Quark masses from lattice QCD

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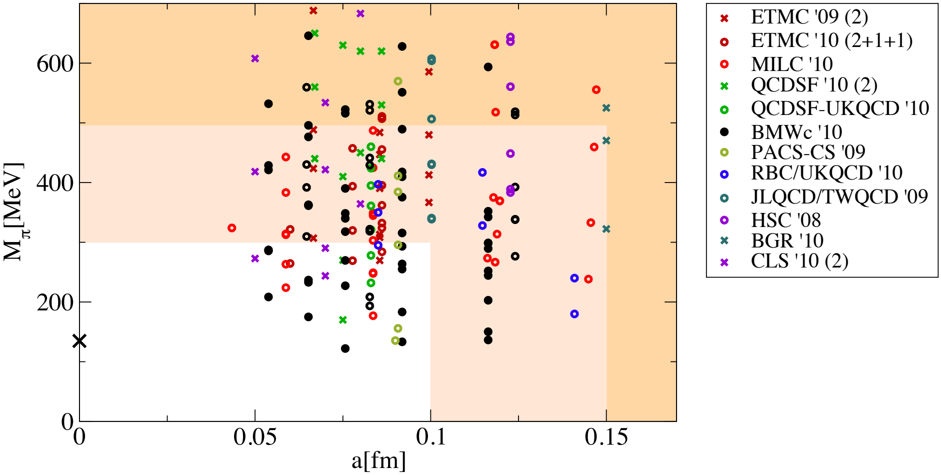
SFB TR55

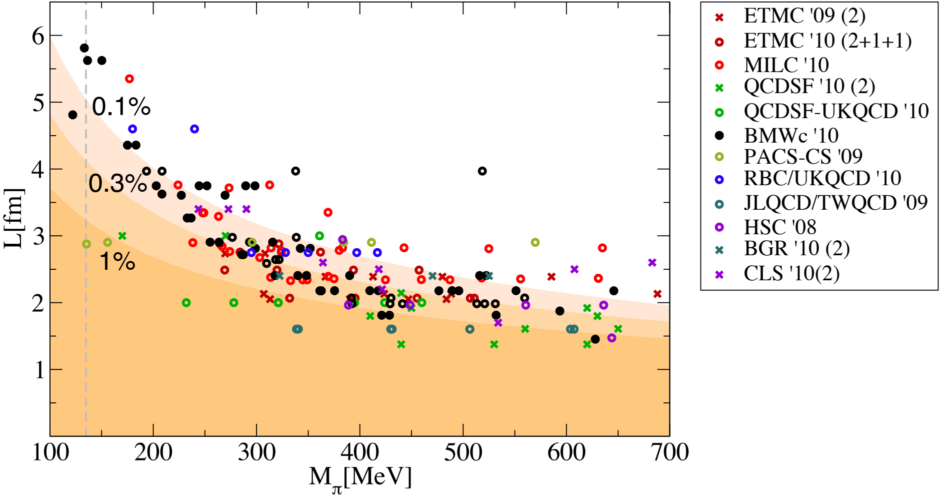
Strategy outline

- Goal:
  - Compute light quark masses ab initio
- Relevance:
  - Fundamental SM parameters
  - Stability of matter depends on their values
  - Not obtainable perturbatively
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Continuum extrapolation
    - Nonperturbative renormalization
    - Infinite volume

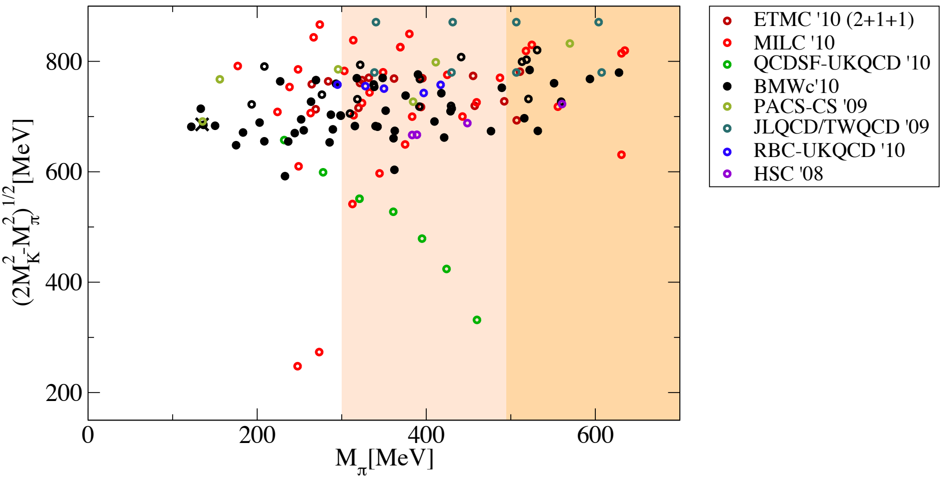
Landscape M_π vs. a



Landscape L vs. M_π



Landscape M_K vs. M_π



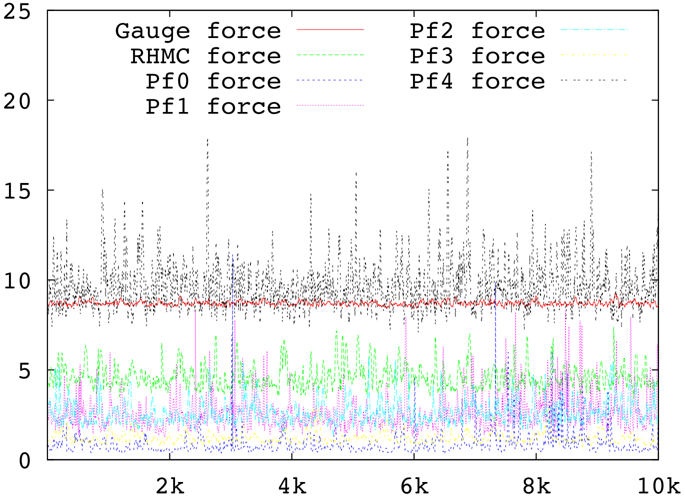
Action details

- Goal:
  - Optimize physics results per CPU time
  - Conceptually clean formulation
- Method
  - Dynamical $2+1$ flavor, Wilson fermions at physical M_π
  - 5 lattice spacings $0.053\text{ fm} < a < 0.125\text{ fm}$
  - Tree level $O(a^2)$ improved gauge action (Lüscher, Weisz, 1985)
  - Tree level $O(a)$ improved fermion action (Sheikholeslami, Wohlert, 1985)
  - UV filtering (APE coll. 1985; Hasenfratz, Knechtli, 2001; Capitani, Durr, C.H., 2006)

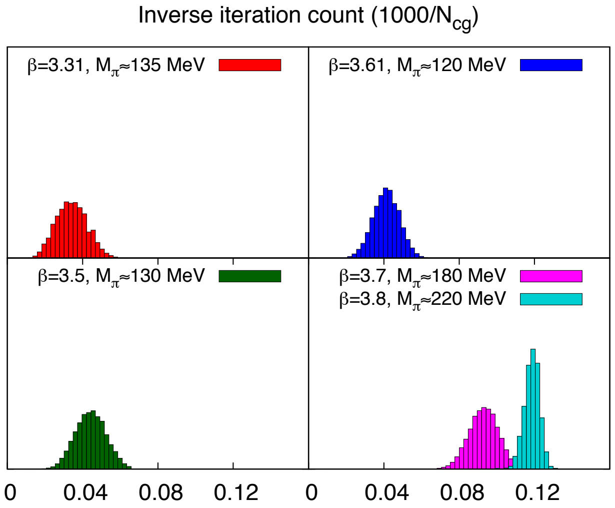
  ➛ Discretization effects of $O(\alpha_s a, a^2)$

  ✓ We fully include both $O(\alpha_s a)$ and $O(a^2)$ into systematic error

Algorithm stability

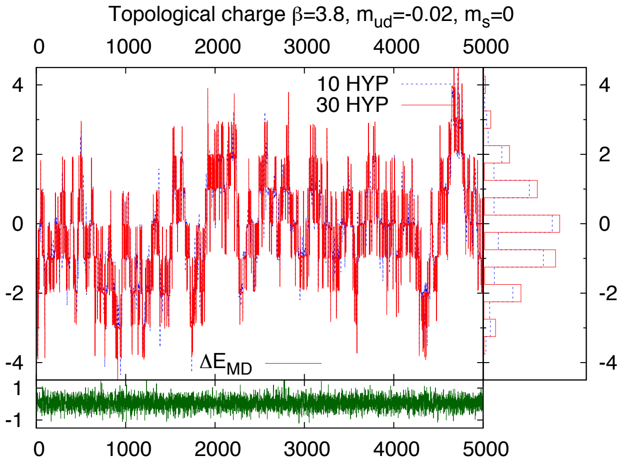


No exceptional configs



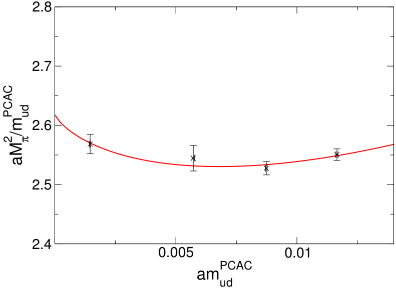
Topological sector sampling

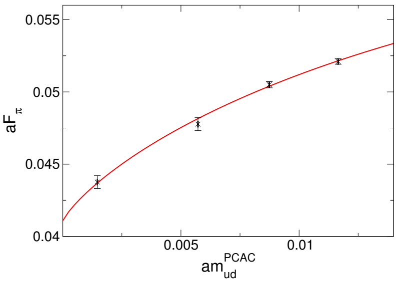
$\tau_{int} \sim O(10)$



Chiral interpolation

- Simultaneous fit to NLO $SU(2)$ χPT (Gasser, Leutwyler, 1984)
- Consistent for $M_\pi \lesssim 400$ MeV





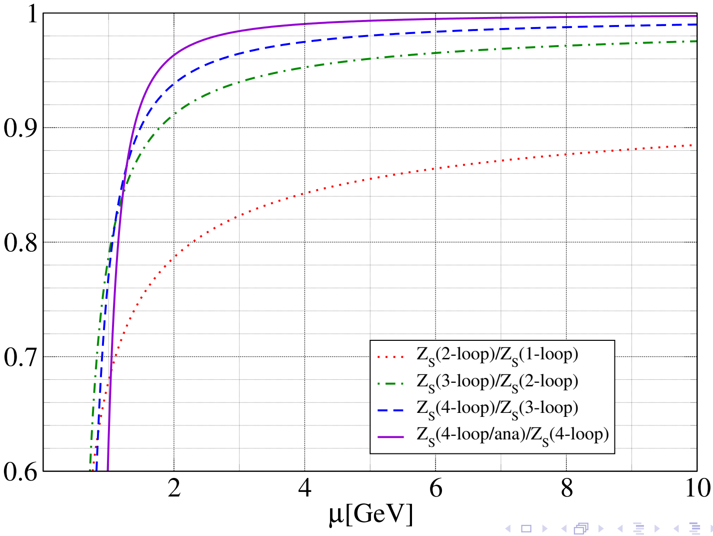
➢ We use 2 safe chiral interpolation ranges: $M_\pi < 340, 380$ MeV

➢ We use $SU(2)$ χPT and Taylor interpolation forms

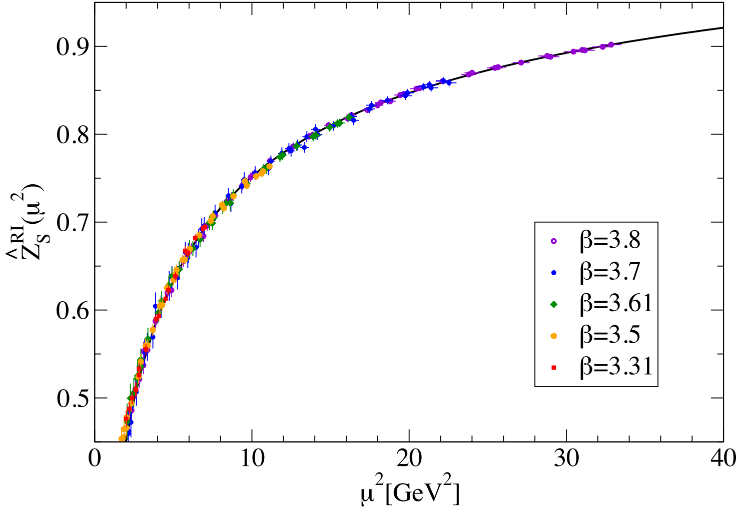
Renormalization strategy

- Goal:
  - Full nonperturbative renormalization
  - Optional accurate conversion to perturbative scheme
- Method:
  - We use RI-MOM scheme (Martinelli et. al., 1993)
    - $O(a)$ correction (Maillart, Niedermayer, 2008)
  - Compute m_q at low scale $\mu \ll 2\pi/a \sim 11 - 24$ GeV
    - $\mu = 2.1 GeV$
    - $\mu = 1.3 GeV$
  - Do continuum non-perturbative running to high scale $\mu' \gg \Lambda_{QCD}$
  - Further conversion in 4-loop PT

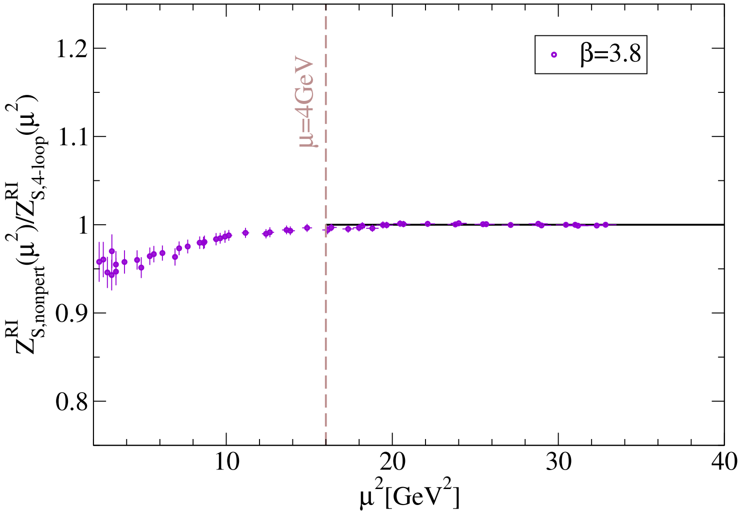
Desired scale in RI-MOM scheme



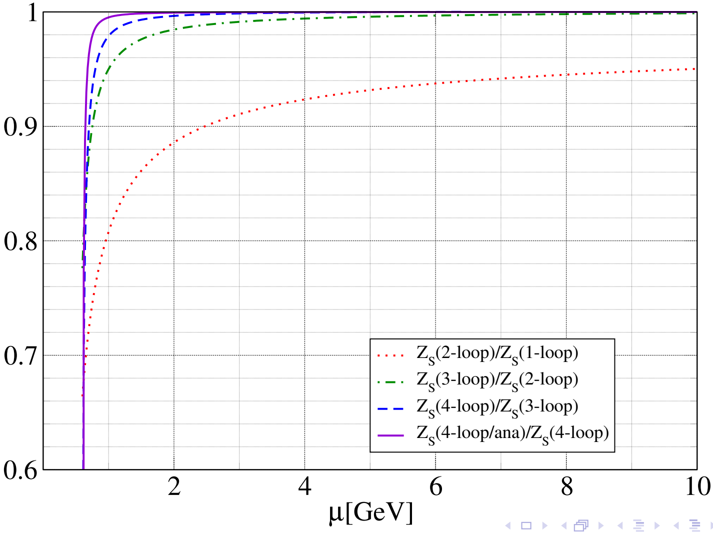
Nonperturbative running



Reaching the perturbative regime



Optional conversion to $\overline{\text{MS}}$



Quark mass definitions

- Lagrangian mass m^{bare}
- $m^{\text{ren}} = \frac{1}{Z_S}(m^{\text{bare}} - m^{\text{bare}}_{\text{crit}})$

- m^{PCAC} from $\frac{\langle \partial_0 A_0 P \rangle}{\langle P(t) P(0) \rangle}$
- $m^{\text{ren}} = \frac{Z_A}{Z_P} m^{\text{PCAC}}$

Better use

- $d = m_s^{\text{bare}} - m_{ud}^{\text{bare}}$
- $d^{\text{ren}} = \frac{1}{Z_S} d$

- $r = m_s^{\text{PCAC}} / m_{ud}^{\text{PCAC}}$
- $r^{\text{ren}} = r$

and reconstruct

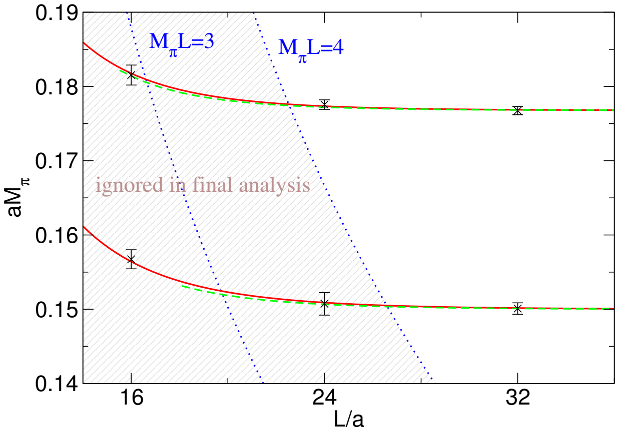
- $m_s^{\text{ren}} = \frac{1}{Z_S} \frac{rd}{r-1}$
- $m_{ud}^{\text{ren}} = \frac{1}{Z_S} \frac{d}{r-1}$

✓ No additive mass renormalization and ambiguity in m_{crit}

✓ Only Z_S multiplicative renormalization (no pion poles)

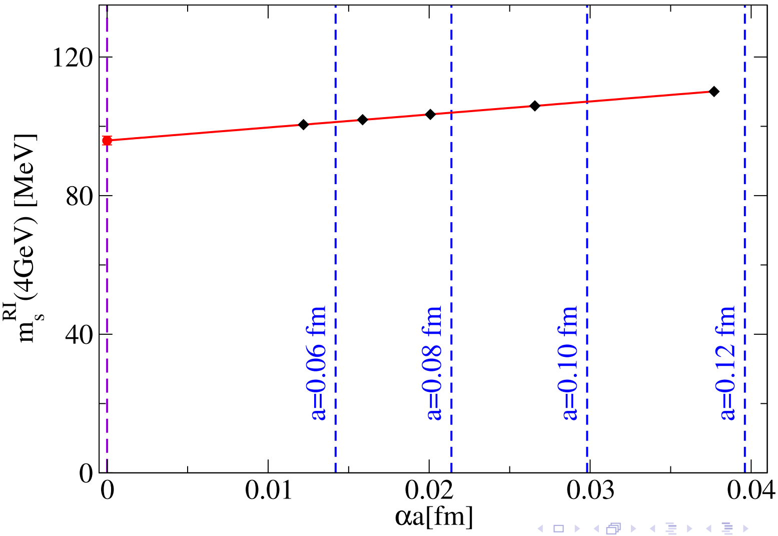
☞ Works with $O(a)$ improvement (we use this)

Tiny finite volume effects

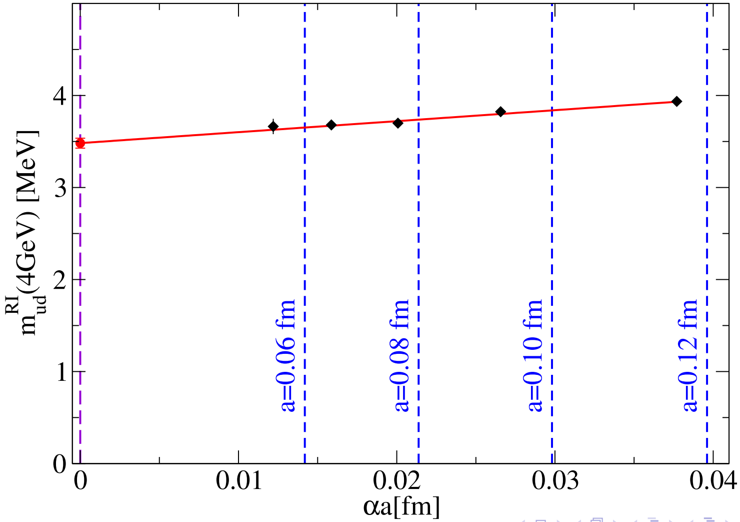


- FV effects tiny
- Dedicated FV runs
- Perfect agreement with FV χPT (Colangelo et. al. 2005)

Strange quark mass



Light quark masses



Individual m_u and m_d

- Goal:
  - Compute m_u and m_d separately
- Method:
  - Need QED and isospin breaking effects in principle
  - Alternative: use dispersive input -Q from $\eta \to \pi\pi\pi$

    $$Q^2 = \frac{1}{2}\left(\frac{m_s}{m_{ud}}\right)^2 \frac{m_d - m_u}{m_{ud}}$$

  - ✓ Transform precise m_s/m_{ud} into $(m_d - m_u)/m_{ud}$
  - We use the conservative $Q = 22.3(8)$ (Leutwyler, 2009)

Systematic error treatment

- Goal:
  - Reliably estimate total systematic error
- Method:
  - 288 full analyses (2000 bootstrap on each)
    - 2 plateaux regions
    - 2 continuum forms: $O(\alpha_s a)$, $O(a^2)$
    - 3 chiral forms: $2 \times SU(2)$, Taylor
    - 2 chiral ranges: $M_\pi < 340, 380$ MeV
    - 3 renormalization matching procedures
    - 2 NP continuum running forms
    - 2 scale setting procedures
  - All analyses weighted by fit quality
    - Mean gives final result
    - Stdev gives systematic error
  - Statistical error from 2000 bootstrap samples

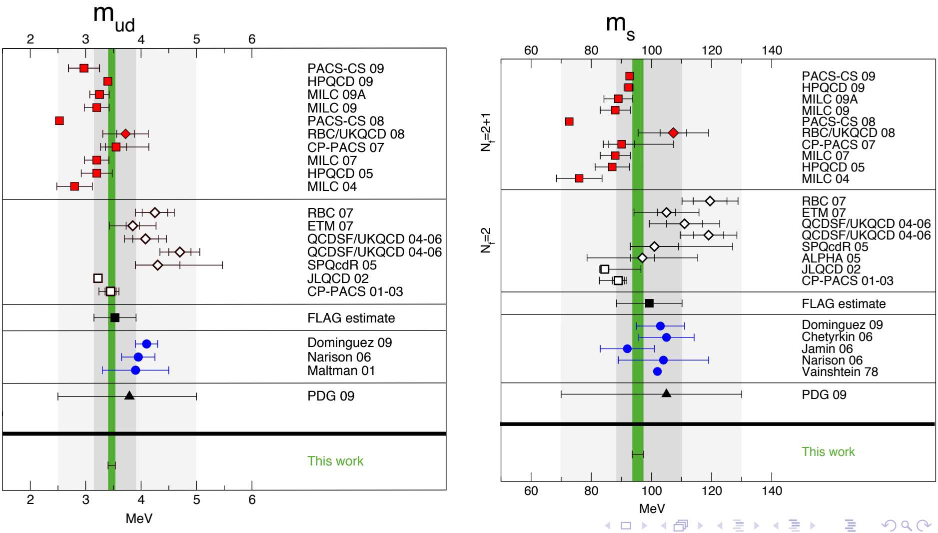
Final result

	RI @ 4 GeV	RGI	$\overline{\text{MS}}$ @ 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_u	2.17(04)(10)	2.86(05)(13)	2.15(03)(10)
m_d	4.84(07)(12)	6.39(09)(15)	4.79(07)(12)

Additional consistency checks:

- ✓ Use m^{PCAC} only, no ratio-difference method
  ☞compatible, slightly larger error
- ✓ Unweighted final result and systematic error
  ☞negligible impact
- ✓ Additional Continuum, chiral and FV terms
  ☞all compatible with 0

Comparison


m_ud
2 3 4 5 6
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Narison 06
Maltman 01
PDG 09
This work
MeV
m_s
60 80 100 120 140
N_f=2+1
N_f=2
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
ALPHA 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Chetyrkin 06
Jamin 06
Narison 06
Vainshtein 78
PDG 09
This work
MeV