Nuclear Force from Quarks and Gluons

Sinya AOKI

University of Tsukuba

“Japan Days” Colloquium, May 2, 2011, University of Wuppertal

The 150th anniversary of the Friendship Treaty between Japan and Germany
Friedrich Albrecht Graf zu Eulenburg

Dampfcorvette "Arcona" in der Bucht von Yokohama

Erste Seite des Freundschafts-, Handels- und Schiffsahrtvertrages zwischen Preußen und Japan, 24.1.1861

Deutsche Gesandtschaft in Tokyo, Eingangstor (1875)

Hochzeitsgesellschaft des Deutschen Generalkonsuls, Herrn von Syburg, in der Deutschen Gesandtschaft 1905

Wilhelm Solf, Erster Botschafter Deutschlands in Japan (1920-28) nach dem Ersten Weltkrieg

Unter den Gästen: Erwin Bälz (hintere Reihe, 2. v. rechts) und Hana Bälz (vorderste Reihe ganz rechts)
It is my great pleasure and honor to give a talk at the colloquium on this special occasion, the “Japan Days”.

First of all, I would like to express my deepest appreciation for supports and encouragements from all over the world, in particular from Germany, to the peoples in Japan. We are still struggling against tragedies caused by the Earthquake and Tsunami. I however strongly believe that we will be able to overcome these difficult situations, together with your great help.
My collaborators

HAL QCD Collaboration

S. Aoki (Tsukuba)
T. Doi (Tsukuba)
T. Hatsuda (Tokyo)
Y. Ikeda (Riken)
T. Inoue (Nihon)
N. Ishii (Tokyo)
K. Murano (KEK)
H. Nemura (Tohoku)
K. Sasaki (Tsukuba)
1. Motivation
What binds protons and neutrons inside a nuclei?

- gravity: too weak
- Coulomb: repulsive between pp
- no force between nn, np

New force (nuclear force)?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force

Yukawa potential

\[ V(r) = \frac{g^2}{4\pi} \frac{e^{-m_\pi r}}{r} \]

1949 Nobel prize
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei
- Structure of neutron star
- Ignition of Type II SuperNova
Phenomenological NN potential
(~40 parameters to fit 5000 phase shift data)
Repulsive core is important

stability of nuclei
maximum mass of neutron star
explosion of type II supernova

Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.

QCD based explanation is needed
Lattice QCD can explain?
Plan of my talk

1. Motivation
2. Strategy in (lattice) QCD to extract “potential”
3. Nuclear potential from lattice QCD
4. More on nuclear potential
5. Hyperon interactions
6. H-dibaryon
7. Conclusion
2. Strategy in (lattice) QCD to extract “potential”

Challenge to Nambu’s statement


“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”
Definition of “Potential” in (lattice) QCD?

Previous attempt

calculate energy of $Qqq + Qqq$ as a function of $r$ between $2Q$. $Q$: static quark, $q$: light quark

Quenched result

Almost no dependence on $r$!

cf. Recent successful result in the strong coupling limit (deForcrand-Fromm, PRL104(2010)112005)
Alternative approach

Consider “elastic scattering”

\[ \text{NN} \rightarrow \text{NN} \]

\[ \text{NN} \rightarrow \text{NN} + \text{others} \]

\[ (\text{NN} \rightarrow \text{NN} + \pi, \text{NN} + \bar{\text{NN}}, \cdots) \]

Elastic threshold \( E_{\text{th}} = 2m_N + \pi \)

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives

\[ S = e^{2i\delta} \]

\[ E = 2\sqrt{k^2 + m_N^2} < E_{\text{th}} \]

- Nambu-Bethe-Salpeter (NBS) Wave function

\[ \varphi_E(r) = \langle 0|N(x + r, 0)N(x, 0)|6q, E\rangle \]

QCD eigen-state with energy E and \#quark = 6

\[ N(x) = \varepsilon_{abc}q^a(x)q^b(x)q^c(x): \text{local operator} \]
NBS wave function satisfies

\[ \varphi_E(r) = e^{ik \cdot r} + \int \frac{d^3p}{(2\pi)^3} e^{ip \cdot r} \frac{E_k + E_p}{8E_p^2} T(p, -p \leftarrow k, -k) \]

\[ + \mathcal{I}(r) \]

in the inelastic contribution

\[ \propto O\left( e^{-\sqrt{E_{th}^2 - E^2} |r|} \right) \]

**Asymptotic behavior**

\[ r = |r| \rightarrow \infty \]

\[ \varphi^l_E(r) \rightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \]

partial wave \( l = 0, 1, 2, \ldots \)

\( \delta_l(k) \) is the scattering phase shift

**Finite volume**

allowed value: \( k_n^2 \)

Lueshcer’s formula

C.-J.D.Lin et al., NPB69 (2001) 467
CP-PACS Coll., PRD71 (2005) 094504

no interaction

interaction range

Finite volume

Lueshcer’s formula

\( L \)
We define a “non-local potential”

\[
[\epsilon_k - H_0] \varphi_E(x) = \int d^3 y U(x, y) \varphi_E(y)
\]

\[
\epsilon_k = \frac{k^2}{2\mu} \quad H_0 = \frac{-\nabla^2}{2\mu}
\]

**Velocity expansion**

\[
U(x, y) = V(x, \nabla) \delta^3(x - y)
\]

Okubo-Marshak (1958)

\[
V(x, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + O(\nabla^2)
\]

\[
S_{12} = \frac{3}{r^2}(\sigma_1 \cdot x)(\sigma_2 \cdot x) - (\sigma_1 \cdot \sigma_2)
\]

We calculate observables such as phase shift and binding energy using this approximated potential.
3. Nuclear potential from lattice QCD
• well-defined statistical system (finite a and L)
• gauge invariant
• fully non-perturbative

Quenched QCD: neglects creation-annihilation of quark-antiquark pair
Full QCD: includes creation-annihilation of quark-antiquark pair

Monte-Carlo simulations
4-pt Correlation function

\[ F(r, t - t_0) = \langle 0 | T \{ N(x + r, t)N(x, t) \} \bar{J}(t_0) | 0 \rangle \]

complete set

\[ F(r, t - t_0) = \langle 0 | T \{ N(x + r, t)N(x, t) \} \sum_{n,s_1,s_2} |2N, W_n, s_1, s_2\rangle \langle 2N, W_n, s_1, s_2|\bar{J}(t_0) | 0 \rangle \]

\[ = \sum_{n,s_1,s_2} A_{n,s_1,s_2} \varphi W_n(r) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2|\bar{J}(0) | 0 \rangle. \]

Large \( t \)

\[ \lim_{(t-t_0)\to\infty} F(r, t - t_0) = A_0 \varphi W_0(r) e^{-W_0(t-t_0)} + O(e^{-W_n\neq0(t-t_0)}) \]
The 1st quenched QCD results

- $a=0.137$ fm, physical size: $(4.4 \textrm{ fm})^4$

- 3 quark masses
  
  $m_{\pi} = 370$ MeV (2000 conf), $m_{\pi} = 527$ MeV (2000 conf)
  
  $m_{\pi} = 732$ MeV (1000 conf)

Blue Gene/L @ KEK (stop operating in this January)

- 10 racks, 57.3 TFlops peak

- 34-48 % of peak performance

- 4000 hours of 512 Node (half-rack, 2.87 TFlops)
Two Nucleon system

S: spin

L : orbital angular momentum

Consider \( L=0, P(\text{parity})=+ \)

\[
2S+1 \quad L_J
\]

\[
\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0
\]

\[
\begin{array}{c}
\uparrow \uparrow \\
\uparrow \downarrow + \downarrow \uparrow \\
\uparrow \downarrow - \downarrow \uparrow \\
\downarrow \downarrow 
\end{array}
\]
$m_\pi \simeq 0.53 \text{ GeV}$


2011年4月24日日曜日
Qualitative features of NN potential are reproduced!


This paper has been selected as one of 21 papers in Nature Research Highlights 2007

\[ E \approx 0 \quad m_\pi \simeq 0.53 \text{ GeV} \]

\[ V(r;{^1S_0}) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r) \]

\[ V(r;{^3S_1}) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r) \]
Remarks

[Q1] Scheme/Operator dependence of the potential

- The potential itself is NOT a physical observable. Therefore it depends on the definition of the wave function, in particular, on the choice of the nucleon operator $N(x)$. *(Scheme-dependence)*
  - cf. running coupling in QCD
- “good” scheme?
  - good convergence of the derivative expansion for the potential.
    - completely local and energy-independent one is the best and must be unique if exists. *(Inverse scattering method)*
[Q2] Energy dependence of the potential

Non-local, E-independent \[ \begin{aligned}
\left( E + \frac{\nabla^2}{2m} \right) \varphi_E(x) &= \int d^3 y \, U(x, y) \varphi_E(y) \\
V_E(x) \varphi_E(x) &= \left( E + \frac{\nabla^2}{2m} \right) \varphi_E(x)
\end{aligned} \]

Local, E-dependent

non-locality can be determined order by order in velocity expansion (cf. ChPT)

\[ V(x, \nabla) = V_C(r) + V_T(r) S_{12} + V_{LS}(r) \mathbf{L} \cdot \mathbf{S} + \{ V_D(r), \nabla^2 \} + \cdots \]

Numerical check in quenched QCD

\[ m_\pi \simeq 0.53 \text{ GeV} \]
\[ a=0.137\text{fm} \]

K. Murano, N. Ishii, S. Aoki, T. Hatsuda


Anti-Periodic B.C.
- PBC  (E~0 MeV)
- APBC  (E~46 MeV)
E-dependence of the local potential turns out to be very small at low energy in our choice of wave function. Quenched QCD

\[ m_\pi \simeq 0.53 \text{ GeV} \]

\[ a=0.137\text{fm} \]

E-good scheme?
4. More on nuclear potential
Tensor potential

\[ (H_0 + V_C(r) + V_T(r)S_{12})\psi(r; 1^+) = E\psi(r; 1^+) \]

mixing between \( ^3S_1 \) and \( ^3D_1 \) through the tensor force

\[ \psi(r; 1^+) = P\psi(r; 1^+) + Q\psi(r; 1^+) \]

\[ P\psi_{\alpha\beta}(r; 1^+) = P^{(A_1)}\psi_{\alpha\beta}(r; 1^+) \quad \text{"projection" to } L=0 \quad ^3S_1 \]

\[ Q\psi_{\alpha\beta}(r; 1^+) = (1 - P^{(A_1)})\psi_{\alpha\beta}(r; 1^+) \quad \text{"projection" to } L=2 \quad ^3D_1 \]

\[ H_0[P\psi](r) + V_C(r)[P\psi](r) + V_T(r)[PS_{12}\psi](r) = E[P\psi](r) \]

\[ H_0[Q\psi](r) + V_C(r)[Q\psi](r) + V_T(r)[QS_{12}\psi](r) = E[Q\psi](r) \]
Figure 4: The central potentials for the spin-singlet channel from the orbital $A^+_{1}$ representation at three different pion masses in quenched QCD. Taken from Ref. [17].

Figure 5: (Left) $(\alpha, \beta) = (2, 1)$ components of the orbital $A^+_{1}$ and non-$A^+_{1}$ wave functions from $J^P = T^+_{1}$ (and $J_z = S_z = 0$) states at $m_\pi \approx 529$ MeV. (Right) The same wave functions but the spherical harmonics components are removed from the non-$A^+_{1}$ part. Taken from Ref. [17].

4.2 Tensor potential

In Fig. 5(Left), we show the $A^+_{1}$ and non-$A^+_{1}$ components of the NBS wave function obtained from the $J^P = T^+_{1}$ (and $J_z = S_z = 0$) states at $m_\pi \approx 529$ MeV, according to eqs. (37) and (38). The $A^+_{1}$ wave function is multivalued as a function of $r$ due to its angular dependence. For example, $(\alpha, \beta) = (2, 1)$ spin component of the $L = 2$ part of the non-$A^+_{1}$ wave function is proportional to the spherical harmonics $Y_{20}(\theta, \phi) \propto 3 \cos^2 \theta - 1$. Fig. 5(Right) shows non-$A^+_{1}$ component divided by $Y_{20}(\theta, \phi)$. It is clear that the multivaluedness is mostly removed, showing that the non-$A^+_{1}$ component is dominated by the $D^+_{1}(L = 2)$ state.
- no repulsive core in the tensor potential.
- the central potential is roughly equal to the effective central potential.
- the tensor potential is still small.
Potentials

Tensor Force and Central Force (t-t₀=5)

$V_T(r)$
$V_C(r)$
$V_C,\text{eff}$

$V_C \simeq V_C,\text{eff}$

$m_\pi \simeq 0.53$ GeV

- no repulsive core in the tensor potential.
- the central potential is roughly equal to the effective central potential.
- the tensor potential is still small.
Quark mass dependence

- the tensor potential increases as the pion mass decreases.
- manifestation of one-pion-exchange?
- the fit below works well.

\[
V_T(r) = b_1(1 - e^{-b_2r^2})^2 \left( 1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2} \right) e^{-m_\rho r} + b_3(1 - e^{-b_4r^2})^2 \left( 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) e^{-m_\pi r}
\]
Full QCD Calculation

PACS-CS gauge configurations (2+1 flavors)

mass [GeV]

$\rho$, $K^*$, $N$, $\Lambda$, $\Sigma$, $\Xi^*$, $\Omega$

- Experiment
- $\pi$, $K$, $\Omega$ input

$a = 0.09$ fm

$L = 2.9$ fm

$m^{\text{min.}}_\pi = 156$ MeV

$m_\pi L = 2.3$

We are almost on the “physical point”.

Calculations with $L=5.8$ fm with $m_\pi \simeq 140$ MeV are on-going.

$m_\pi L > 4$

“Real QCD”

cf. BMW collaboration

2011年4月24日日曜日
both repulsive core at short distance and the tensor potential are enhanced in full QCD.

- the attraction at medium distance is shifted to outer region, while the magnitude remains almost unchanged.

These differences may be caused by dynamical quark effects. For more definite conclusion on this point, a more controlled comparison is needed.

\[ V(r) \text{ [MeV]} \]

\[ r \text{ [fm]} \]

\[ a \approx 0.091 \text{ fm} \quad L \approx 2.9 \text{ fm} \quad a \approx 0.137 \text{ fm} \quad L \approx 4.4 \text{ fm} \]
Phase shift from $V(r)$ in full QCD

$a = 0.1$ fm, $L = 2.9$ fm

$1S_0$ and $3S_1$ states with different masses.
They have reasonable shapes. The strength is much weaker, though.

calculation at physical quark mass is important. (future work)
5. Hyperon interactions

Hyperon: Baryon (3 quark state) which contains one or more strange quarks.

$p = (uud)$, $n = (udd)$

$\Lambda = (uds)_{I=0}$

$\Sigma^+ = (uus)$, $\Sigma^0 = (uds)_{I=1}$, $\Sigma^- = (dds)$

$\Xi^0 = (udd)$, $\Xi^- = (dds)$

nucleon(N)

hyperon(Y)
Octet Baryon interactions

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

• prediction from lattice QCD
• difference between NN and YN?
Octet Baryon interactions

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN?
Octet Baryon interactions

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

Also in GSI

- prediction from lattice QCD
- difference between NN and YN?
Octet Baryon interactions

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC
- prediction from lattice QCD
- difference between NN and YN?

also in GSI

J-PARC (Tokai, Japan)
damaged by the Earthquake

2011年4月24日日曜日
3D Nuclear chart

Strangeness

S = -2

Proton Number

S = -1

Neutron Number

3 known

40 known

~3000 known

2D (N - Z) Nuclear Chart
Baryon-Baryon interactions in an SU(3) symmetric world

\[ m_u = m_d = m_s \]

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]

Symmetric \hspace{2cm} Anti-symmetric

6 independent potential in flavor-basis

\[ V^{(27)}(r), \ V^{(8s)}(r), \ V^{(1)}(r), \ V^{(10^*)}(r), \ V^{(10)}(r), \ V^{(8a)}(r) \]
Potentials in full QCD

Spin-singlet

Inoue et al. (HAL QCD Coll.), PTP124(2010)591

\[ a=0.12 \text{ fm}, L=2 \text{ fm} \]

BG/L@KEK

27, 10*: same behaviors as NN potentials

<table>
<thead>
<tr>
<th>flavor multiplet</th>
<th>baryon pair (isospin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>{NN}(I=1), {NΣ}(I=3/2), {ΣΣ}(I=2), {ΣΞ}(I=3/2), {ΞΞ}(I=1)</td>
</tr>
<tr>
<td>8_s</td>
<td>none</td>
</tr>
<tr>
<td>1</td>
<td>none</td>
</tr>
<tr>
<td>10*</td>
<td>[NN](I=0), [ΣΞ](I=3/2)</td>
</tr>
<tr>
<td>10</td>
<td>[NΣ](I=3/2), [ΞΞ](I=0)</td>
</tr>
<tr>
<td>8_a</td>
<td>[NΞ](I=0)</td>
</tr>
</tbody>
</table>
8s: strong repulsive core. repulsion only.

1: attractive instead of repulsive core! attraction only. bound state?

10: strong repulsive core. weak attraction.

8a: weak repulsive core. strong attraction.
Potential matrix in “particle basis” (S=-2, I=0, spin-singlet)

\[
V_{ij}(r) = \sum_X U_{iX} V^{(X)}(r) U^\dagger_{Xj}
\]

- attraction appeared in flavor-singlet channel can not be easily seen due to the strong repulsion in 8s channel. \( \Sigma \Sigma \) is the most repulsive due to the large coupling to 8s.
- \( N \Xi \) has the strong attraction due to the large coupling to the singlet.
- off-diagonal parts are comparable to diagonal parts in magnitude.
  - full matrix is needed.
- “physics” can be easily seen in the “flavor” basis.
6. H-dibaryons

H-dibaryon:
a possible six quark state(uuddss) predicted by the model but not observed yet.
L=3 fm is enough for the potential.
pion mass dependence

lighter the pion mass, stronger the attraction

fit the potential at L=4 fm by

\[ V(r) = a_1 e^{-a_2 r^2} + a_3 \left( 1 - e^{-a_4 r^2} \right)^2 \left( \frac{e^{-a_5 r}}{r} \right)^2 \]

solve Schroedinger equation with this potential in the infinite volume.

One bound state (H-dibaryon) exists!
An H-dibaryon exists in the flavor SU(3) limit! Binding energy = 30-40 MeV, weak quark mass dependence.
Deuteron

H-dibayon

Distance $r$ (fm)

Potential energy [MeV]

Potential

Wave function

Distance $r$ (fm)

Potential energy [MeV]

Potential

Wave function
Real world?

SU(3) limit

\[ \Lambda \Lambda - N \Xi - \Sigma \Sigma \]

30-40 MeV

Real world

\[ \Sigma \Sigma \]

2386 MeV

129 MeV

\[ N \Xi \]

2257 MeV

25 MeV

\[ H \]

2232 MeV

2011年4月24日日曜日
7. Conclusions

- the potential method is new but very useful to investigate baryon interactions in (lattice) QCD.
- the method can be easily also applied to meson-baryon and meson-meson interactions.
- three body force can be analyzed.
- various extensions of the method will be looked for.
Three nucleon force (TNF)

Linear setup

(1,2) pair \(1S_0, 3S_1, 3D_1\)  S-wave only

Triton\((I = 1/2, J^P = 1/2^+)\)

\[ \phi_S = \frac{1}{\sqrt{2}}(-\psi_{1S_0} + \psi_{3S_1}) \]
\[ \phi_M = \frac{1}{\sqrt{2}}(\psi_{1S_0} + \psi_{3S_1}) \]

Scalar/isoscalar TNF is observed at short distance.

Further study is needed to confirm this result.
QCD meets Nuclei!

“The achievement is both a computational tour de force and a triumph for theory.” (Nature Research Highlight 2007)
A possible collaboration between Japan-Germany in near future

BMW collaboration

HAL QCD collaboration

HAL will ride on the BMW?