Nuclear Force from Quarks and Gluons

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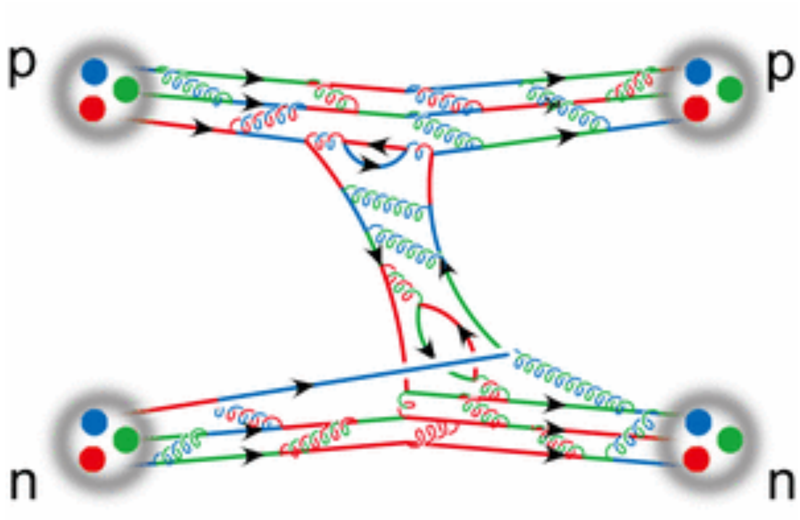


“Japan Days” Colloquium, May 2, 2011, University of Wuppertal

The 150th anniversary of the Friendship Treaty
between Japan and Germany

http://dj150.jp/d/geschichte.html

Friedrich Albrecht Graf zu Eulenburg

Dampfcorvette "Arcona" in der Bucht von Yokohama

Erste Seite des Freundschafts-, Handels- und Schiffahrtsvertrages zwischen Preußen und Japan, 24.1.1861


Freundschafts-, Handels- und Schiffahrts-
Vertrag zwischen Preußen und Japan

Vom 24. Januar 1861

Deutsche Gesandtschaft in Tokyo, Eingangstor (1875)

Wilhelm Solf, Erster Botschafter Deutschlands in Japan (1920-28) nach dem Ersten Weltkrieg

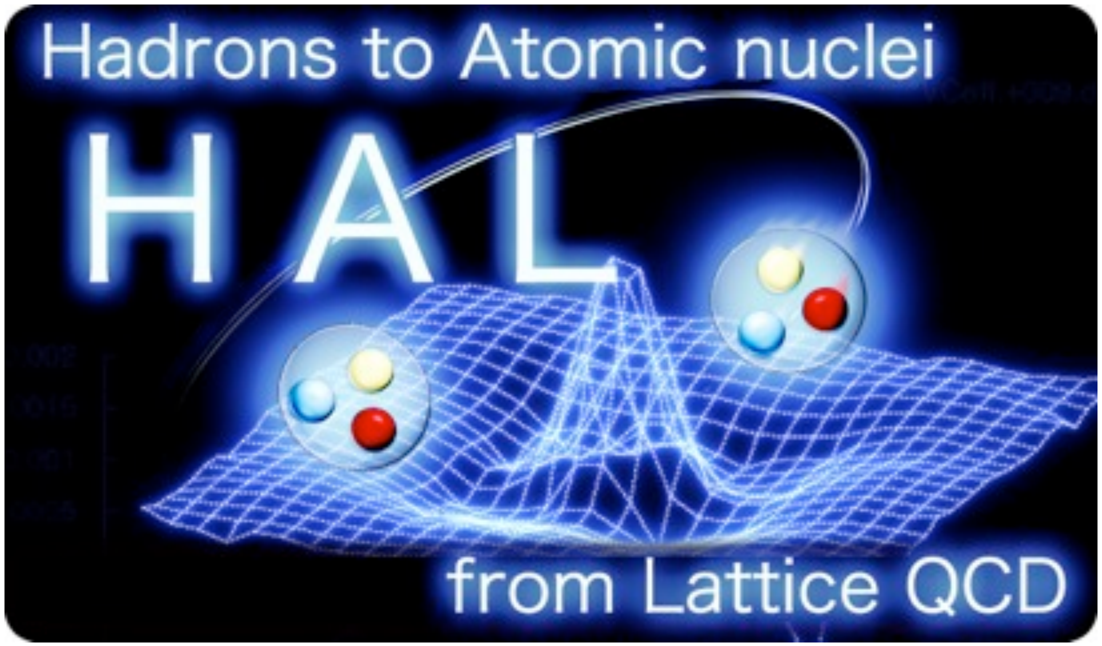
Hochzeitsgesellschaft des Deutschen Generalkonsuls, Herrn von Syburg, in der Deutschen Gesandtschaft 1905
Unter den Gästen: Erwin Bälz (hintere Reihe, 2. v. rechts) und Hana Bälz (vorderste Reihe ganz rechts)

It is my great pleasure and honor to give a talk at the colloquium on this special occasion, the “Japan Days”.

First of all, I would like to express my deepest appreciation for supports and encouragements from all over the world, in particular from Germany, to the peoples in Japan.
We are still struggling against tragedies caused by the Earthquake and Tsunami. I however strongly believe that we will be able to overcome these difficult situations, together with your great help.

My collaborators

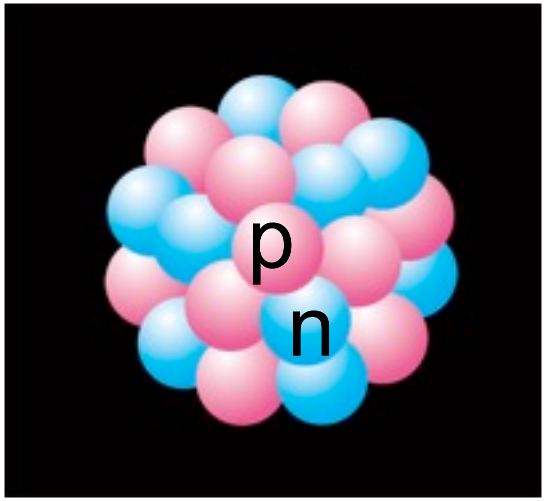
HAL QCD Collaboration



S. Aoki (Tsukuba)
T. Doi (Tsukuba)
T. Hatsuda (Tokyo)
Y. Ikeda (Riken)
T. Inoue (Nihon)
N. Ishii(Tokyo)
K. Murano (KEK)
H. Nemura (Tohoku)
K. Sasaki (Tsukuba)

1. Motivation

What binds protons and neutrons inside a nuclei ?



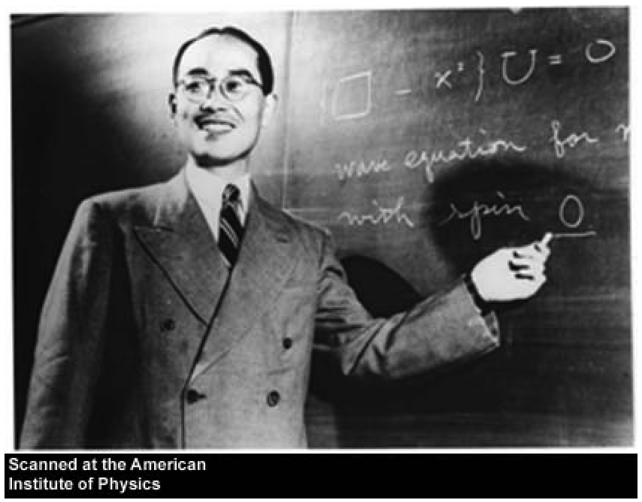
gravity: too weak

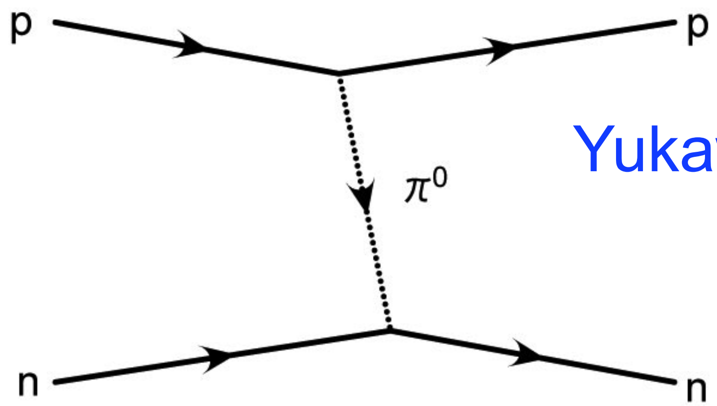
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force





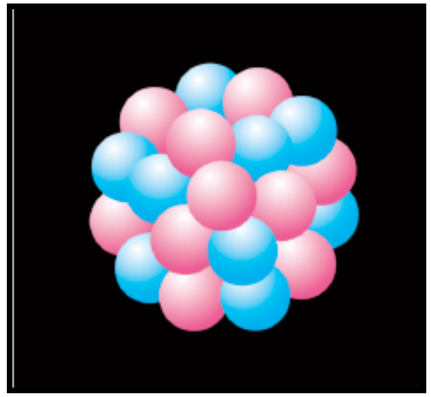
Yukawa potential

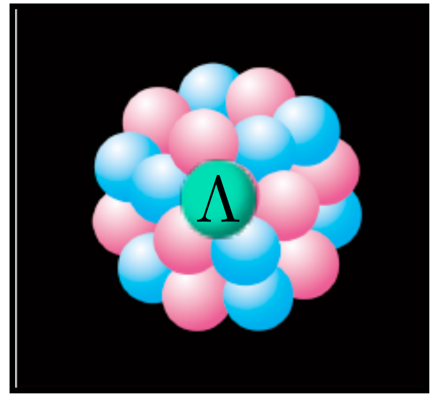
$$V(r) = \frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

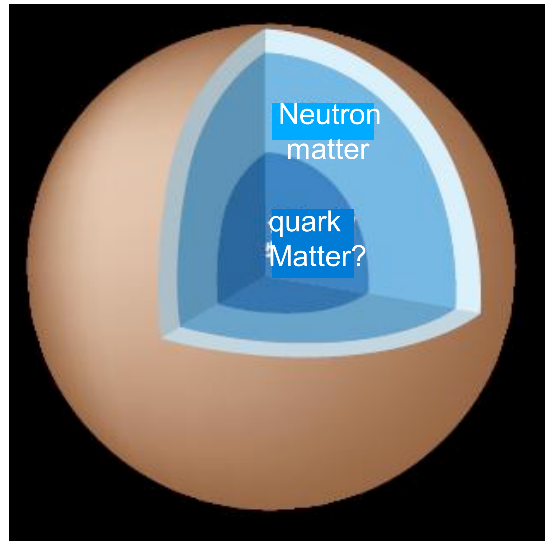
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei





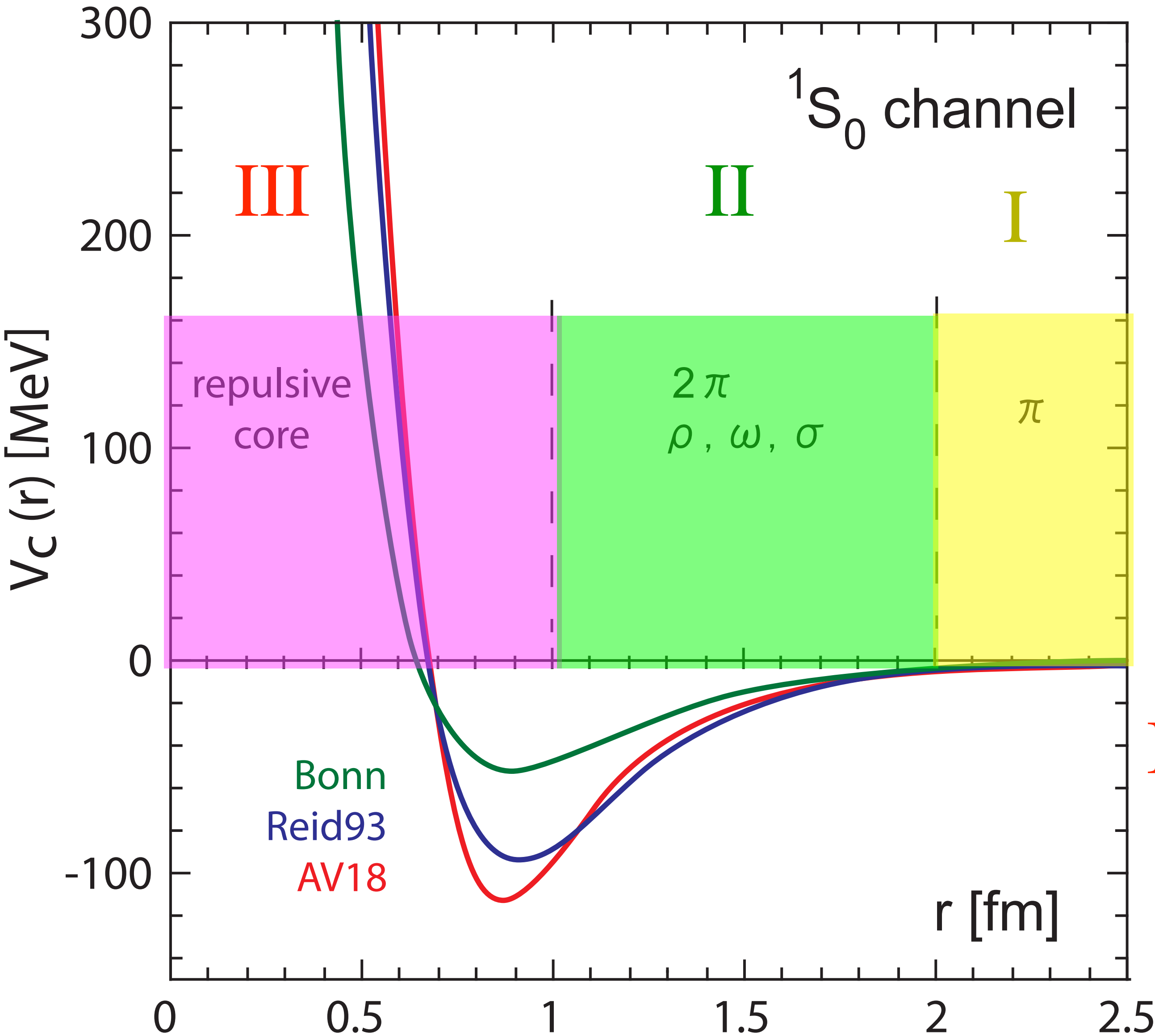
- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

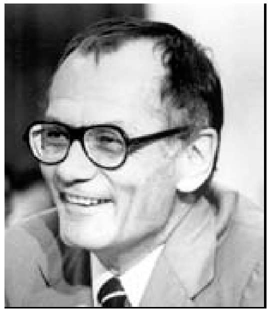
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

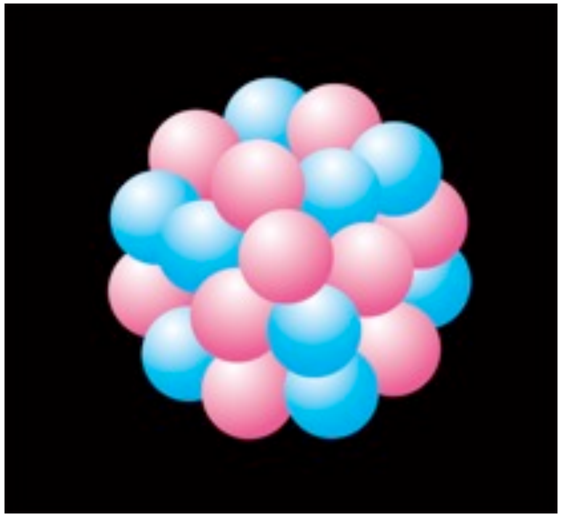
III Repulsive core

Jastrow(1951)

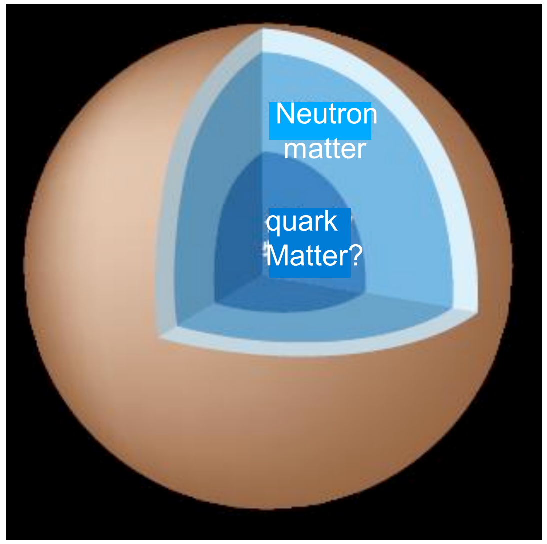


Repulsive core is important

stability of nuclei



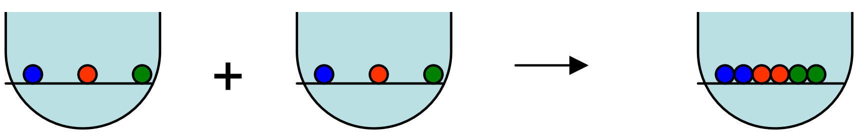
maximum mass of neutron star

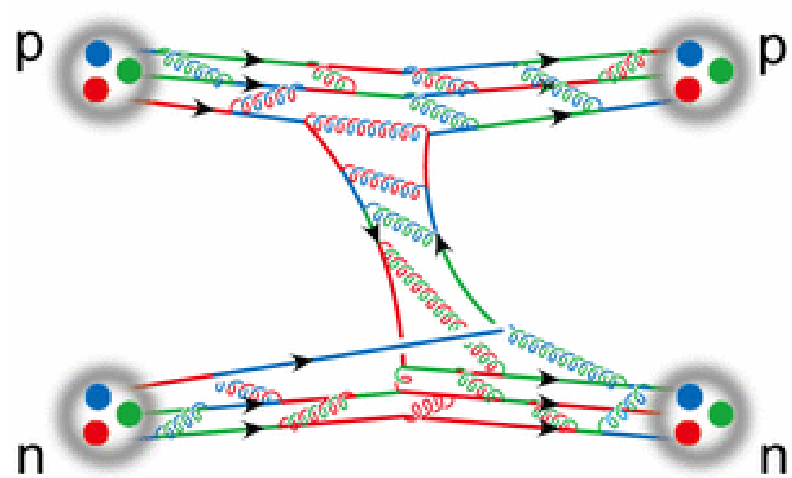


explosion of type II supernova

Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.



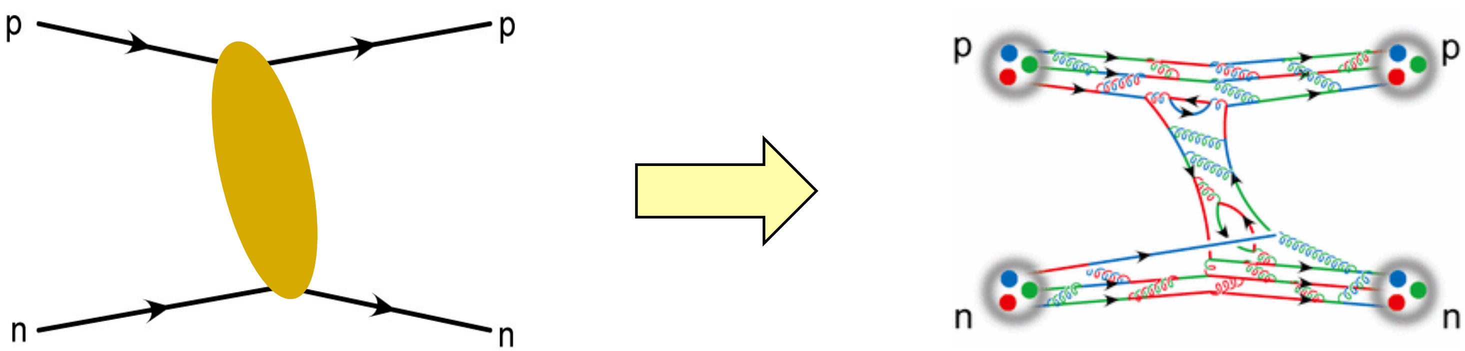


QCD based explanation is needed
Lattice QCD can explain ?

Plan of my talk

1. Motivation
2. Strategy in (lattice) QCD to extract “potential”
3. Nuclear potential from lattice QCD
4. More on nuclear potential
5. Hyperon interactions
6. H-dibaryon
7. Conclusion

2. Strategy in (lattice) QCD to extract “potential”



Challenge to Nambu’s statement

Y. Nambu, “Quarks : Frontiers in Elementary Particle Physics”, World Scientific (1985)

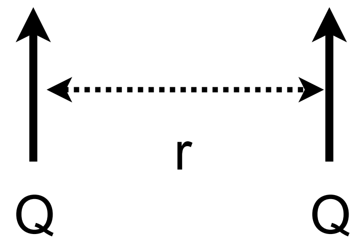
> **“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”**

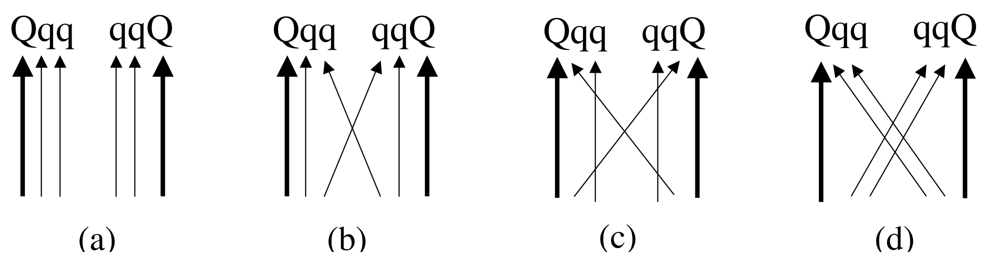
Definition of “Potential” in (lattice) QCD ?

Previous attempt

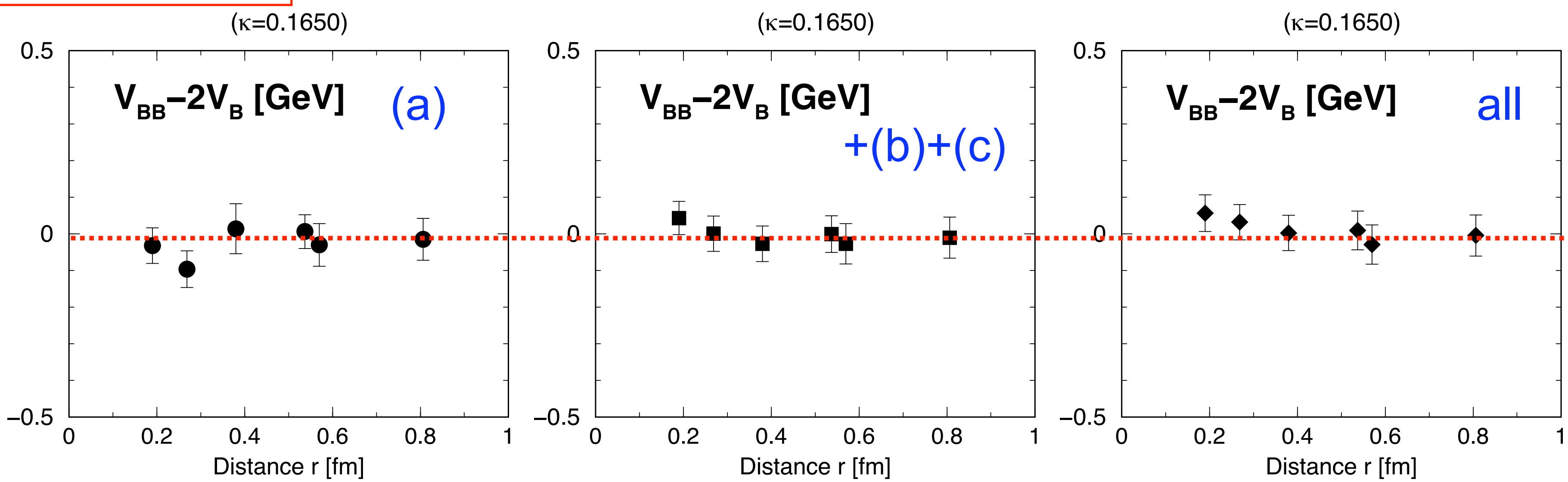
Takahashi-Doi-Suganuma, AIP Conf.Proc. 842,249(2006)

calculate energy of Qqq +Qqq as a function of r between 2Q.
Q:static quark, q: light quark





Quenched result



Almost no dependence on r !

cf. Recent successful result in the strong coupling limit
(deForcrand-Fromm, PRL104(2010)112005)

Alternative approach

Consider "elastic scattering"

$$NN \to NN$$

~~$NN \to NN + \text{others}$~~

~~$(NN \to NN + \pi, NN + \bar{N}N, \cdots)$~~

Elastic threshold $E_{\rm th} = 2m_N + \pi$

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives

$$S = e^{2i\delta}$$

$$E = 2\sqrt{\mathbf{k}^2 + m_N^2} < E_{\rm th}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r}, 0) N(\mathbf{x}, 0) | \underline{6q, E} \rangle$$

QCD eigen-state with energy E and #quark =6

$$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x): \ \text{local operator}$$

NBS wave function satisfies

off-shell T-matrix

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p}, -\mathbf{p} \leftarrow \mathbf{k}, -\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon}$$

$$+ \underline{\ \mathcal{I}(\mathbf{r})}$$

inelastic contribution $\propto O(e^{-\sqrt{E_{th}^2 - E^2}|\mathbf{r}|})$

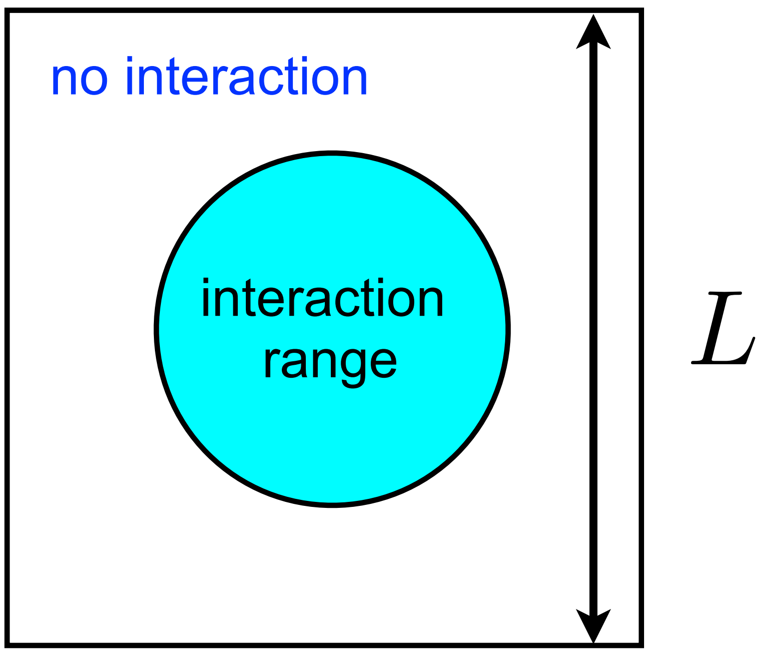
C.-J.D.Lin et al., NPB69(2001) 467
CP-PACS Coll., PRD71 (2005) 094504

Asymptotic behavior $\quad r = |\mathbf{r}| \to \infty$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

partial wave $\quad l = 0, 1, 2, \cdots$

$\delta_l(k)$ is the scattering phase shift



Finite volume

allowed value: k_n^2 $\quad \Rightarrow \quad \delta_l(k_n)$

Lueshcer's formula

Our proposal

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

We define a "non-local potential"

$$[\epsilon_k - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad \epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

Velocity expansion

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

Okubo-Marshak (1958)

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$

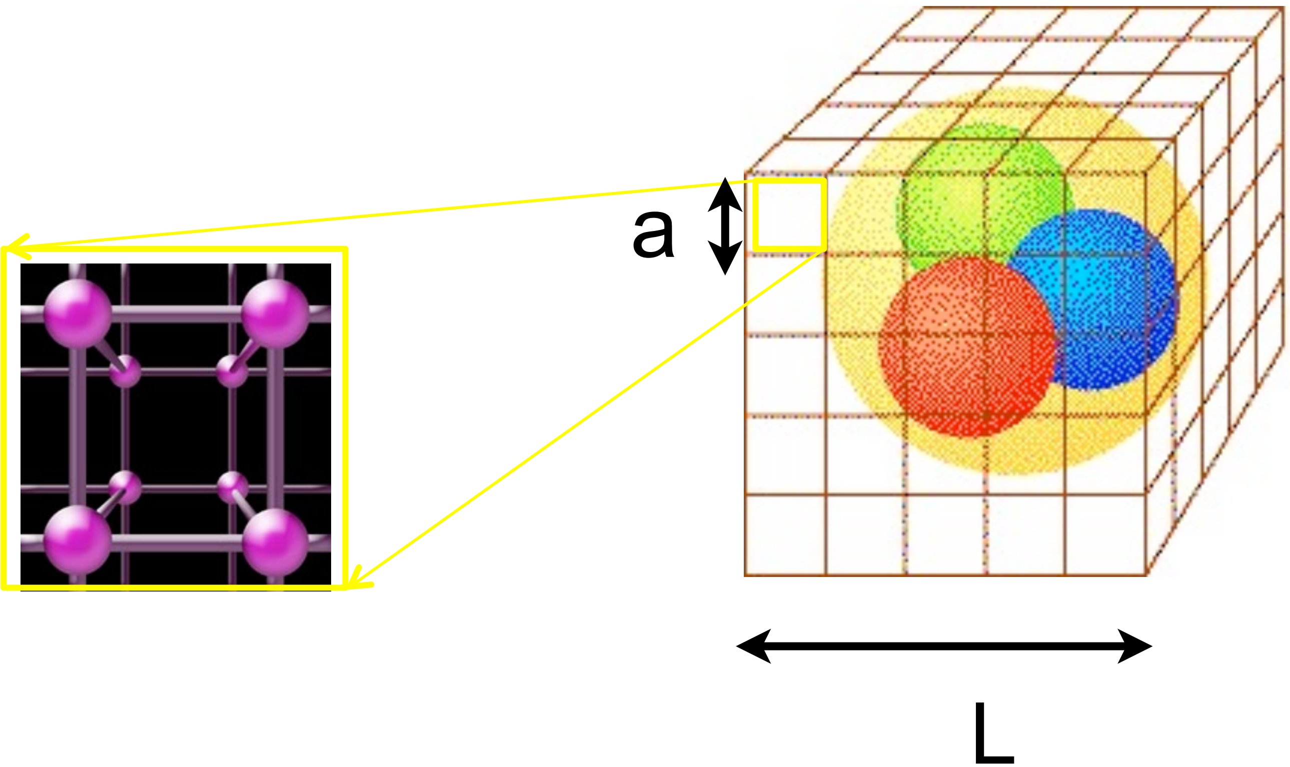
spins

We calculate observables such as phase shift and binding energy using this approximated potential.

3. Nuclear potential from lattice QCD

Lattice QCD

Zoltan's talk



- well-defined statistical system (finite a and L)
- gauge invariant
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

NBS wave function from lattice QCD

4-pt Correlation function

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r}, t) N(\mathbf{x}, t)\} \overline{\mathcal{J}}(t_0) | 0 \rangle$$

source for NN

complete set

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0 | T\{N(\mathbf{x}+\mathbf{r}, t) N(\mathbf{x}, t)\} \sum_{n, s_1, s_2} |2N, W_n, s_1, s_2\rangle \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(t_0) | 0 \rangle \\ &= \sum_{n, s_1, s_2} A_{n, s_1, s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n (t-t_0)}, \quad A_{n, s_1, s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0) | 0 \rangle . \end{aligned}$$

Large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0 (t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

The 1st quenched QCD results

- a=0.137 fm , physical size: (4.4 fm)^4
- 3 quark masses $m_\pi = 370\ \mathrm{MeV}(2000\ \mathrm{conf}),\ m_\pi = 527\ \mathrm{MeV}\ (2000\ \mathrm{conf})$

$$m_\pi = 732\ \mathrm{MeV}(1000\ \mathrm{conf})$$

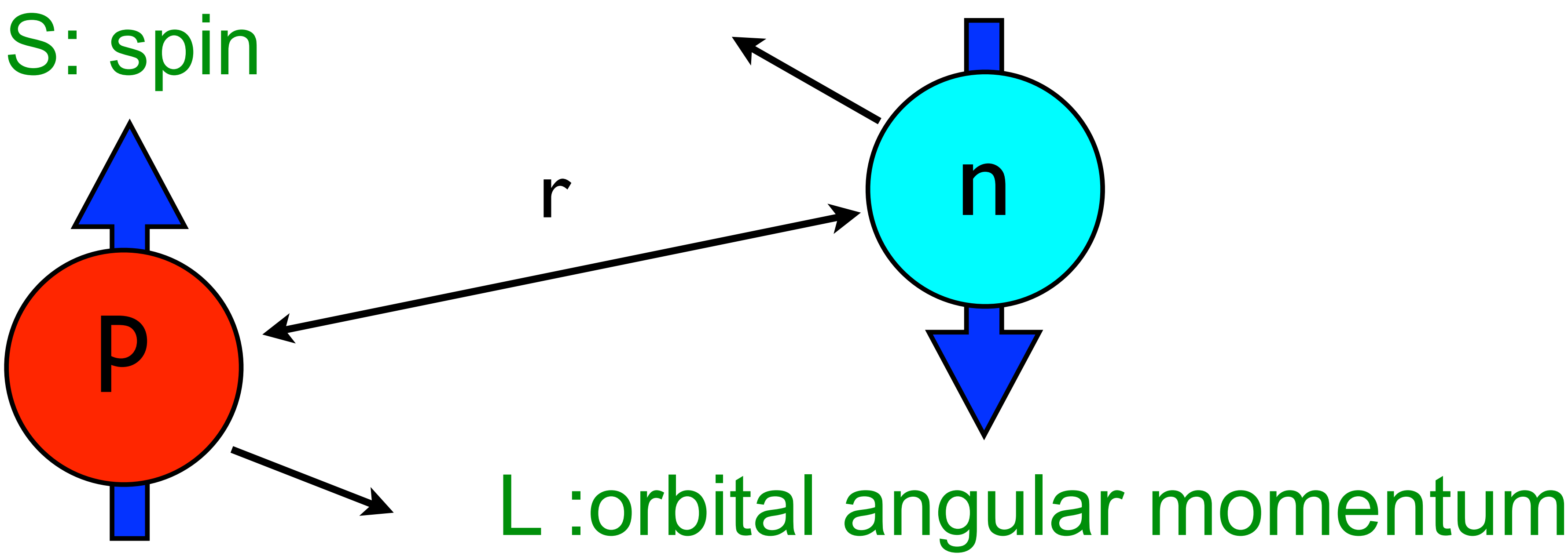
Blue Gene/L @ KEK(stop operating in this January)

10 racks, 57.3 TFlops peak

34-48 % of peak performance

4000 hours of 512 Node
(half-rack, 2.87TFlops)

Two Nucleon system



Consider L=0, P(parity)=+

spin $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$$^{2S+1}L_J$$

$\uparrow\uparrow$

$\uparrow\downarrow + \downarrow\uparrow \qquad \uparrow\downarrow - \downarrow\uparrow$

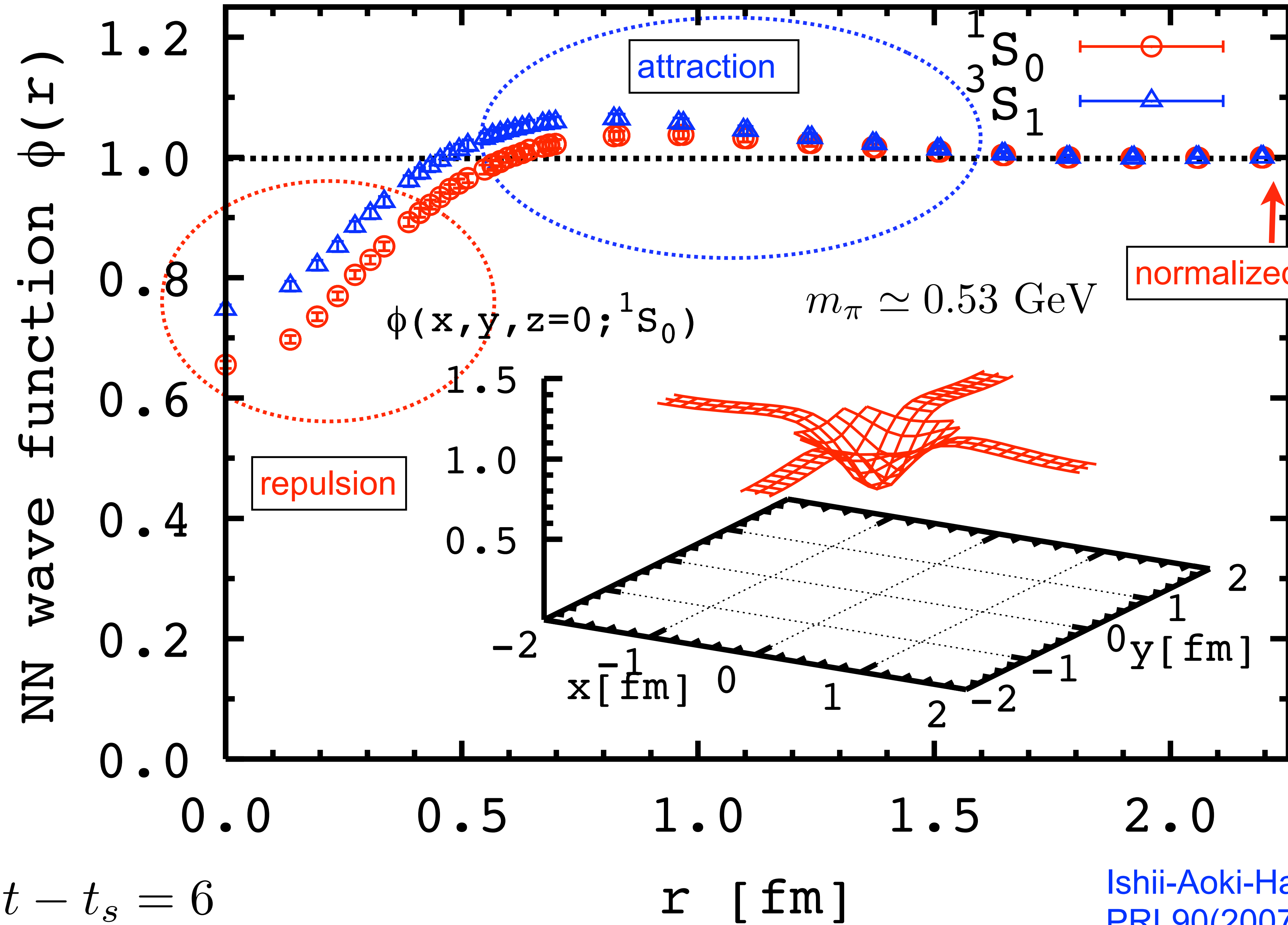
$\downarrow\downarrow$

→ $^3S_1 \qquad ^1S_0$

NN wave function

Quenched QCD

a=0.137fm



$t - t_s = 6$

Ishii-Aoki-Hatsuda,
PRL90(2007)0022001

(quenched) potentials

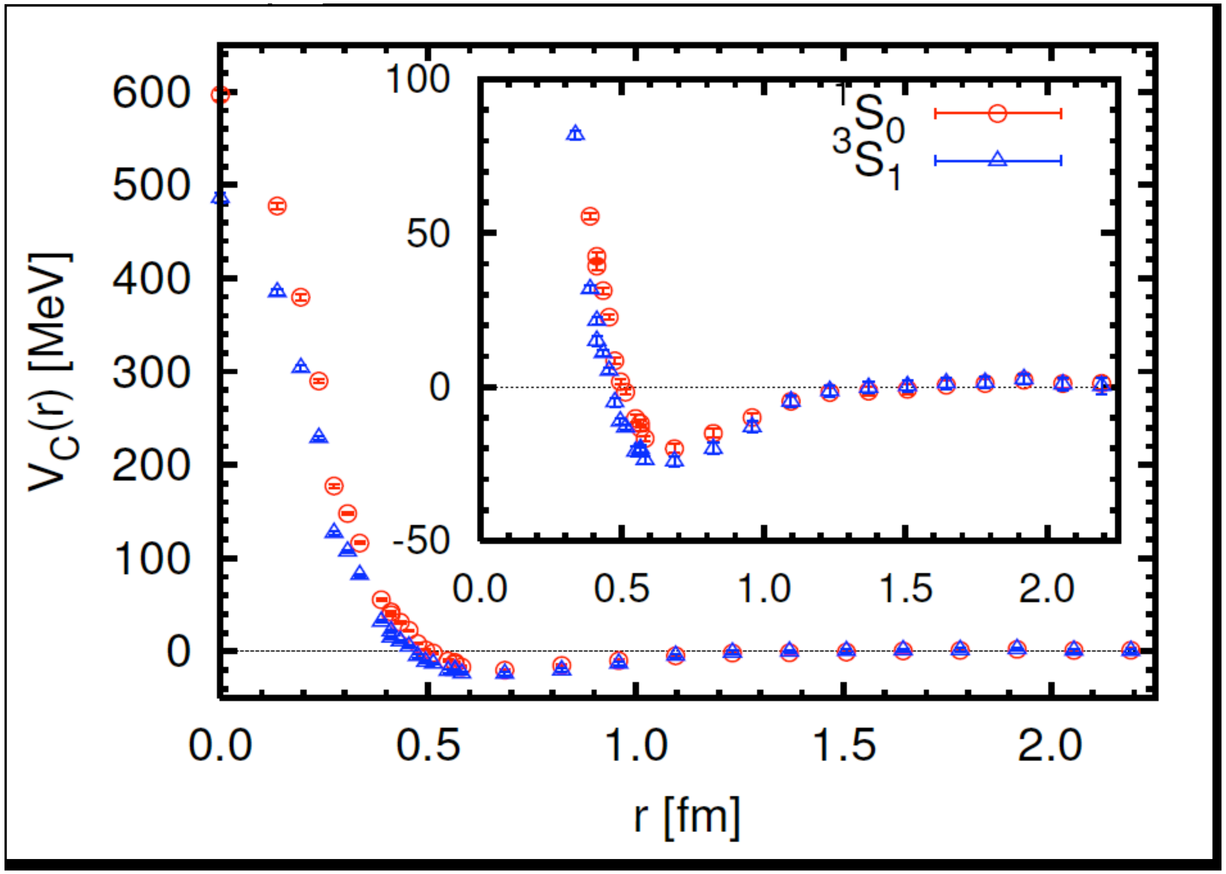
LO (effective) central Potential

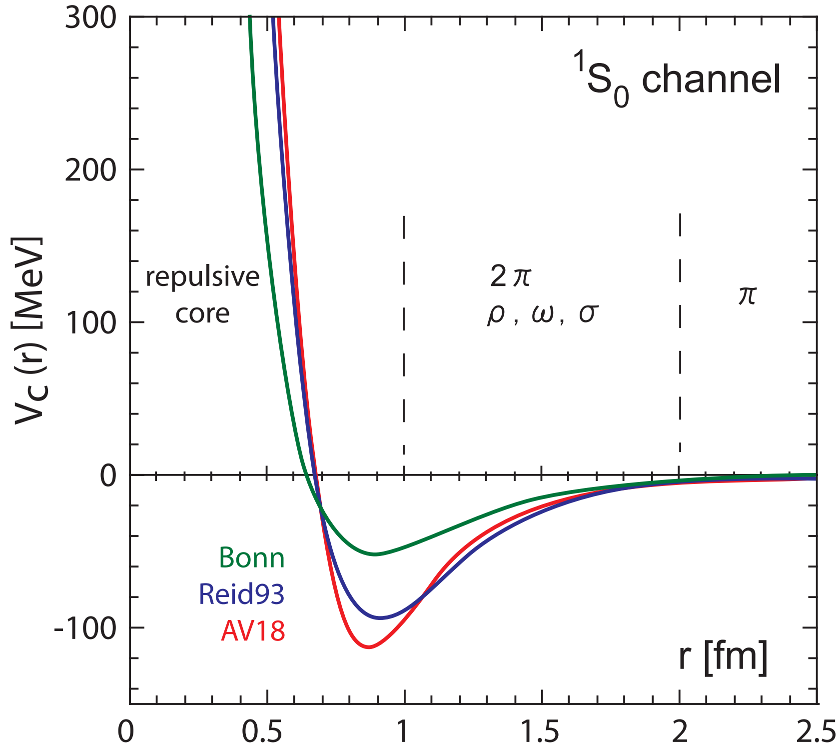
$$V(r;{}^1S_0) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r)$$

$$V(r;{}^3S_1) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r)$$

$E \simeq 0$ $\quad m_\pi \simeq 0.53\ \text{GeV}$

a=0.137 fm L=4.4fm





Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007

Remarks

[Q1] Scheme/Operator dependence of the potential

- The potential itself is NOT a physical observable. Therefore it depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  cf. running coupling in QCD
- "good" scheme ?
  - good convergence of the derivative expansion for the potential.
    - completely local and energy-independent one is the best and must be unique if exists. (Inverse scattering method)

[Q2] Energy dependence of the potential

Non-local, E-independent ⟺ Local, E-dependent

$$\left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

non-locality can be determined order by order in velocity expansion (cf. ChPT)

$$V(\mathbf{x},\nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r),\nabla^2\} + \cdots$$

Numerical check in quenched QCD

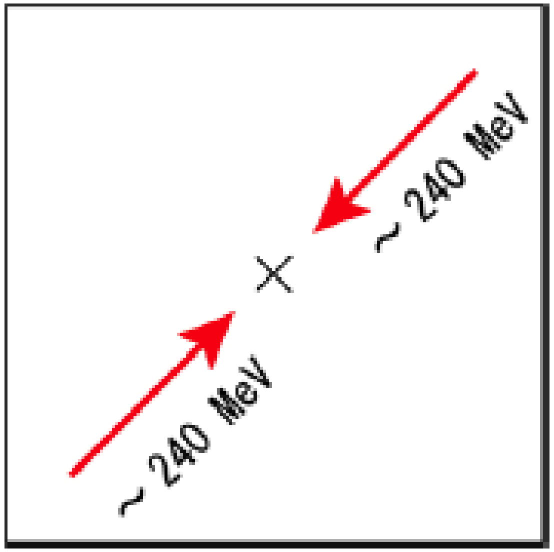
$m_\pi \simeq 0.53$ GeV

a=0.137fm

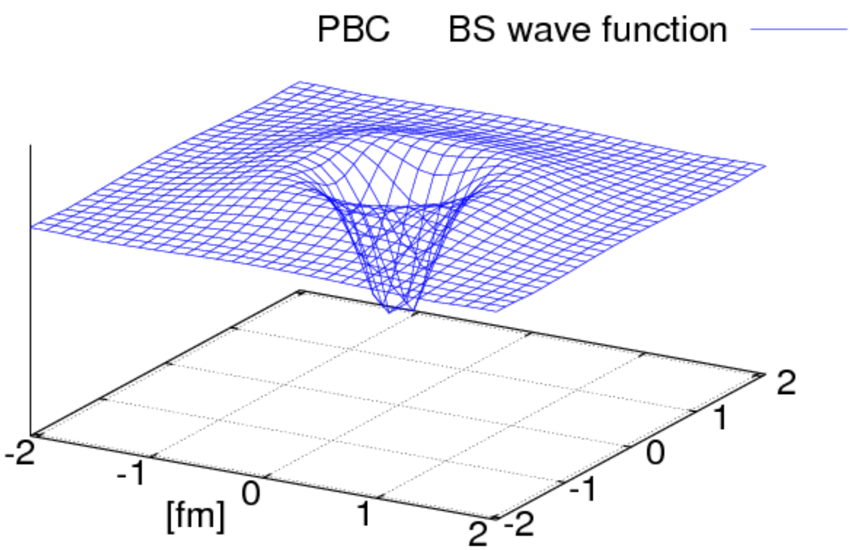
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

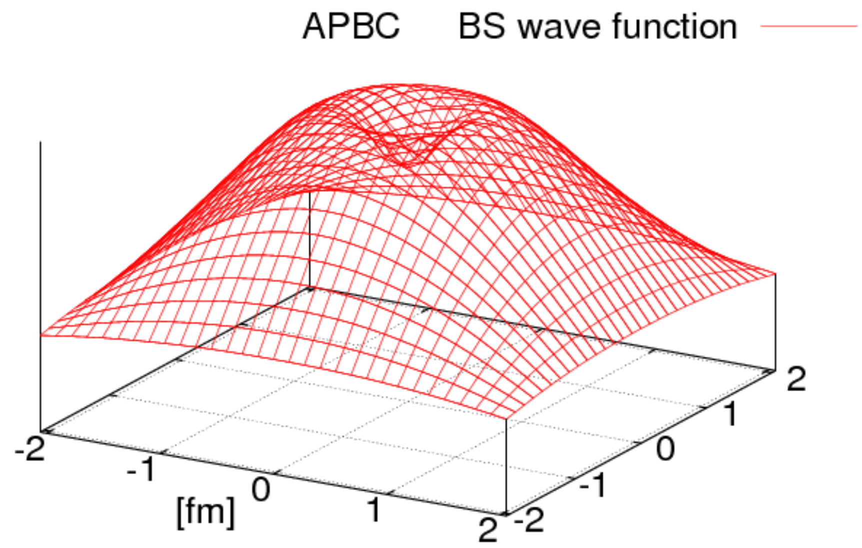
PoS Lattice2009 (2009)126.

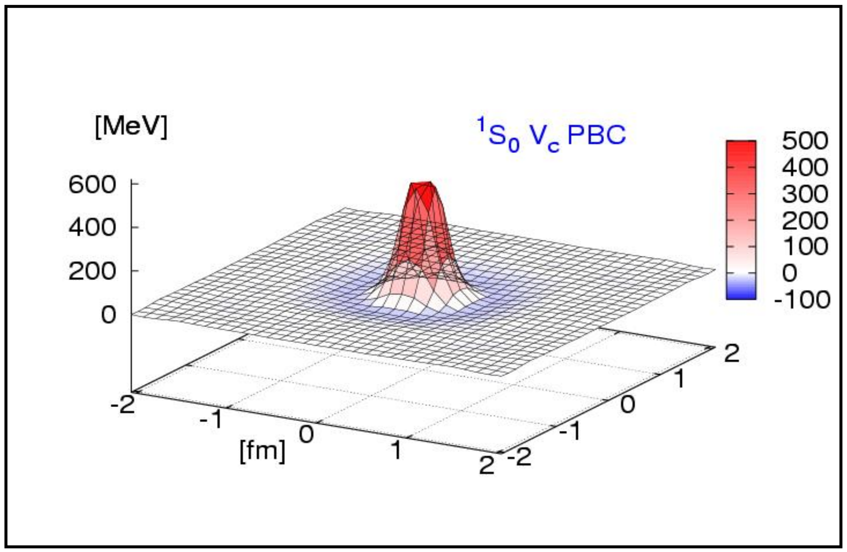
Anti-Periodic B.C.

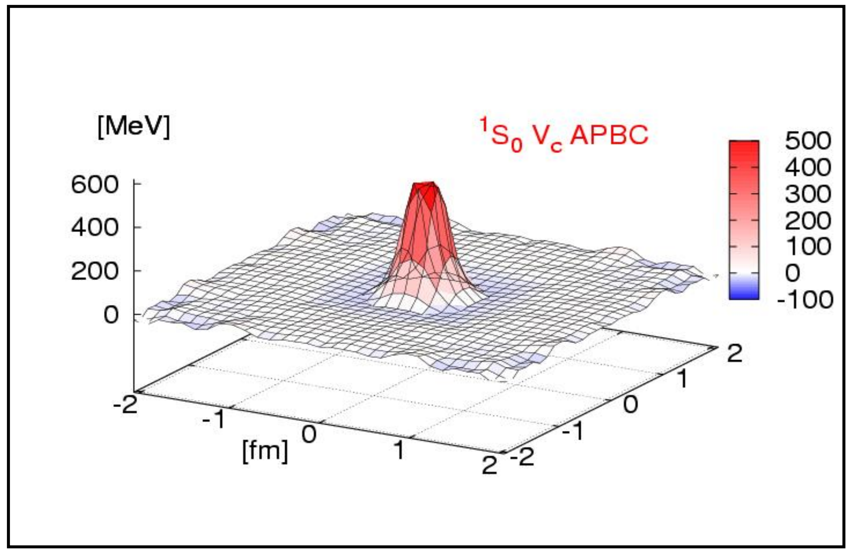


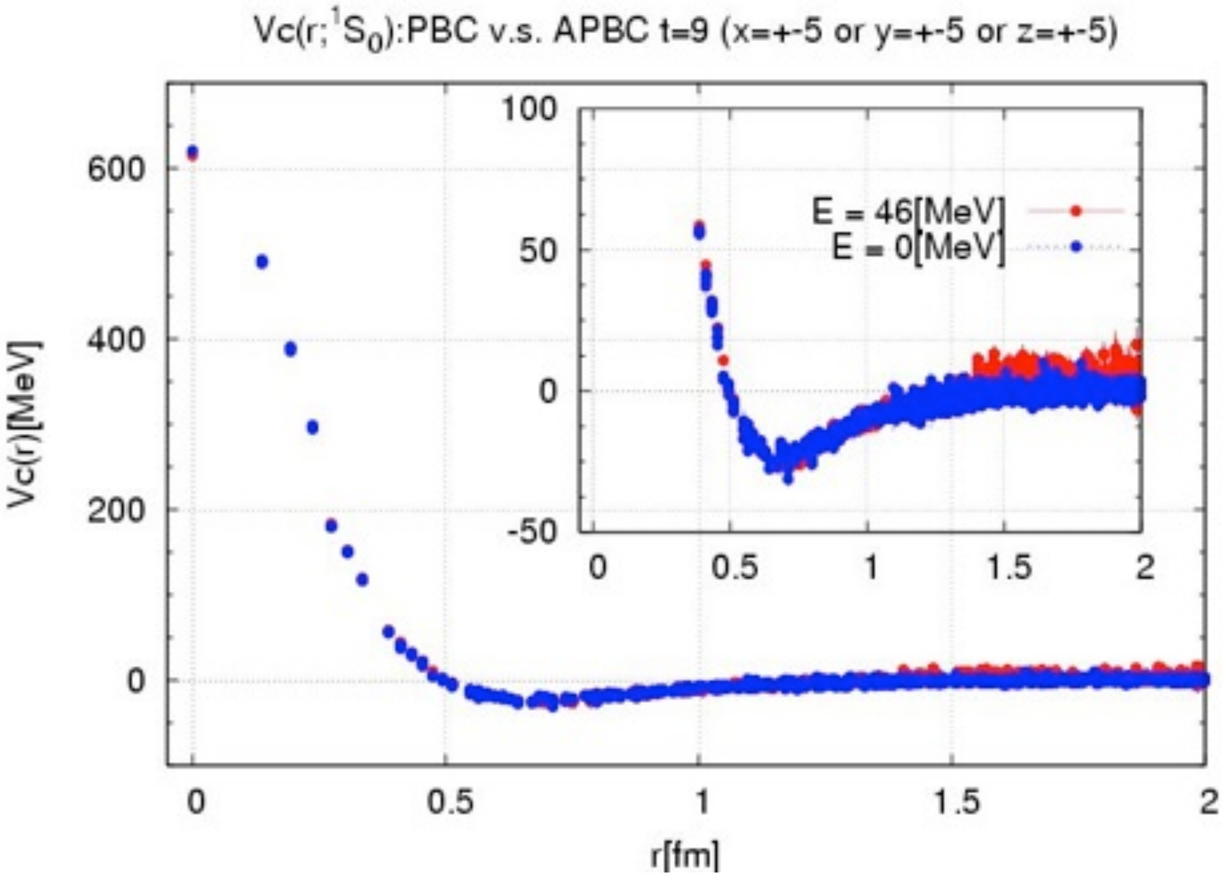
● PBC (E∼0 MeV) ● APBC (E∼46 MeV)









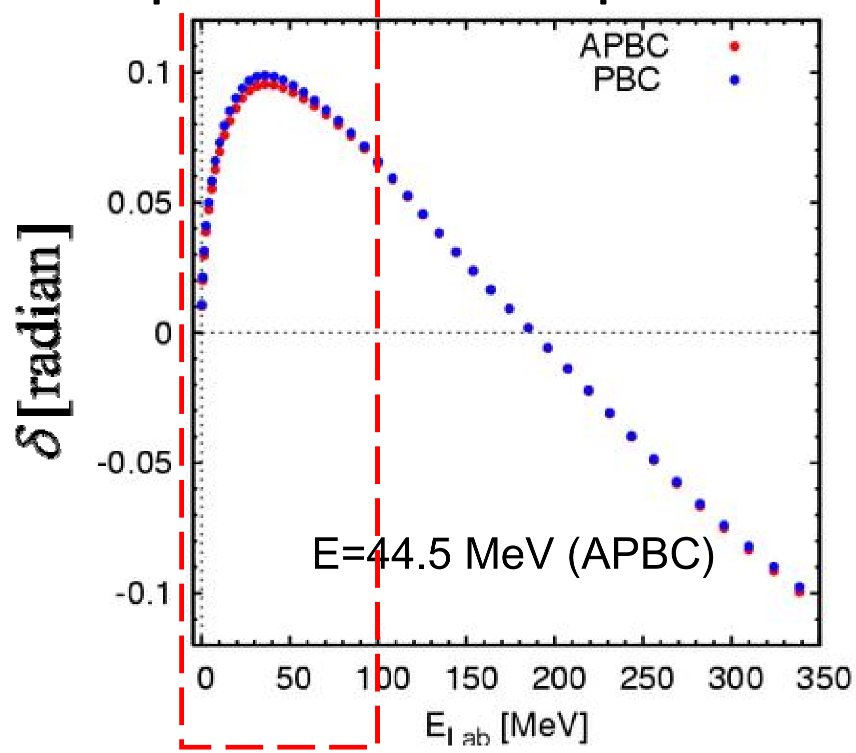


Quenched QCD

$m_\pi \simeq 0.53$ GeV

a=0.137fm

phase shifts from potentials



E-dependence of the local potential turns out to be very small at low energy in our choice of wave function.

good scheme ?

4. More on nuclear potential

Tensor potential

J=1, S=1

$$(H_0 + V_C(r) + V_T(r)S_{12})\psi(\mathbf{r};1^+) = E\psi(\mathbf{r};1^+)$$

mixing between ${}^3S_1$ and ${}^3D_1$ through the tensor force

$$\psi(\mathbf{r};1^+) = \mathcal{P}\psi(\mathbf{r};1^+) + \mathcal{Q}\psi(\mathbf{r};1^+)$$

$$\mathcal{P}\psi_{\alpha\beta}(\mathbf{r};1^+) = P^{(A_1)}\psi_{\alpha\beta}(\mathbf{r};1^+)$$ "projection" to L=0 ${}^3S_1$

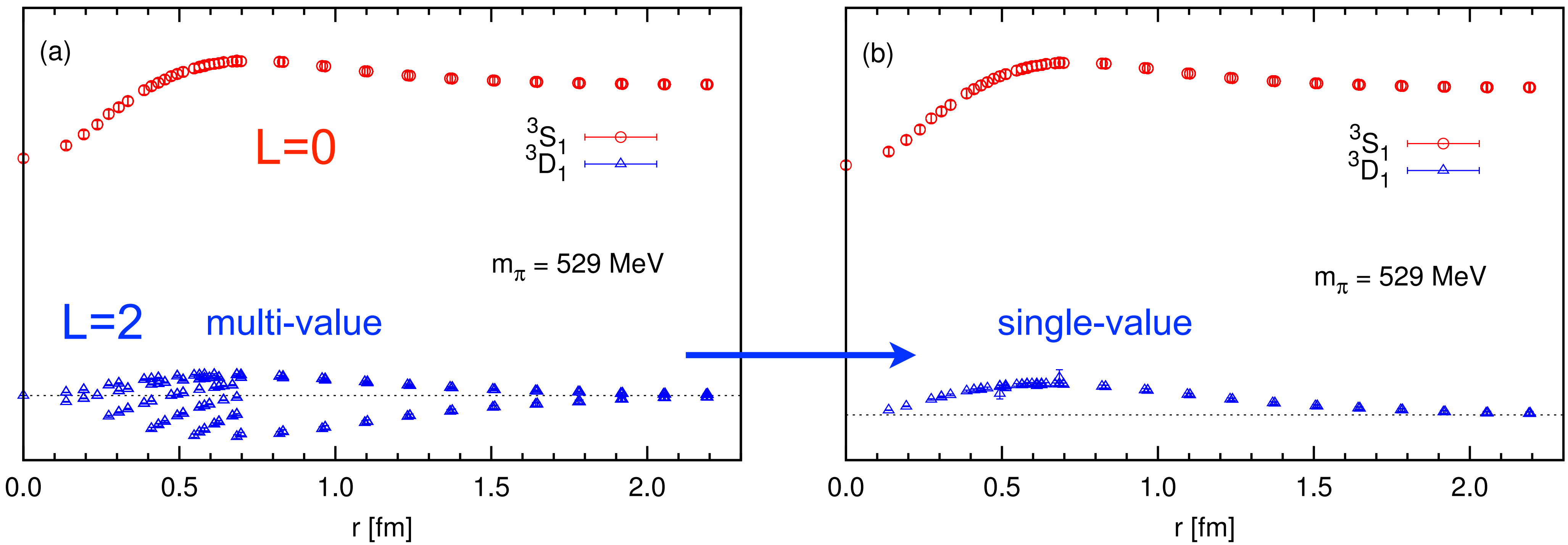
$$\mathcal{Q}\psi_{\alpha\beta}(\mathbf{r};1^+) = (1 - P^{(A_1)})\psi_{\alpha\beta}(\mathbf{r};1^+)$$ "projection" to L=2 ${}^3D_1$

$$\begin{aligned} H_0[\mathcal{P}\psi](\mathbf{r}) + V_C(r)[\mathcal{P}\psi](\mathbf{r}) + V_T(r)[\mathcal{P}S_{12}\psi](\mathbf{r}) &= E[\mathcal{P}\psi](\mathbf{r}) \\ H_0[\mathcal{Q}\psi](\mathbf{r}) + V_C(r)[\mathcal{Q}\psi](\mathbf{r}) + V_T(r)[\mathcal{Q}S_{12}\psi](\mathbf{r}) &= E[\mathcal{Q}\psi](\mathbf{r}) \end{aligned}$$

Wave functions

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

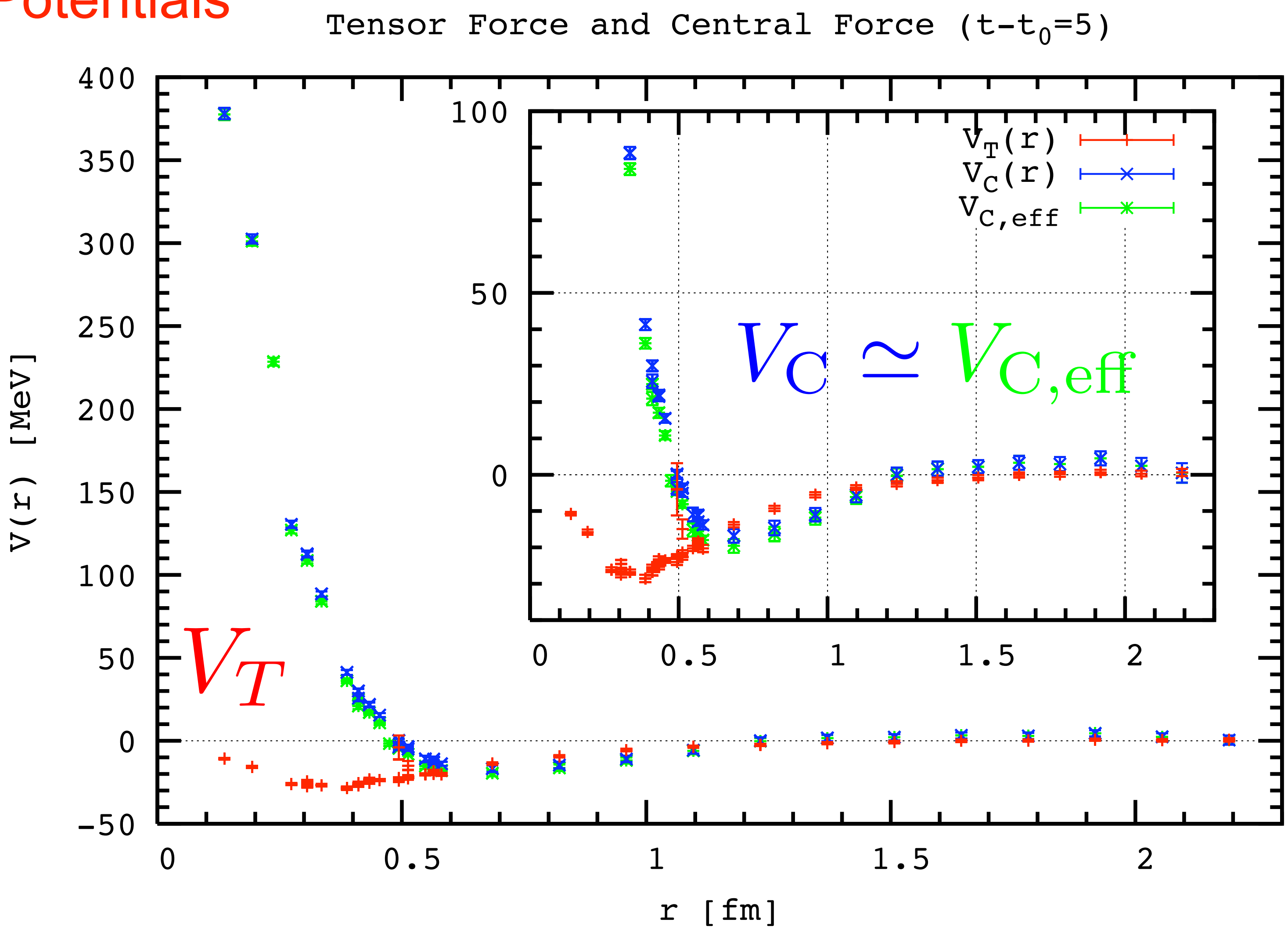
Quenched



$$\text{divided by } Y_{20}(\theta, \phi)$$

Potentials

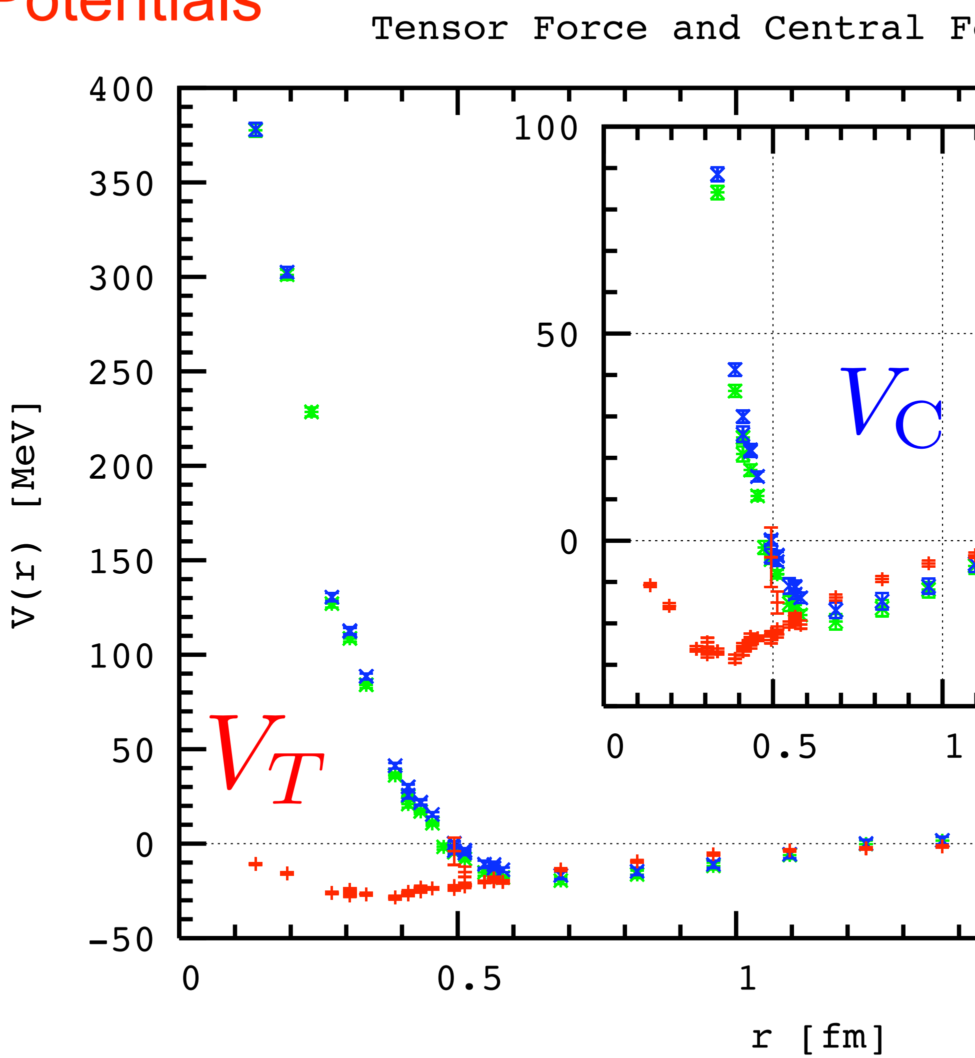
$m_\pi \simeq 0.53$ GeV



- no repulsive core in the tensor potential.
- the central potential is roughly equal to the effective central potential.
  - the tensor potential is still small.

Potentials

$m_\pi \simeq 0.53$ GeV



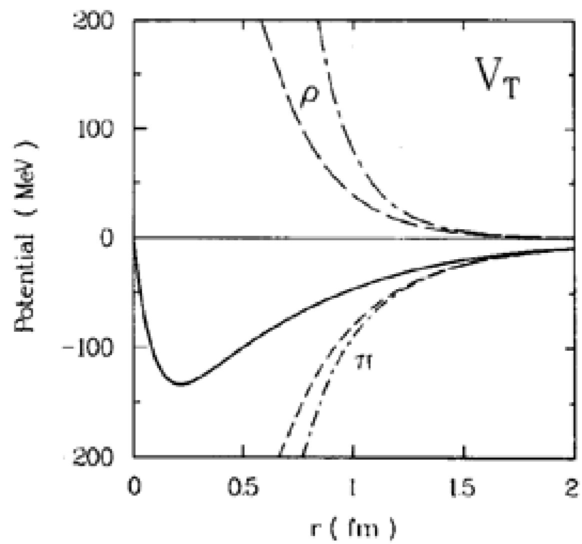
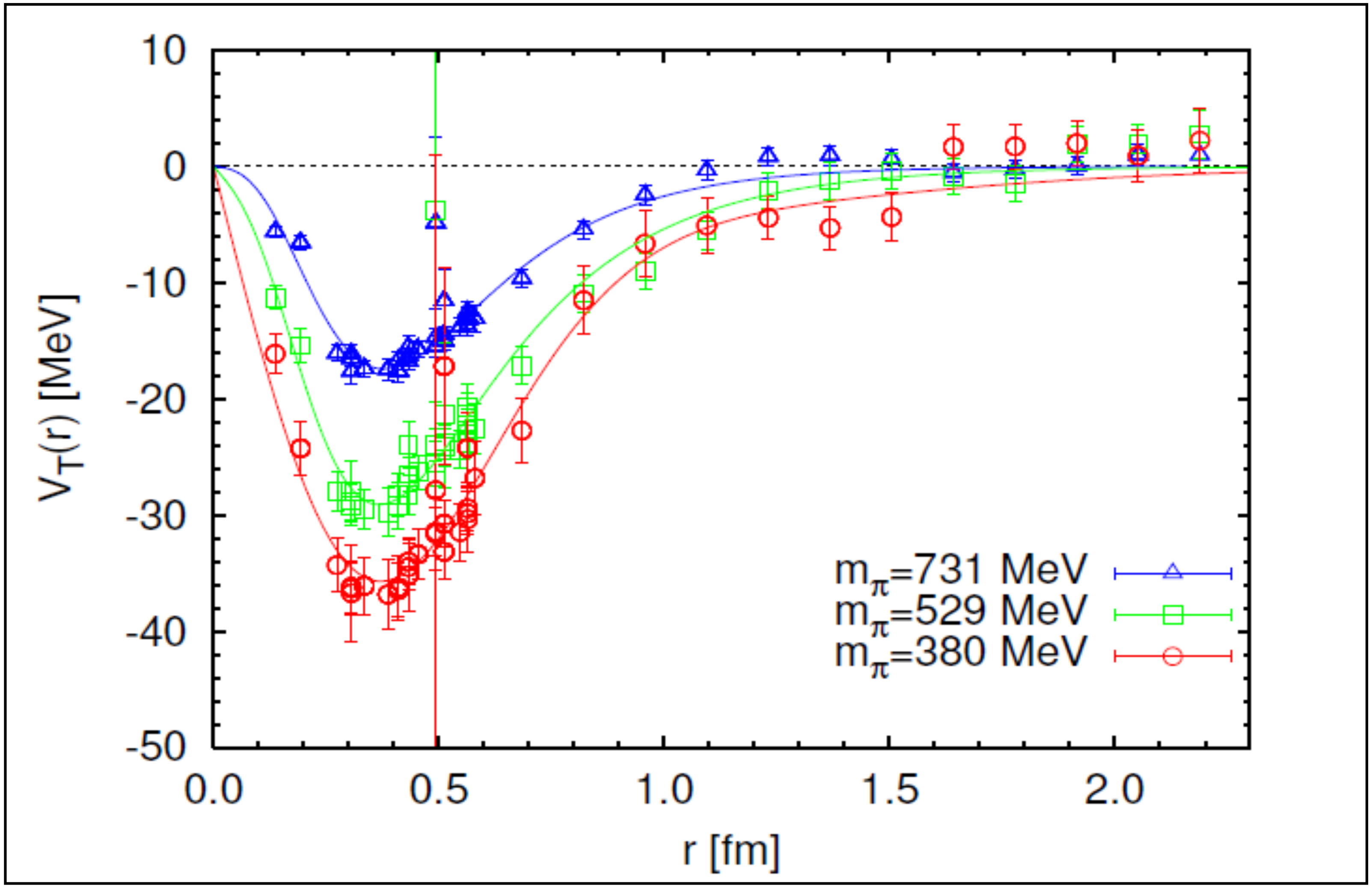


Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

from R.Machleidt, Adv.Nucl.Phys.**19**

- no repulsive core in the tensor pot
- the central potential is roughly equ... to the effective central potential.
  - the tensor potential is still small.

Quark mass dependence



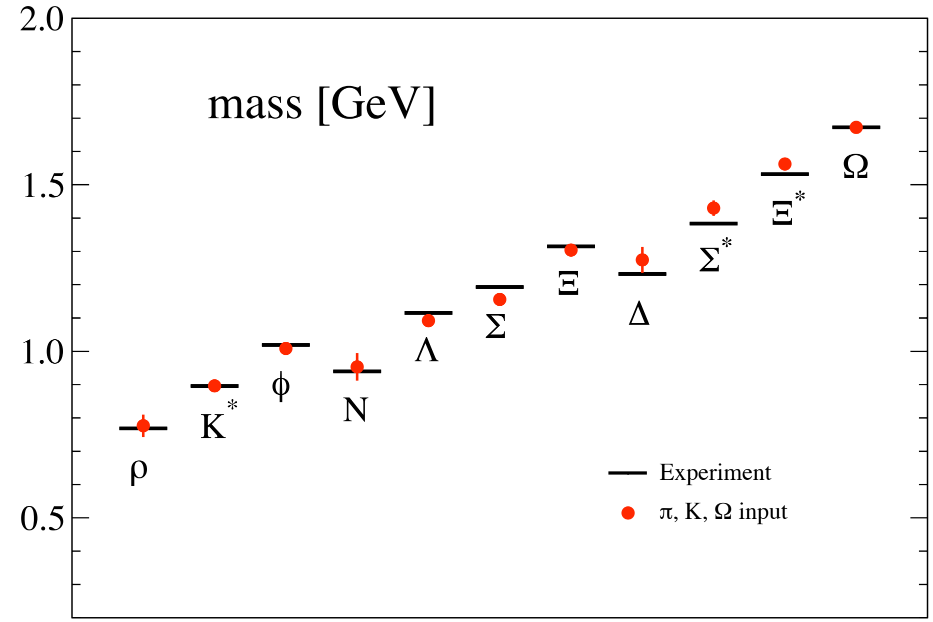
- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?
  - the fit below works well.

$$V_T(r) = b_1(1-e^{-b_2 r^2})^2 \left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r} + b_3(1-e^{-b_4 r^2})^2\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}$$

Full QCD Calculation

PACS-CS gauge configurations(2+1 flavors)

Phys. Rev. D79(2009)034503

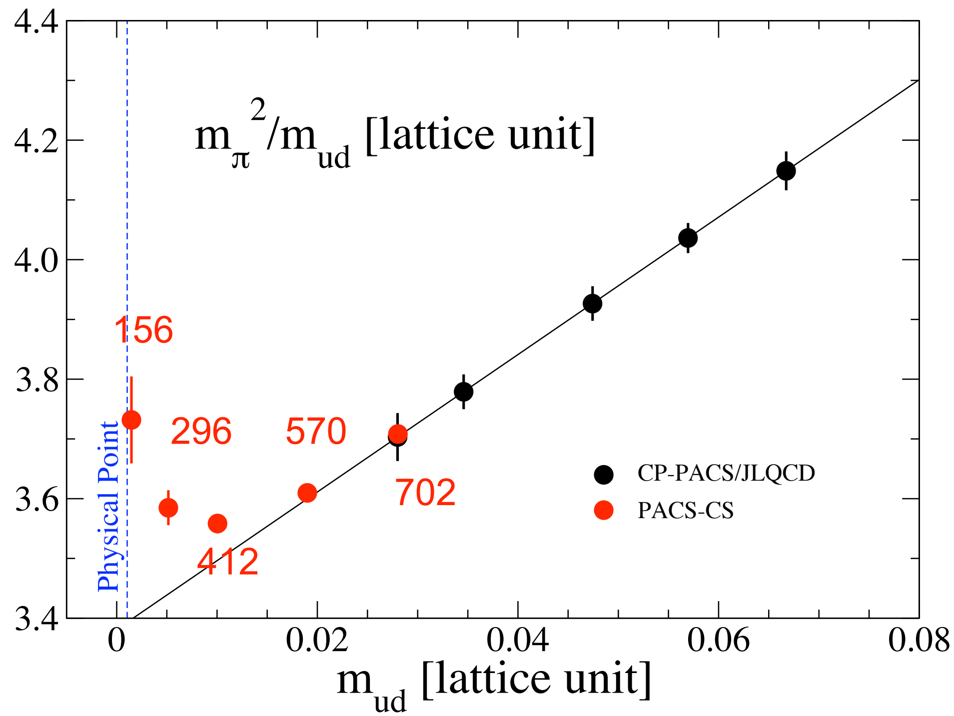


$$a = 0.09\ \text{fm}$$

$$L = 2.9\ \text{fm}$$

$$m_\pi^{\text{min.}} = 156\ \text{MeV} \qquad m_\pi L = 2.3$$

We are almost on the “physical point”.



Calculations with L=5.8 fm with

$m_\pi \simeq 140$ MeV are on-going.

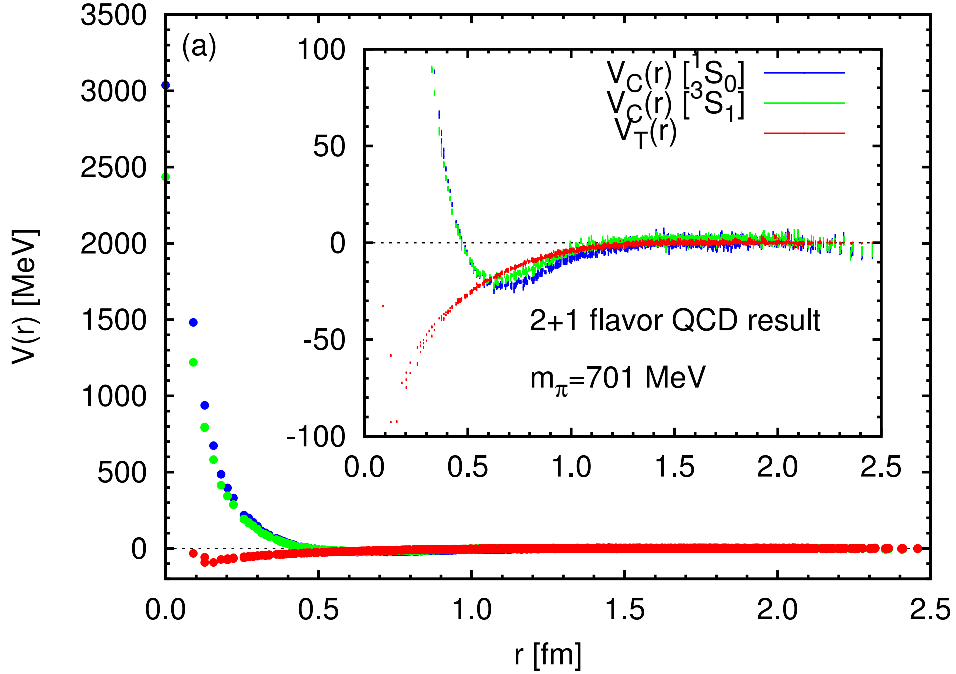
$$m_\pi L > 4$$

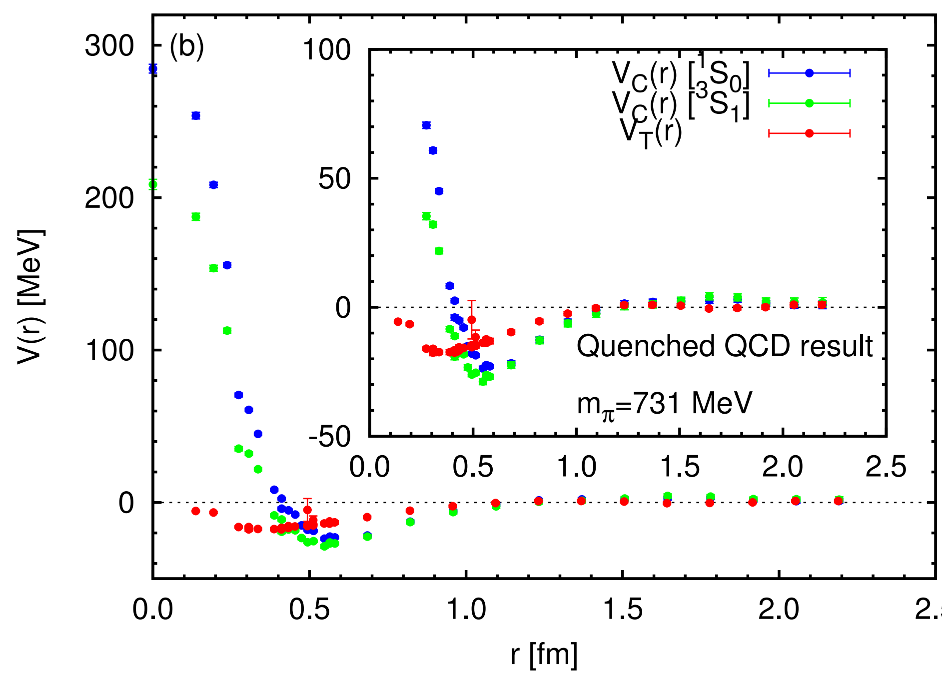
“Real QCD”

cf. BMW collaboration

full QCD

quenched QCD

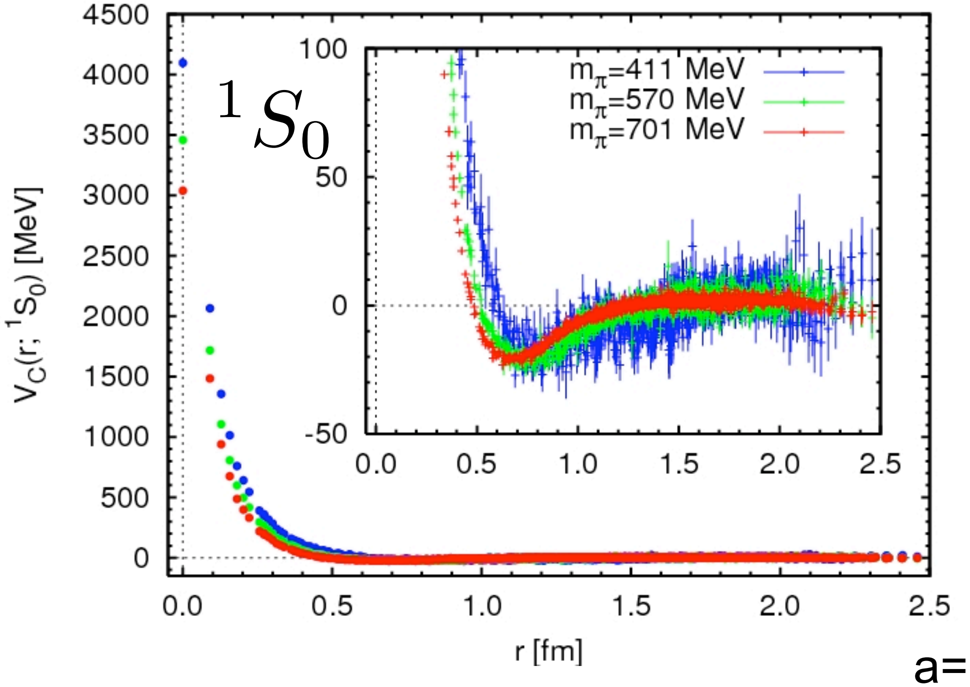


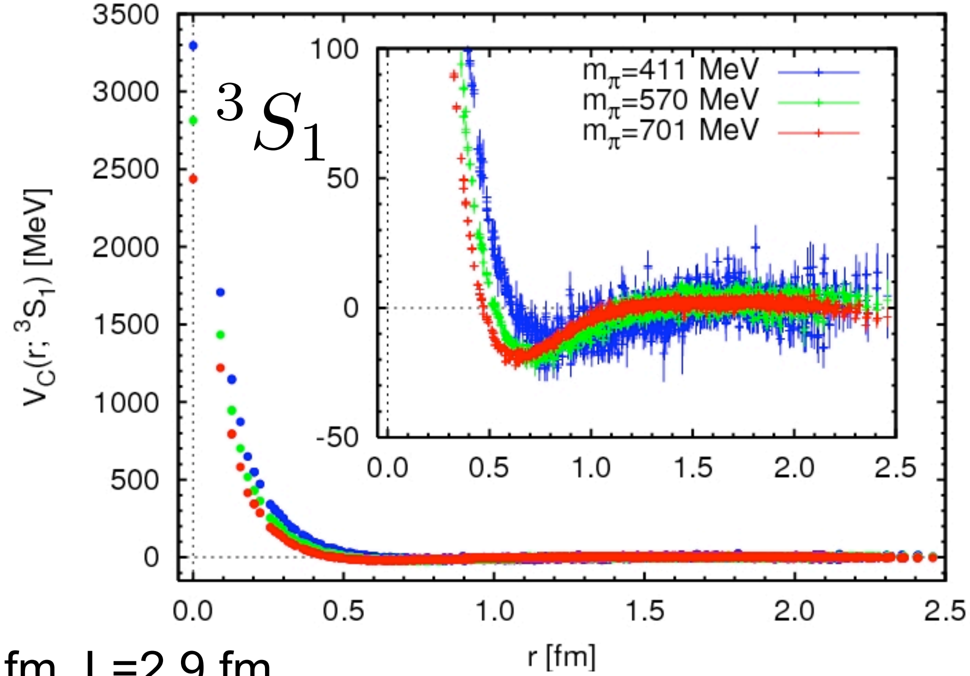


$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm $\quad a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

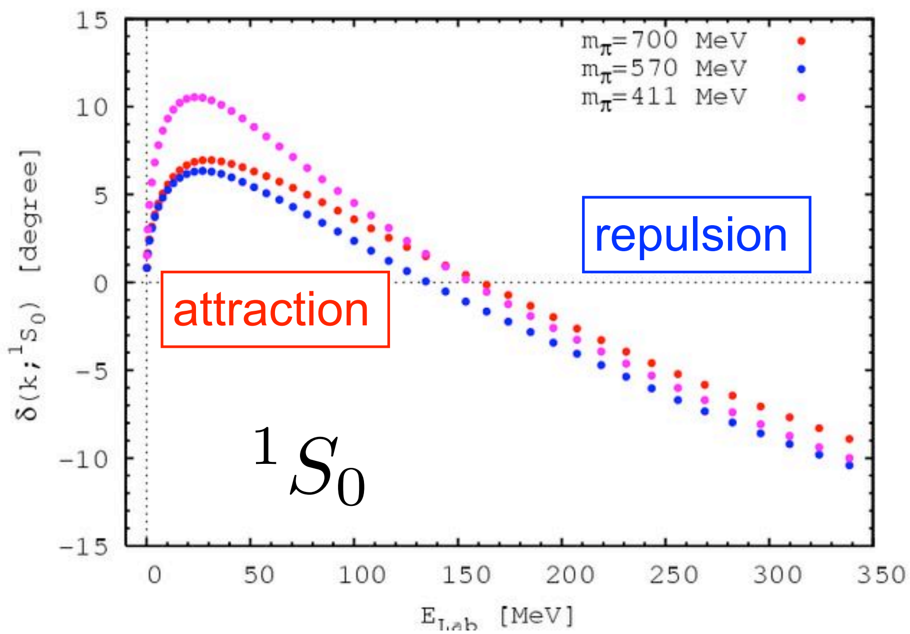
- both repulsive core at short distance and the tensor potential are enhanced in full QCD.
- the attraction at medium distance is shifted to outer region, while the magnitude remains almost unchanged.
- these differences may be caused by dynamical quark effects.
    - a more controlled comparison is needed.

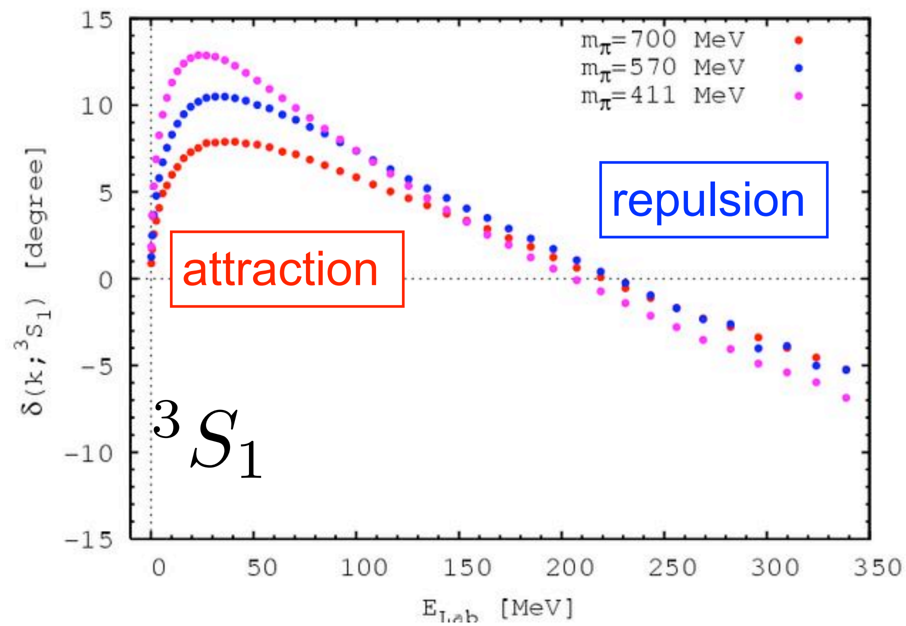
Phase shift from V(r) in full QCD

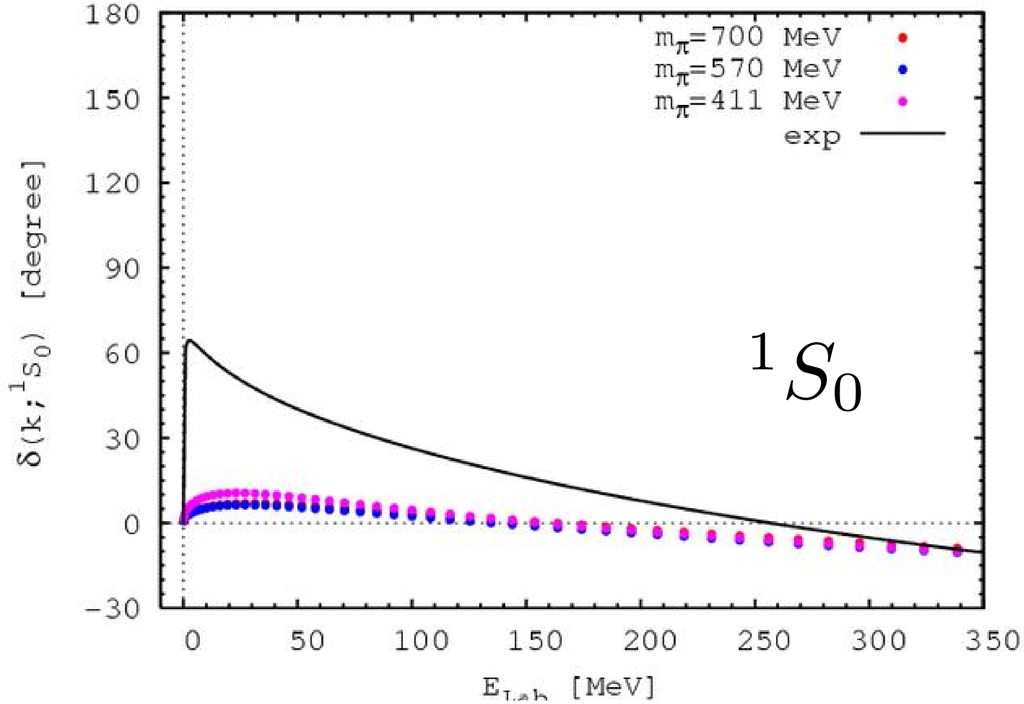


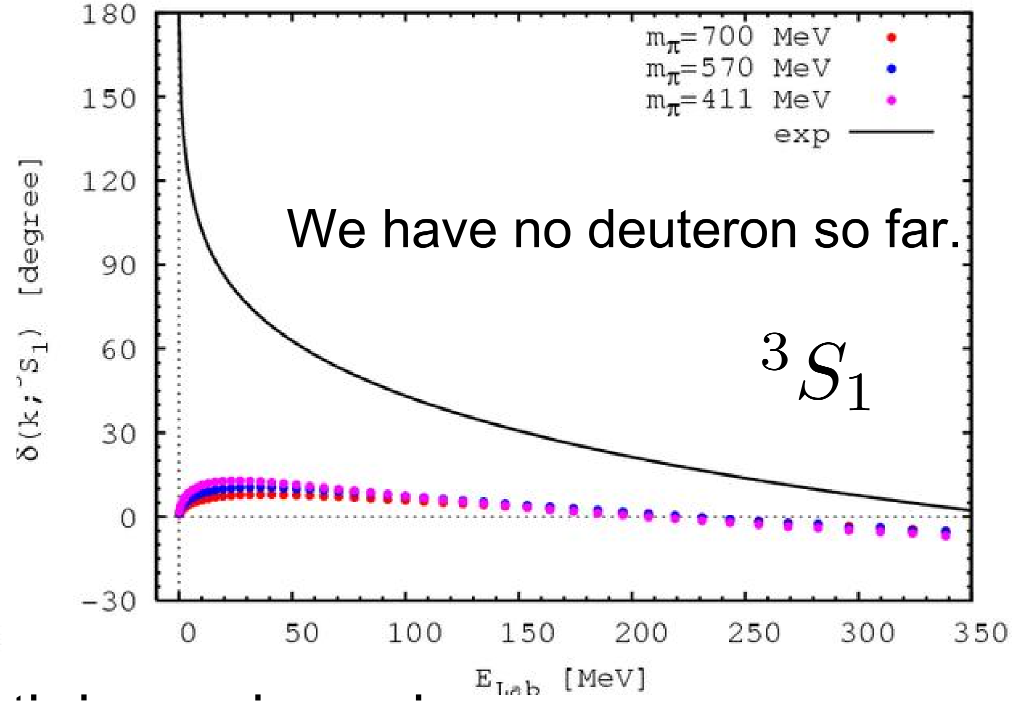


a=0.1 fm, L=2.9 fm





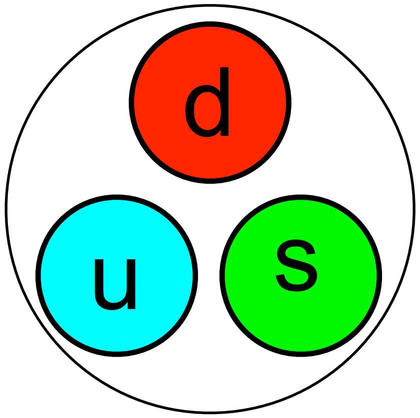




They have reasonable shapes. The strength is much weaker, though.

calculation at physical quark mass is important. (future work)

5. Hyperon interactions



Hyperon:

Baryon (3 quark state) which contains one or more strange quarks.

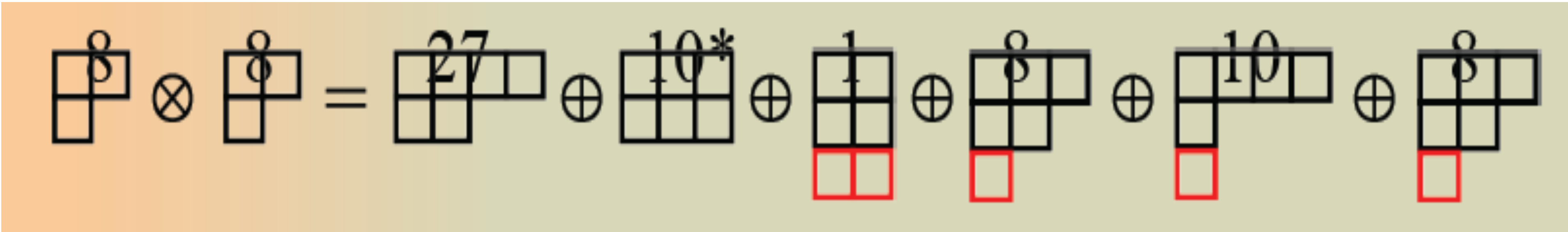
$$p = (uud), n = (udd)$$ nucleon(N)

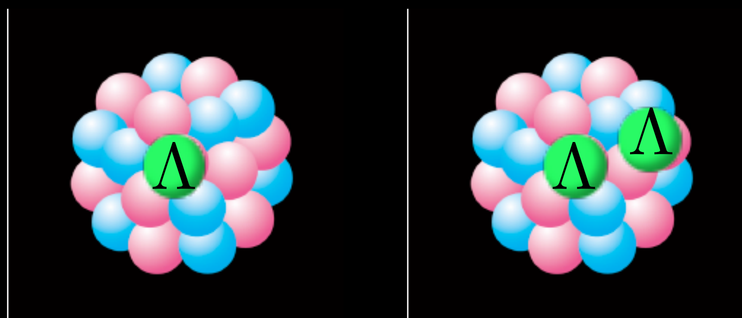
$$\Lambda = (uds)_{I=0}$$

$$\Sigma^+ = (uus), \Sigma^0 = (uds)_{I=1}, \Sigma^- = (dds)$$ hyperon(Y)

$$\Xi^0 = (udd), \Xi^- = (dds)$$

Octet Baryon interactions



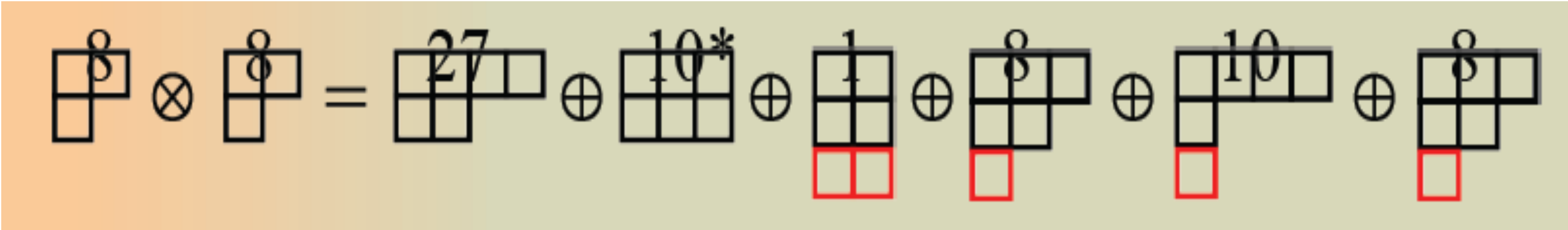


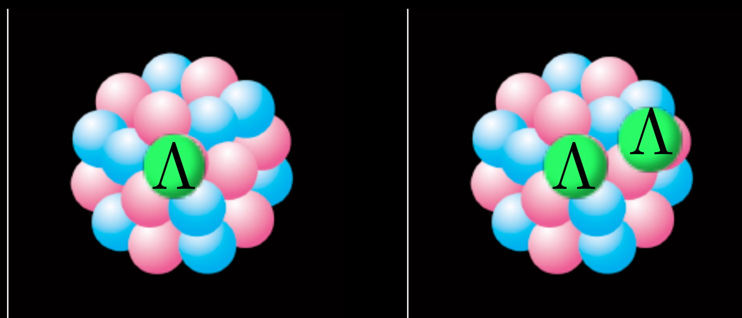
- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

also in GSI

- prediction from lattice QCD
- difference between NN and YN ?

Octet Baryon interactions





- no phase shift available for YN and YY scattering
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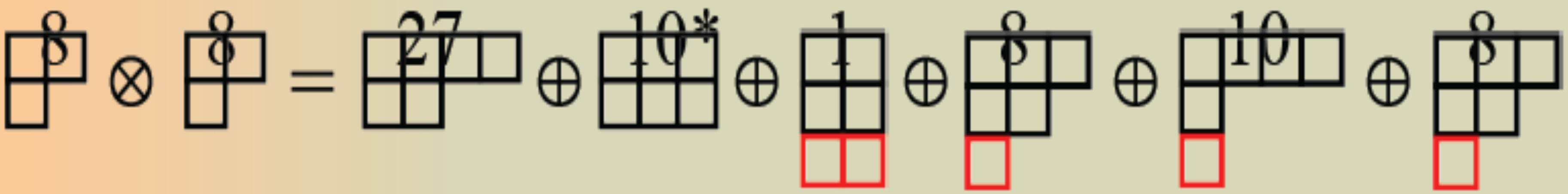
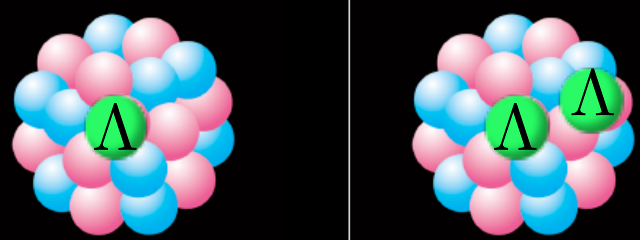
Octet Baryon interactions





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also in GSI

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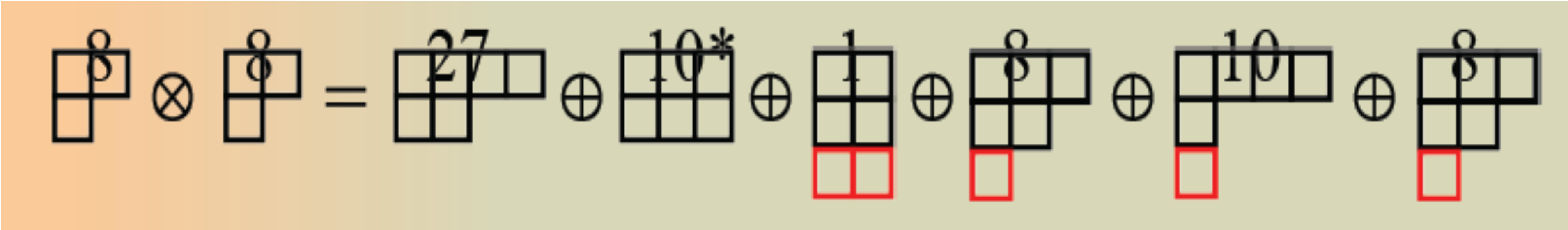
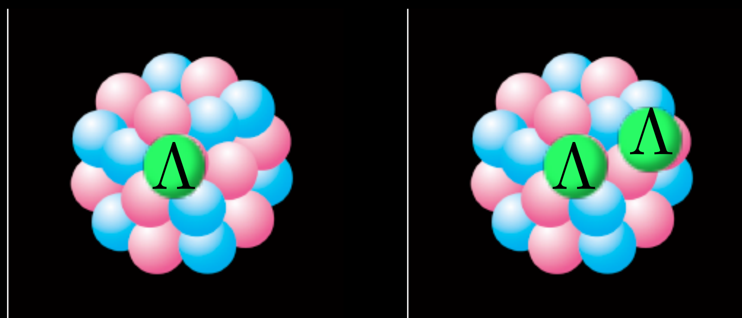
Octet Baryon interactions





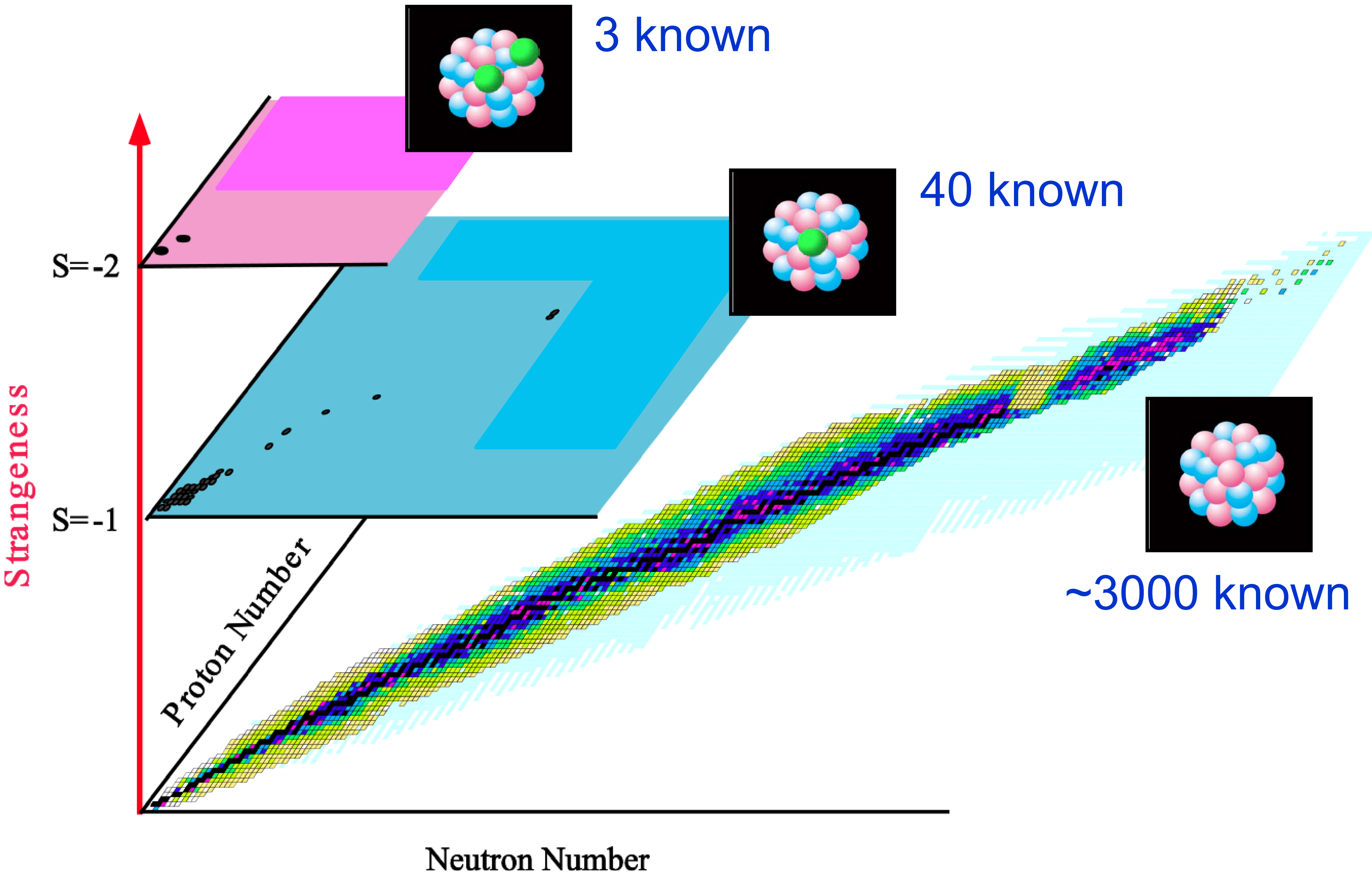
- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

damaged by the Earthquake

also in GSI

- prediction from lattice QCD
- difference between NN and YN ?

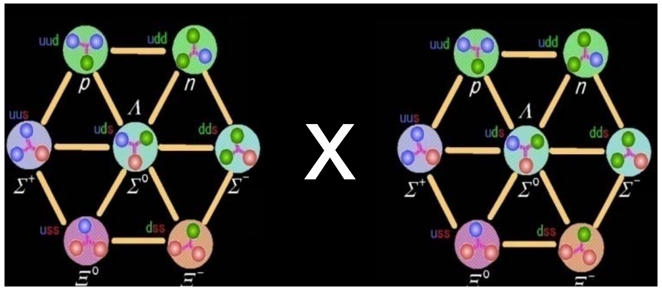
3D Nuclear chart



Baryon-Baryon interactions in an SU(3) symmetric world

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric Anti-symmetric

6 independent potential in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \leftarrow {}^1S_0$$

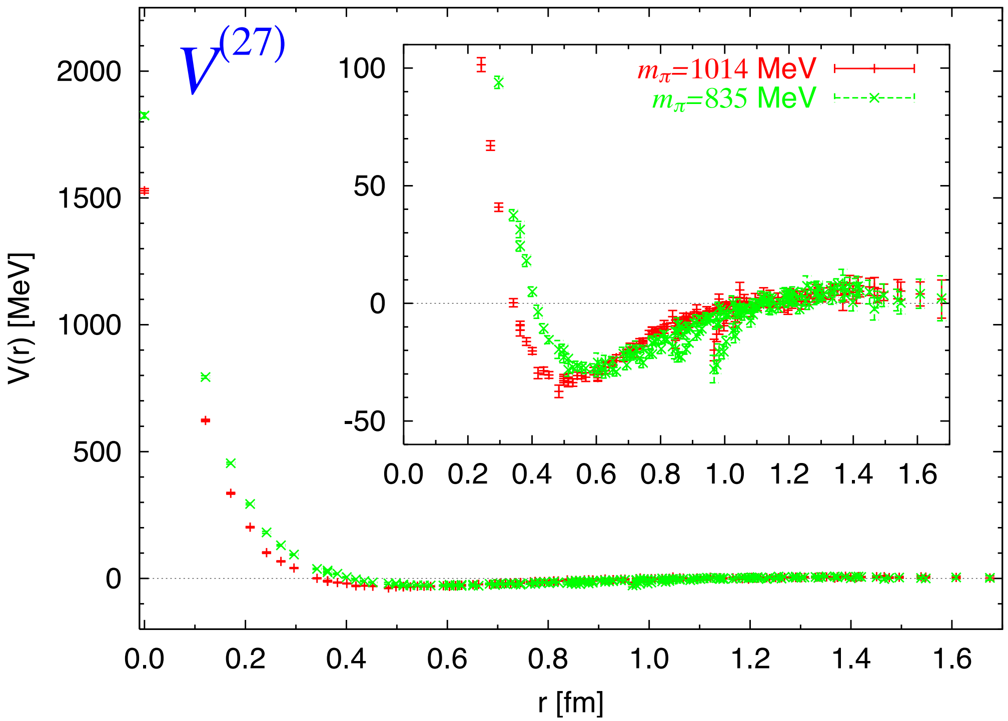
$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \leftarrow {}^3S_1$$

Potentials in full QCD

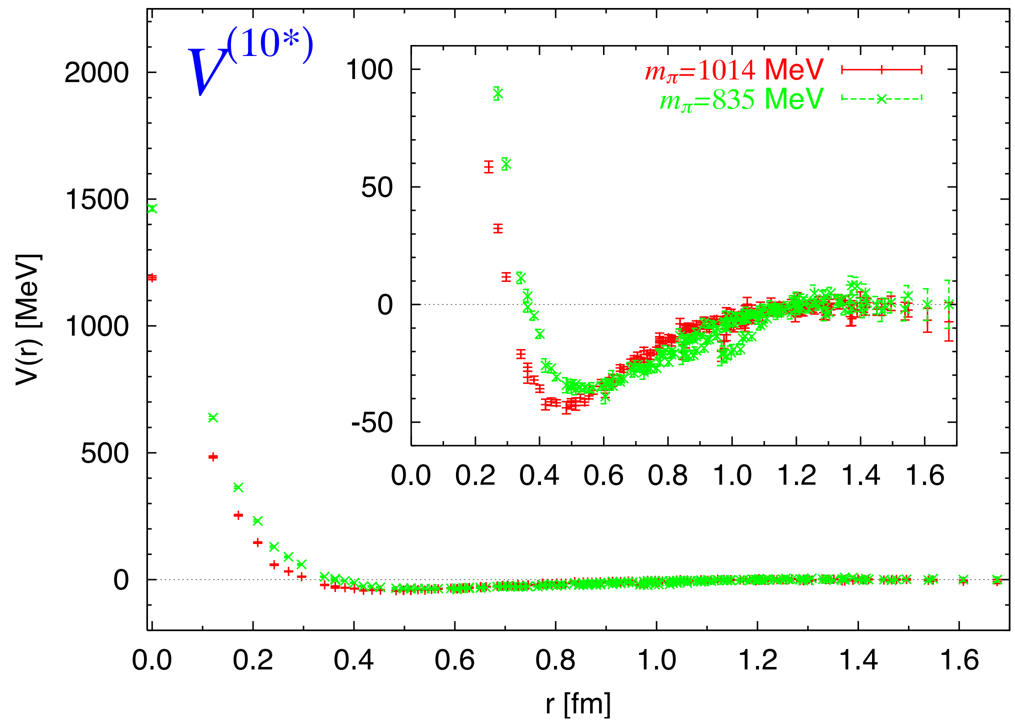
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591

a=0.12 fm, L=2 fm BG/L@KEK

Spin-singlet

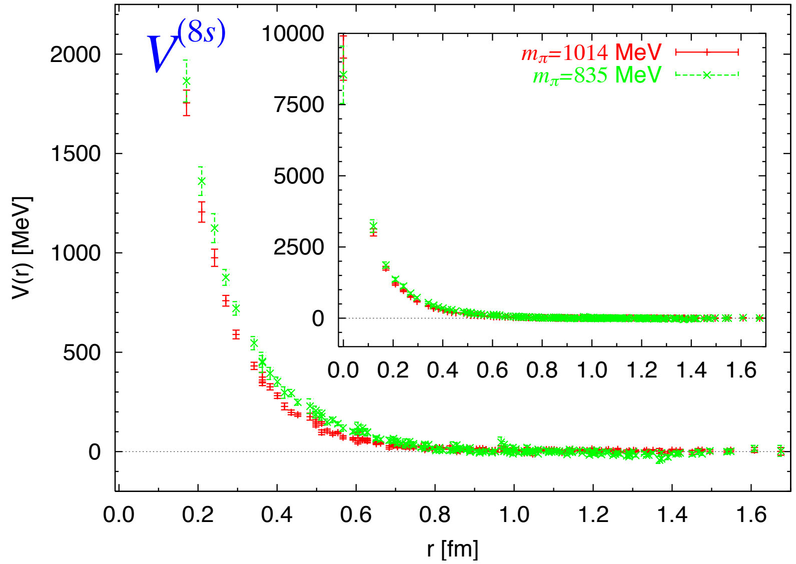


Spin-triplet

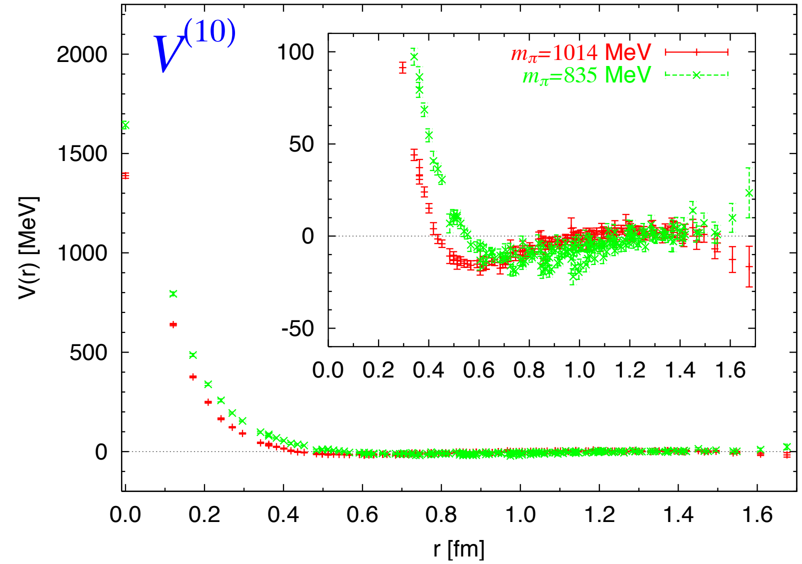


27, 10*: same behaviors as NN potentials

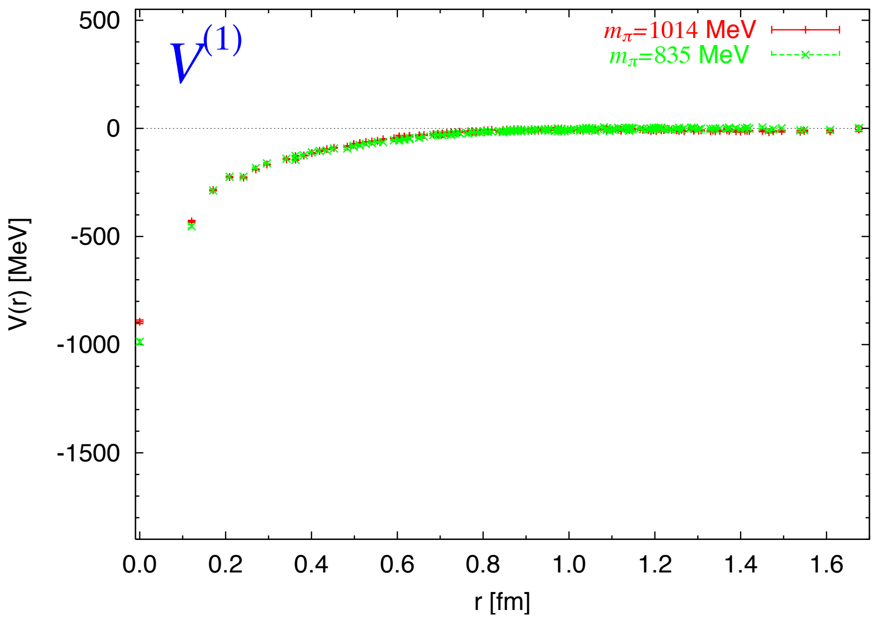
flavor multiplet	baryon pair (isospin)
27	{NN}(I=1), {NΣ}(I=3/2), {ΣΣ}(I=2), {ΣΞ}(I=3/2), {ΞΞ}(I=1)
$\mathbf{8}_s$	none
1	none
$\mathbf{10}^*$	[NN](I=0), [ΣΞ](I=3/2)
10	[NΣ](I=3/2), [ΞΞ](I=0)
$\mathbf{8}_a$	[NΞ](I=0)



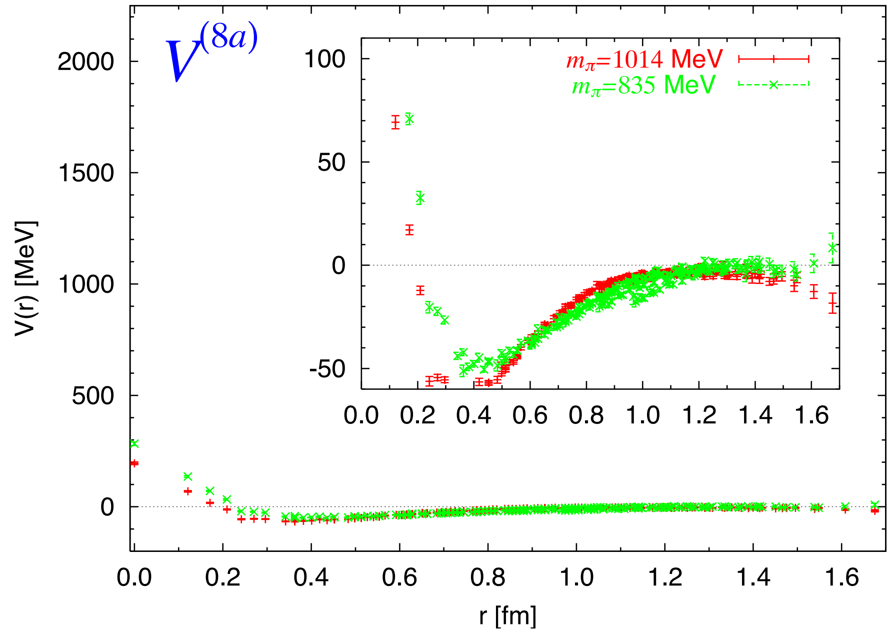
8s: strong repulsive core.
repulsion only.



10: strong repulsive core.
weak attraction.



1: attractive instead of repulsive core ! attraction only . bound state ?



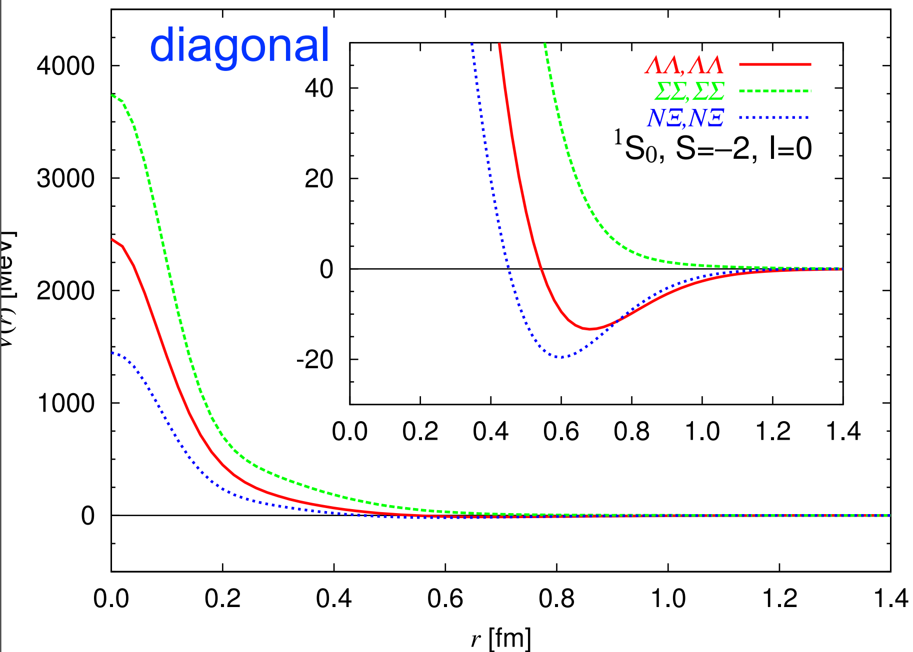
8a: weak repulsive core.
strong attraction.

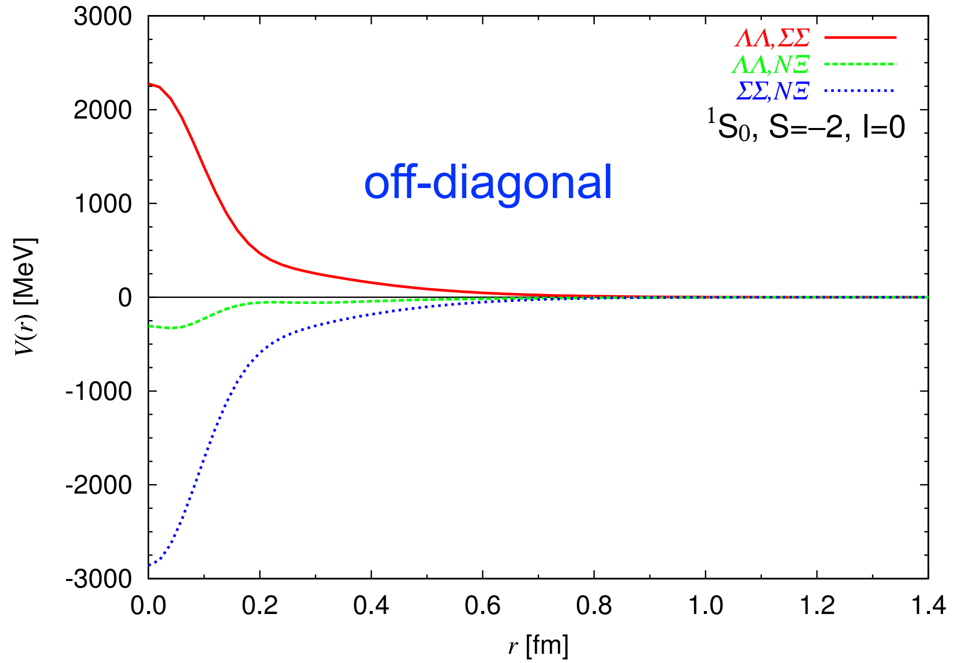
Potential matrix in “particle basis” (S=-2, I=0, spin-singlet)

particle   flavor

$$V_{ij}(r) = \sum_X U_{iX} V^{(X)}(r) U^\dagger_{Xj}$$

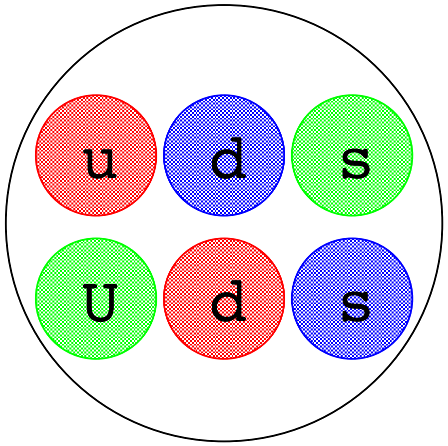
$$\begin{pmatrix} |\Lambda\Lambda\rangle \\ |\Sigma\Sigma\rangle \\ |N\Xi\rangle \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} |\mathbf{27}\rangle \\ |\mathbf{8}_s\rangle \\ |\mathbf{1}\rangle \end{pmatrix}$$





- attraction appeared in flavor-singlet channel can not be easily seen due to the strong repulsion in 8s channel. ΣΣ is the most repulsive due to the large coupling to 8s.
- NΞ has the strong attraction due to the large coupling to the singlet.
- off-diagonal parts are comparable to diagonal parts in magnitude.
  - full matrix is needed.
- “physics” can be easily seen in the “flavor” basis.

6. H-dibaryons

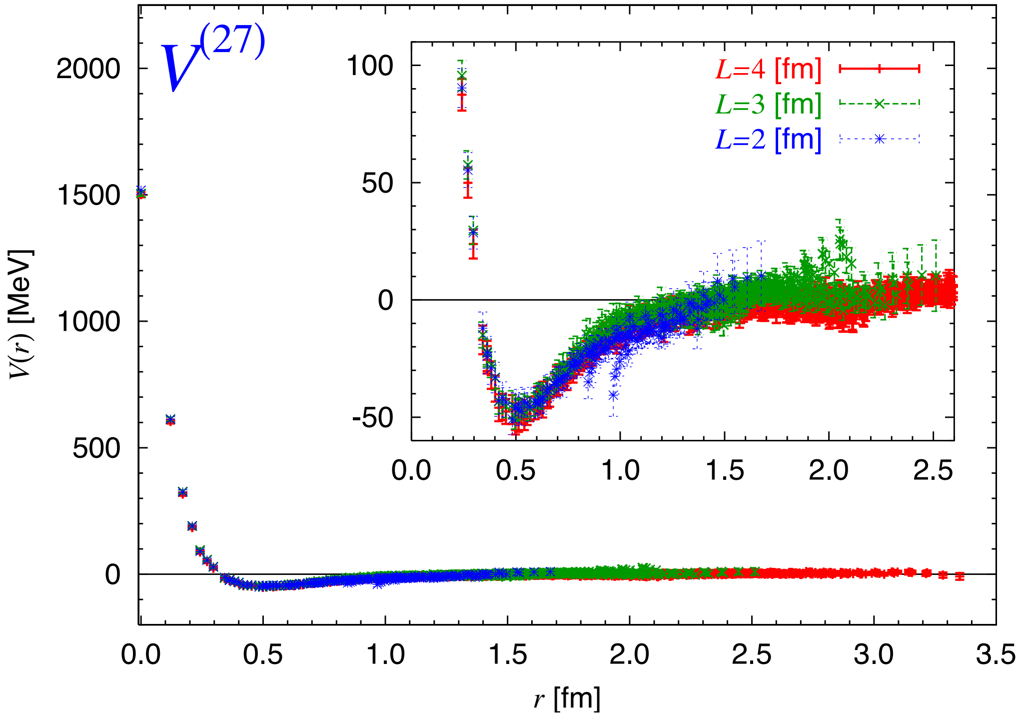


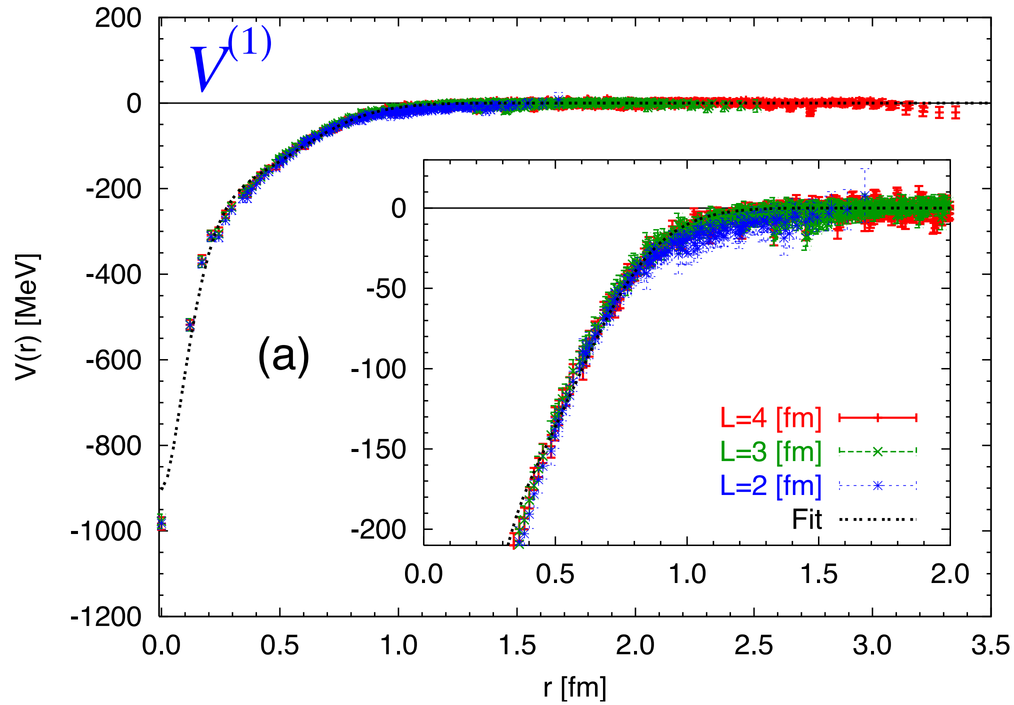
H-dibaryon:
a possible six quark state(uuddss)
predicted by the model but not observed yet.

Bound H dibaryon in flavor SU(3) limit

HAL QCD: Inoue *et al.*, Phys. Rev. Lett. 106(2011) 162002

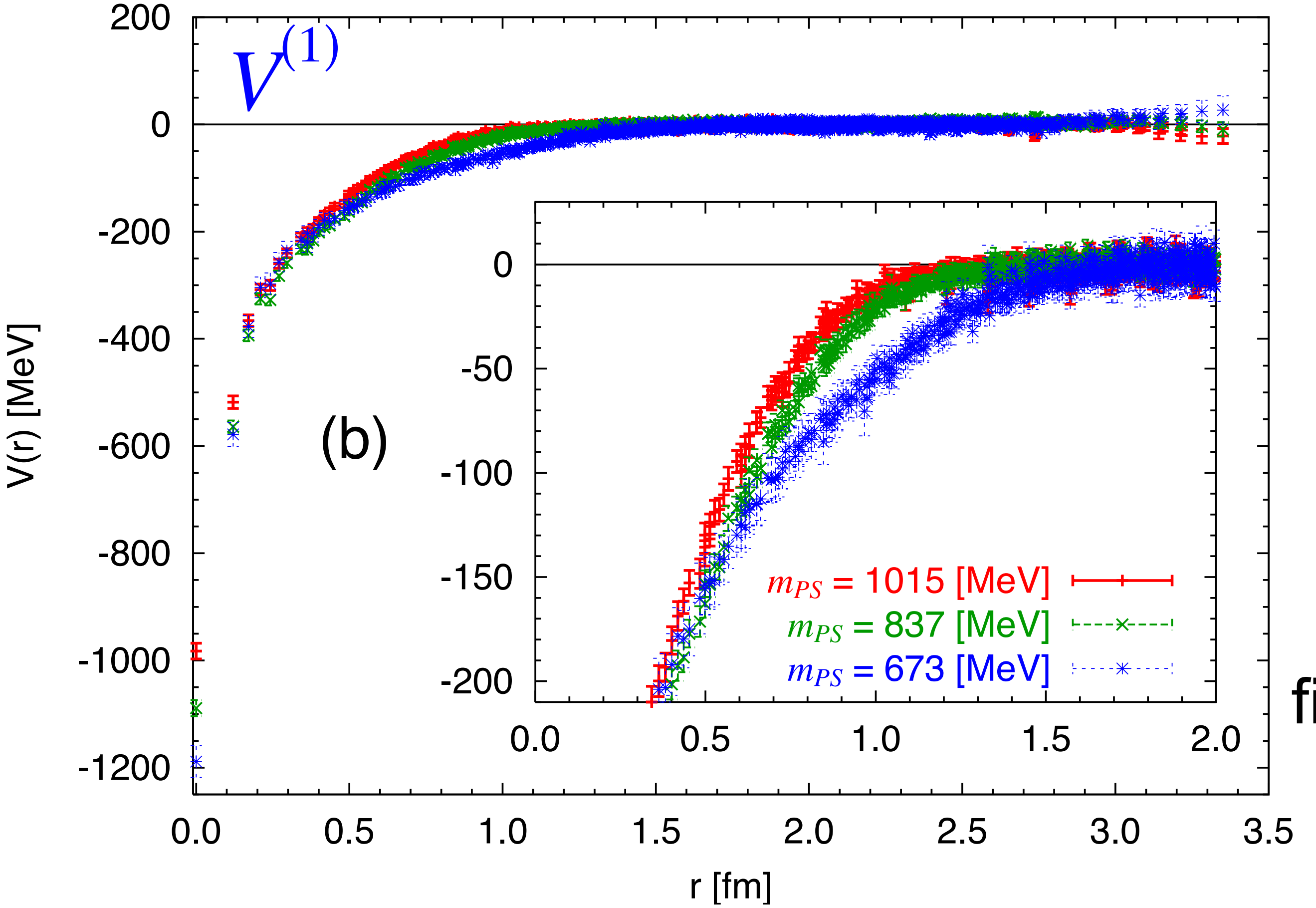
volume dependence





L=3 fm is enough for the potential.

pion mass dependence



lighter the pion mass, stronger the attraction

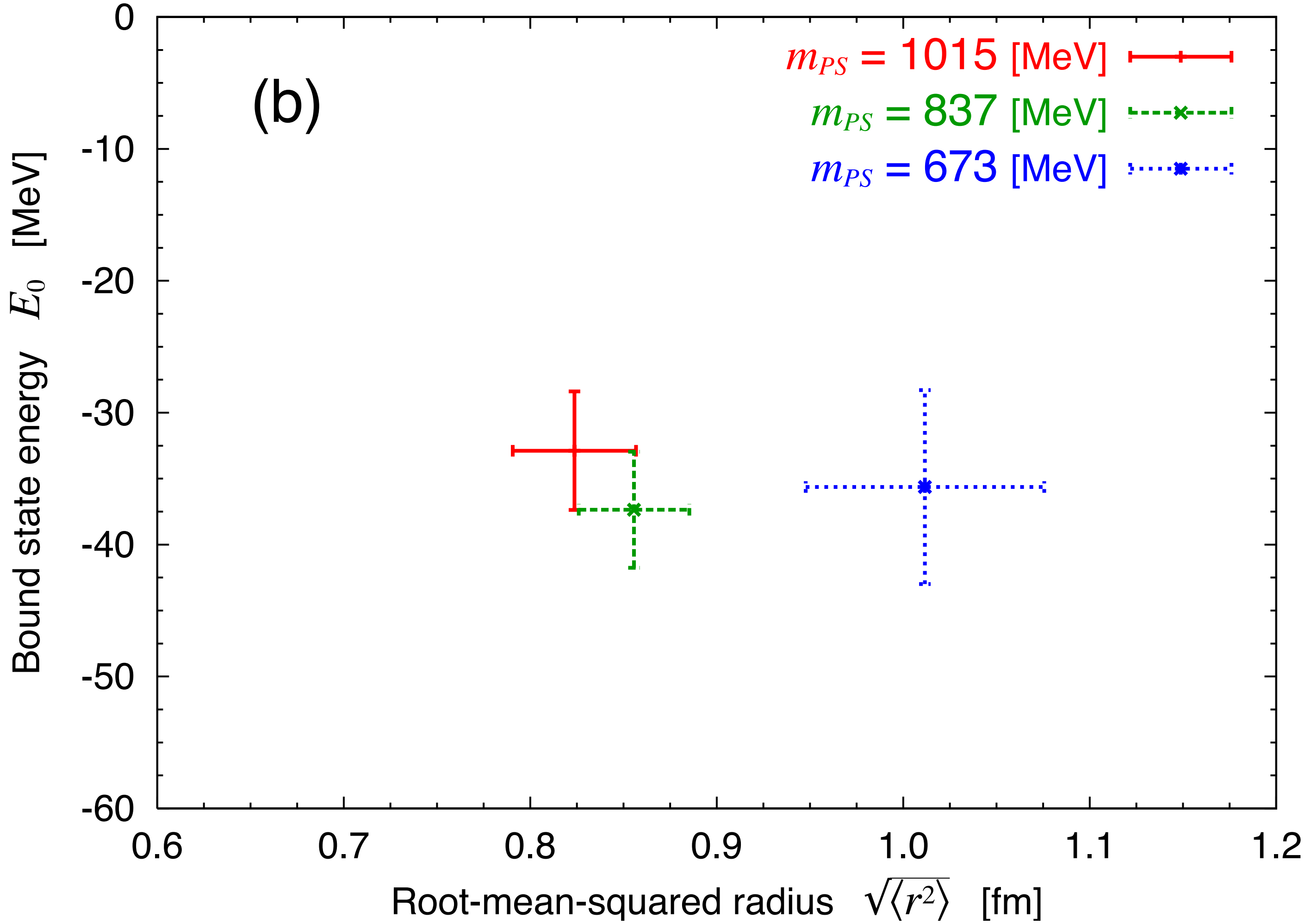
fit the potential at L=4 fm by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

solve Schroedinger equation with this potential in the infinite volume.

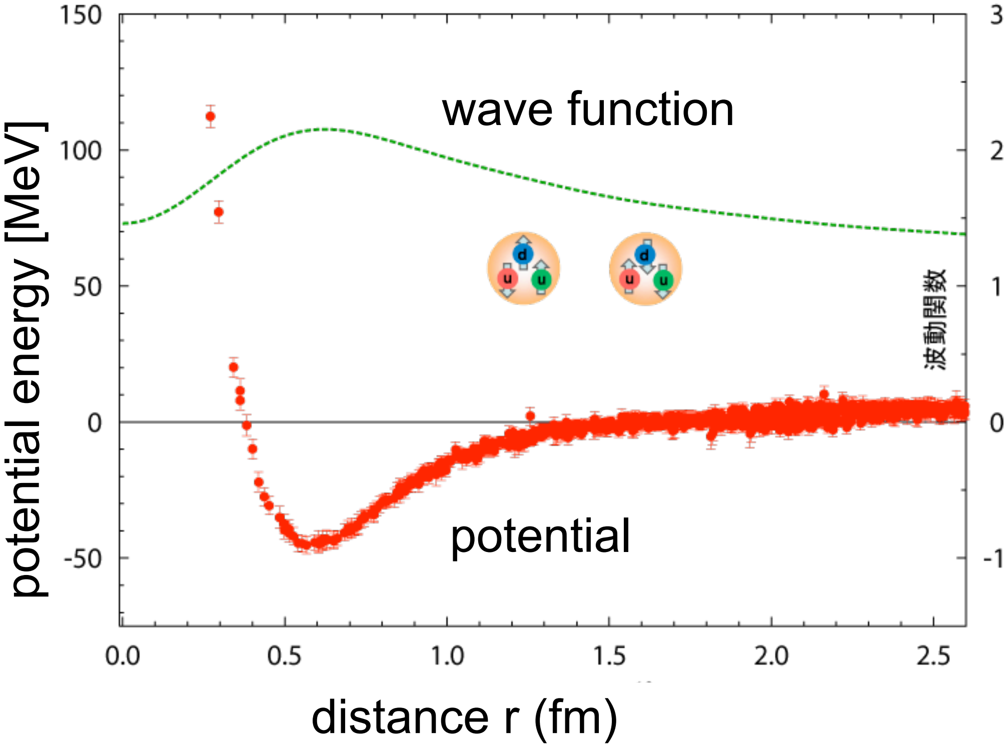
One bound state (H-dibaryon) exists !

pion mass dependence

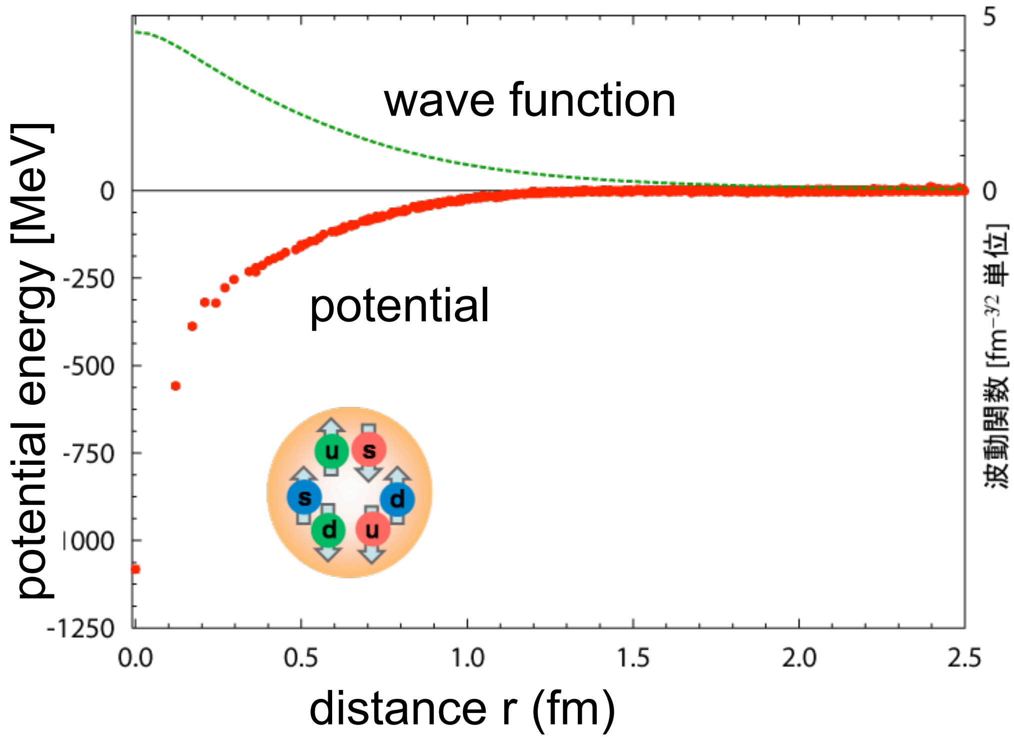


An H-dibaryon exists in the flavor SU(3) limit !
Binding energy = 30-40 MeV,
weak quark mass dependence.

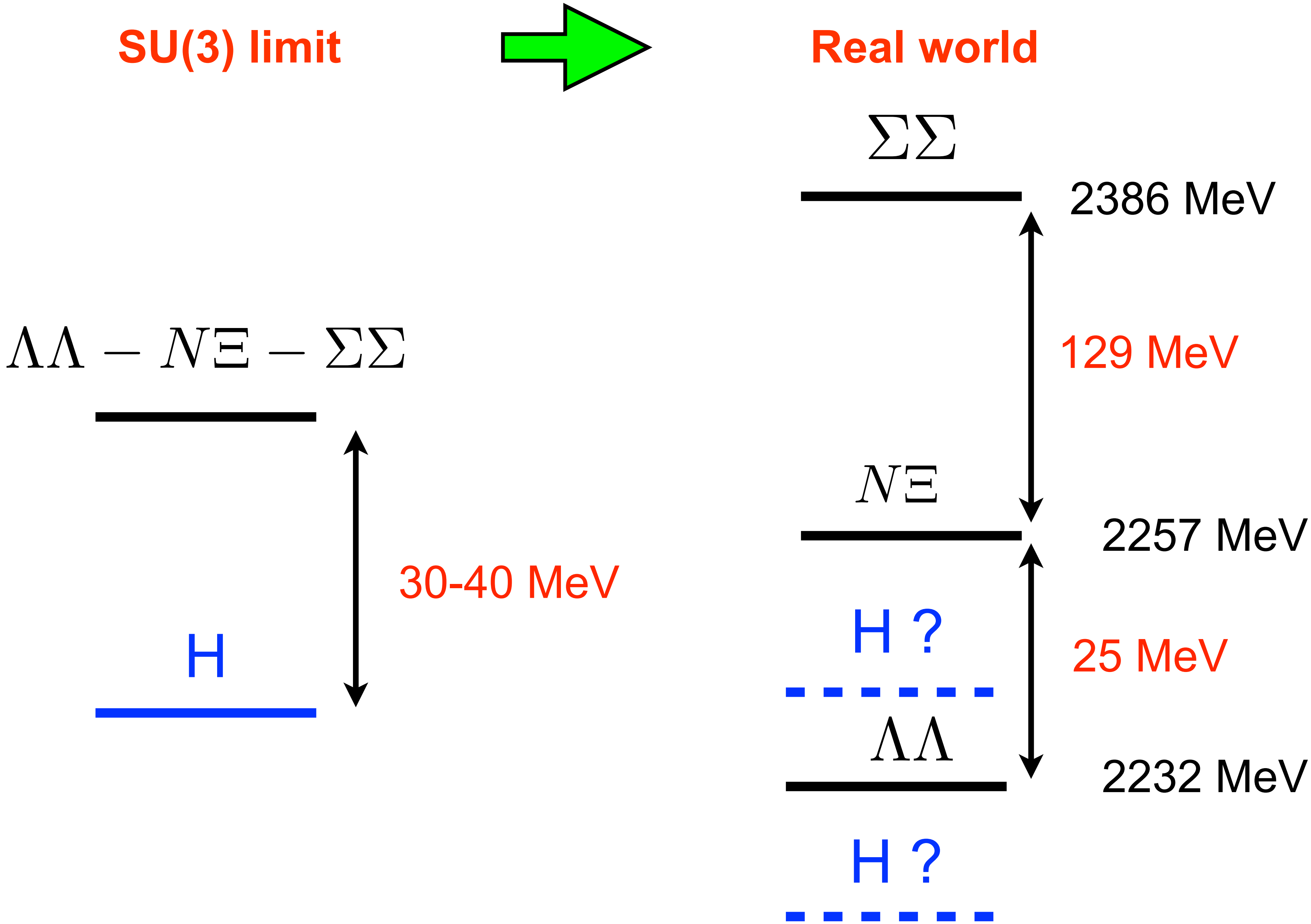
Deuteron



H-dibayon



Real world ?



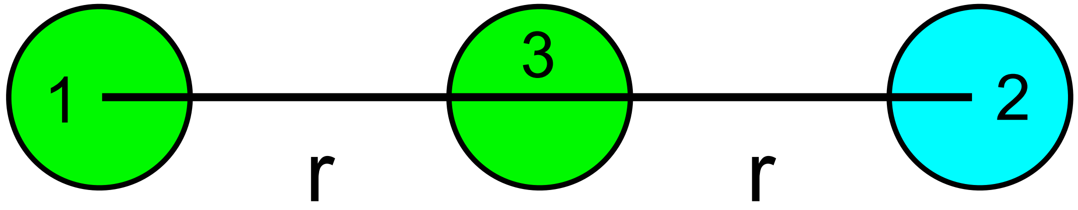
SU(3) limit
Real world
$\Lambda\Lambda - N\Xi - \Sigma\Sigma$
30-40 MeV
H
$\Sigma\Sigma$
2386 MeV
129 MeV
$N\Xi$
2257 MeV
H ?
25 MeV
$\Lambda\Lambda$
2232 MeV
H ?

7. Conclusions

- the potential method is new but very useful to investigate baryon interactions in (lattice) QCD.
- the method can be easily also applied to meson-baryon and meson-meson interactions.
- three body force can be analyzed.
- various extensions of the method will be looked for.

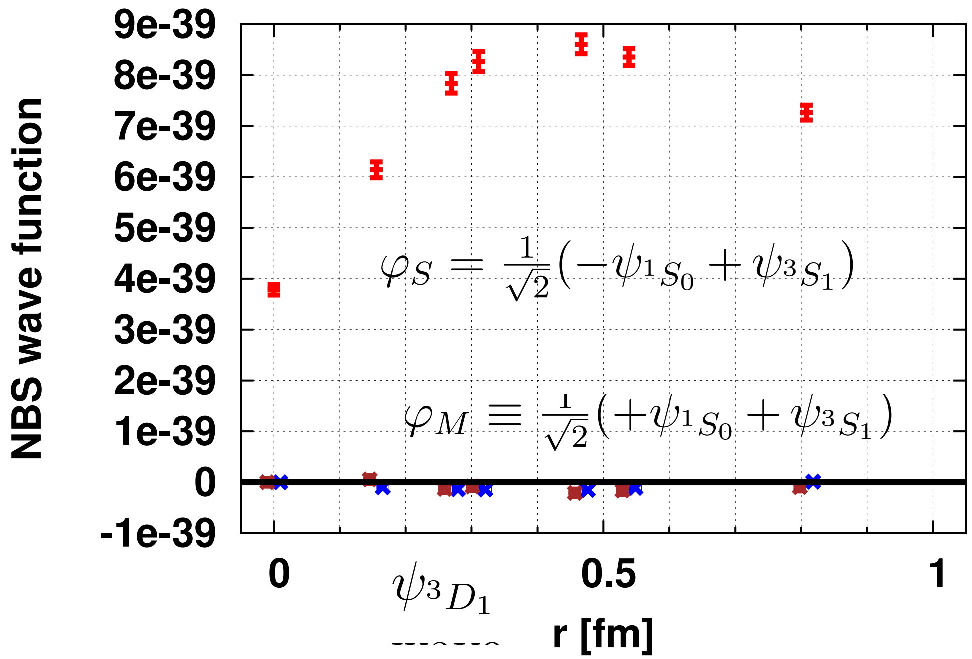
Three nucleon force (TNF)

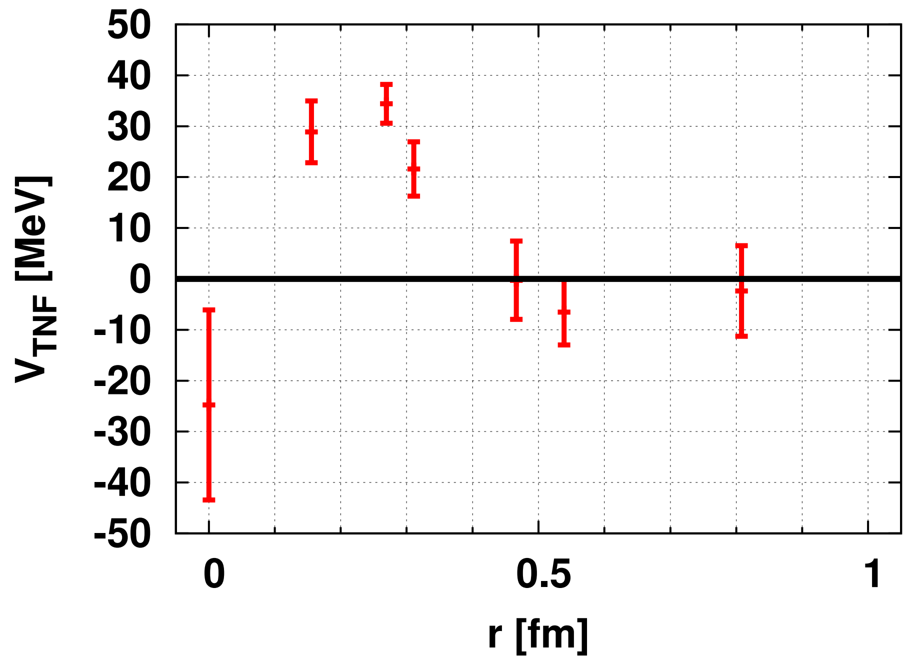
Linear setup



(1,2) pair $\quad {}^1S_0,\ {}^3S_1,\ {}^3D_1 \quad$ S-wave only

$\mathrm{Triton}(I = 1/2,\ J^P = 1/2^+)$

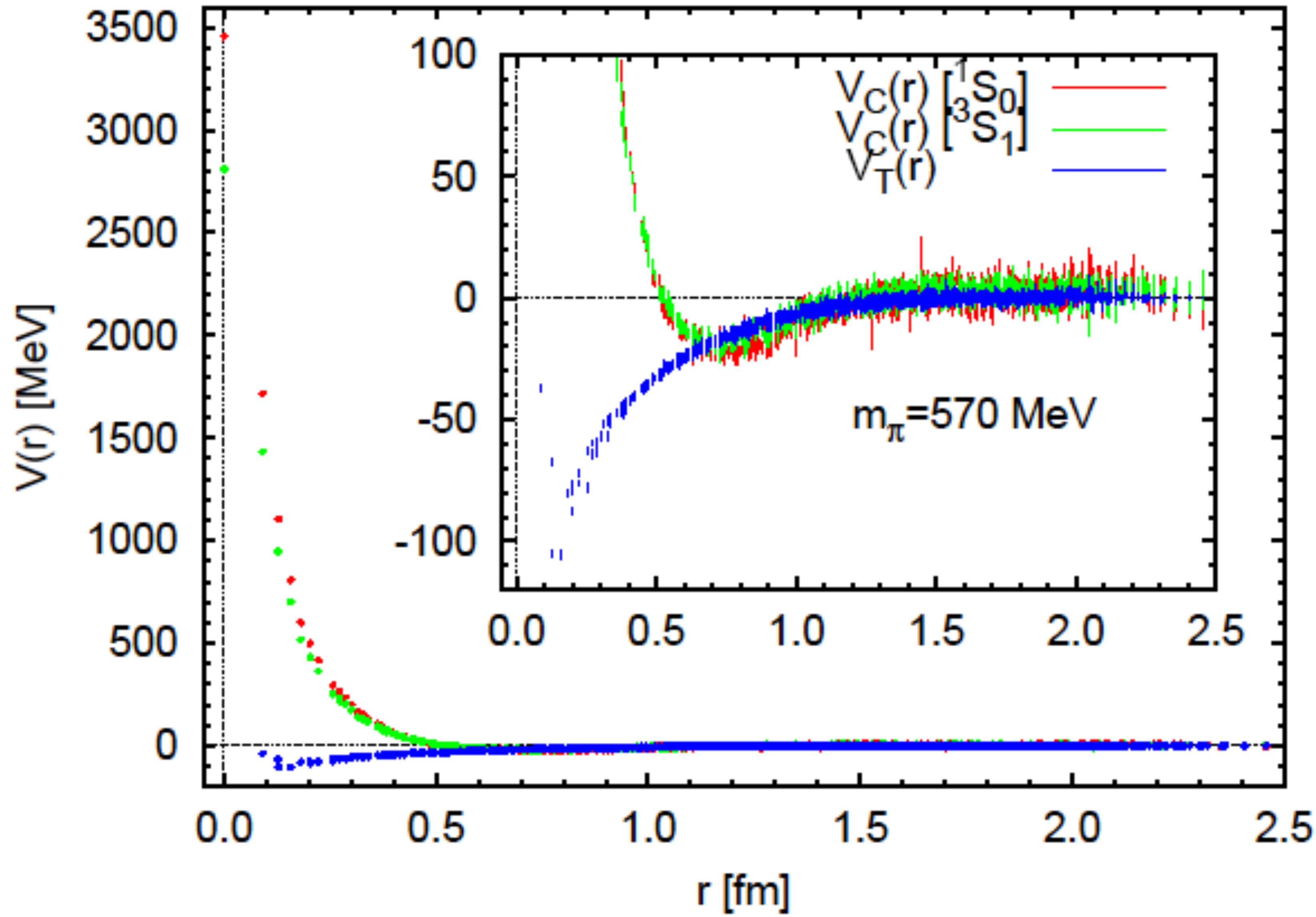




scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

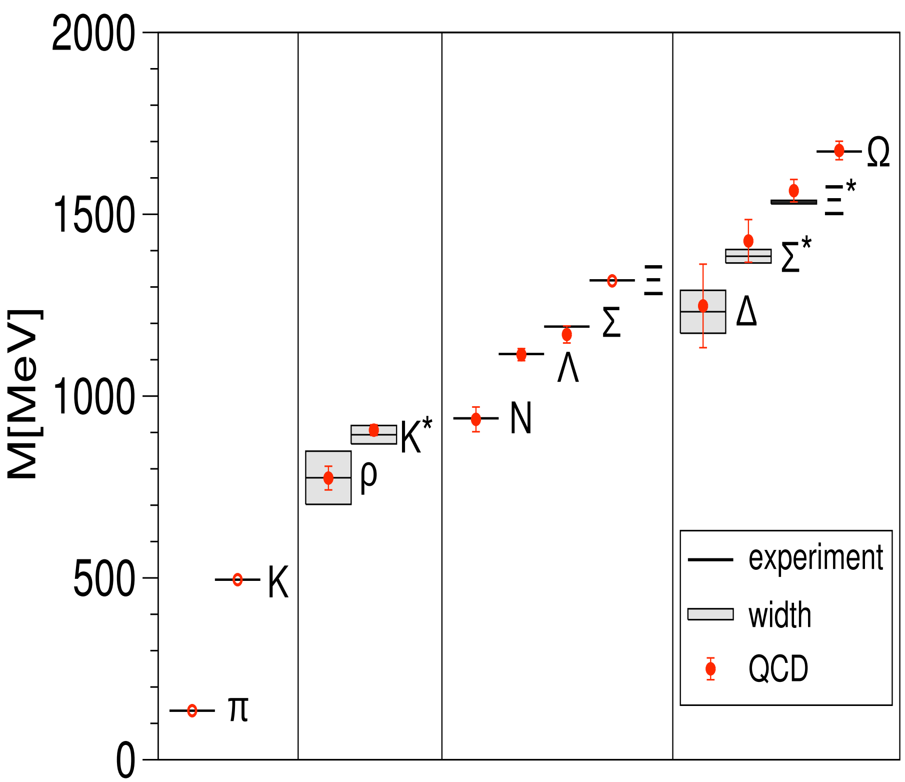
QCD meets Nuclei !



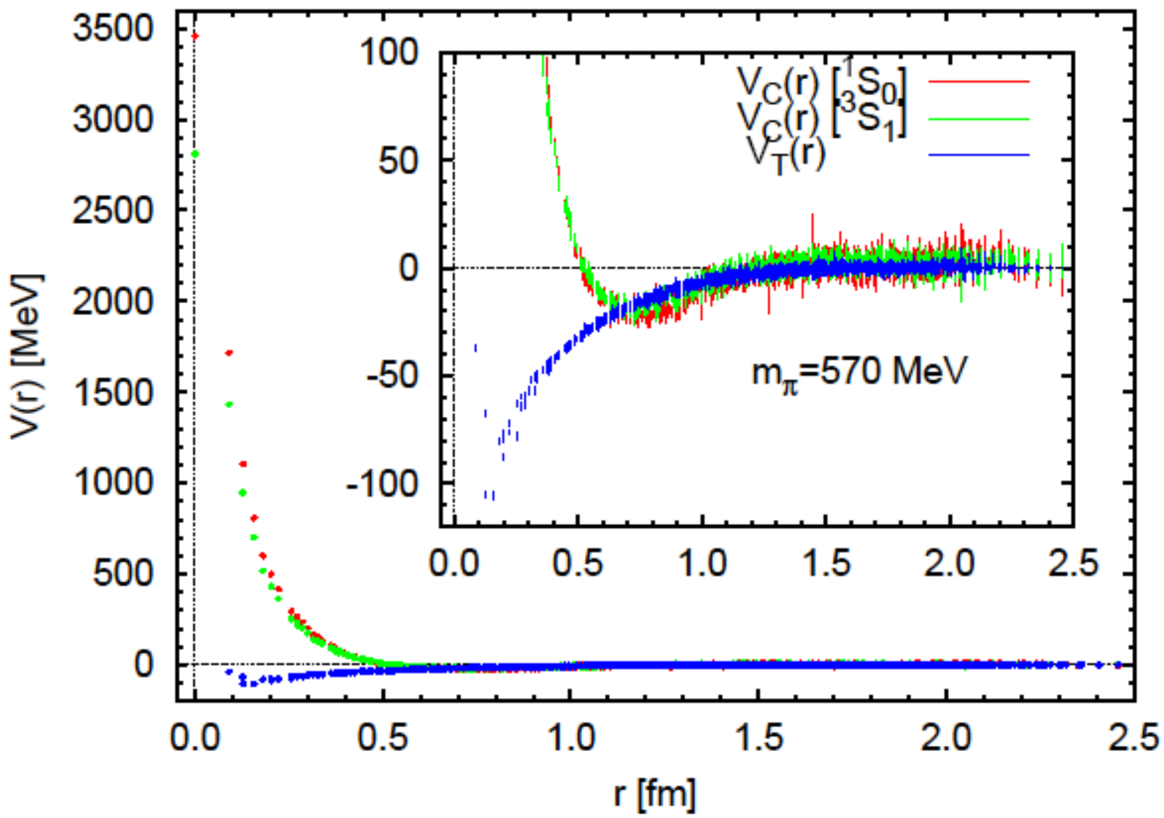
"The achievement is both a computational *tour de force* and a triumph for theory." (Nature Research Highlight 2007)

A possible collaboration between Japan-Germany in near future

BMW collaboration



HAL QCD collaboration



HAL will ride on the BMW ?