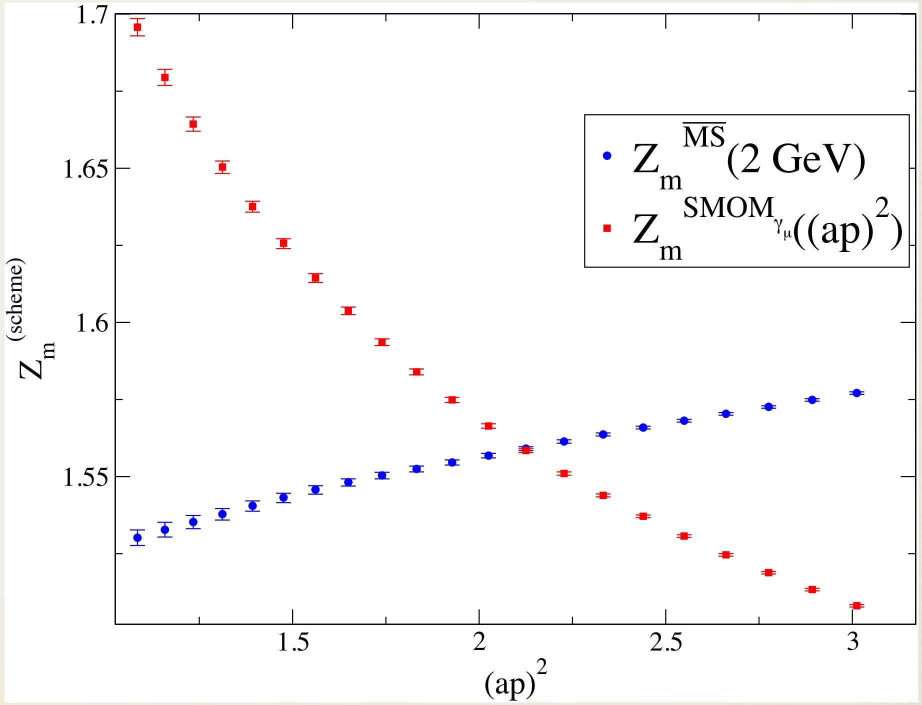
Outlook

- more chiral DSDR:
  - smaller masses (down to unitary m_π≃180 MeV) larger volume, but, coarser lattice
    - with twisted mass to reduce m_{res}
  - in principle, just adding another ensemble with different a^2
  - no need of New NPR if 32^3 is used again as the primary ensemble
  - but...

Outlook

* promising new idea of twisted boundary RI/MOM scheme by R. Arthur, P. Boyle.
    * fix base momentum & twist boundary to smoothly change the momentum
    * ➡ no O(4) breaking effect in p^2 dependence → much precise handling possible





* step scaling tested
* Further improvements are expected!!!

VIELEN DANK!