Running coupling constant of ten-flavor QCD with the Schroedinger functional method

arXiv:1011.2577 (will appear in PRD)

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in collaboration with

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Symposium on Lattice gauge theory at U. of Wuppertal 2/5/2011

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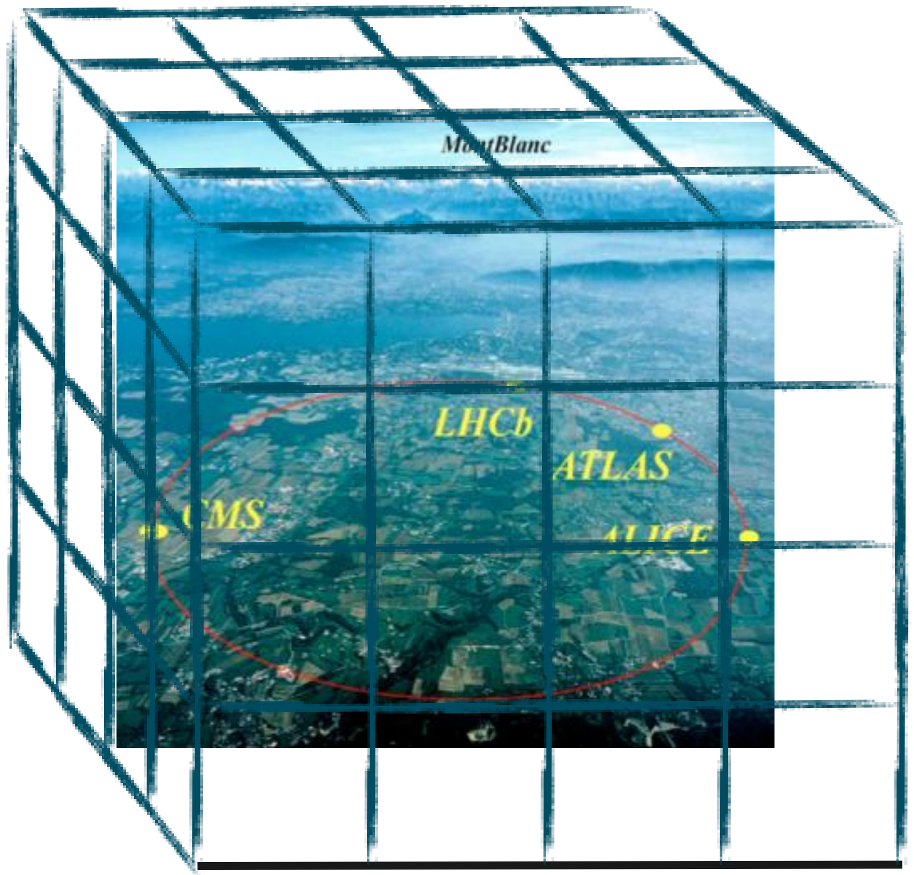
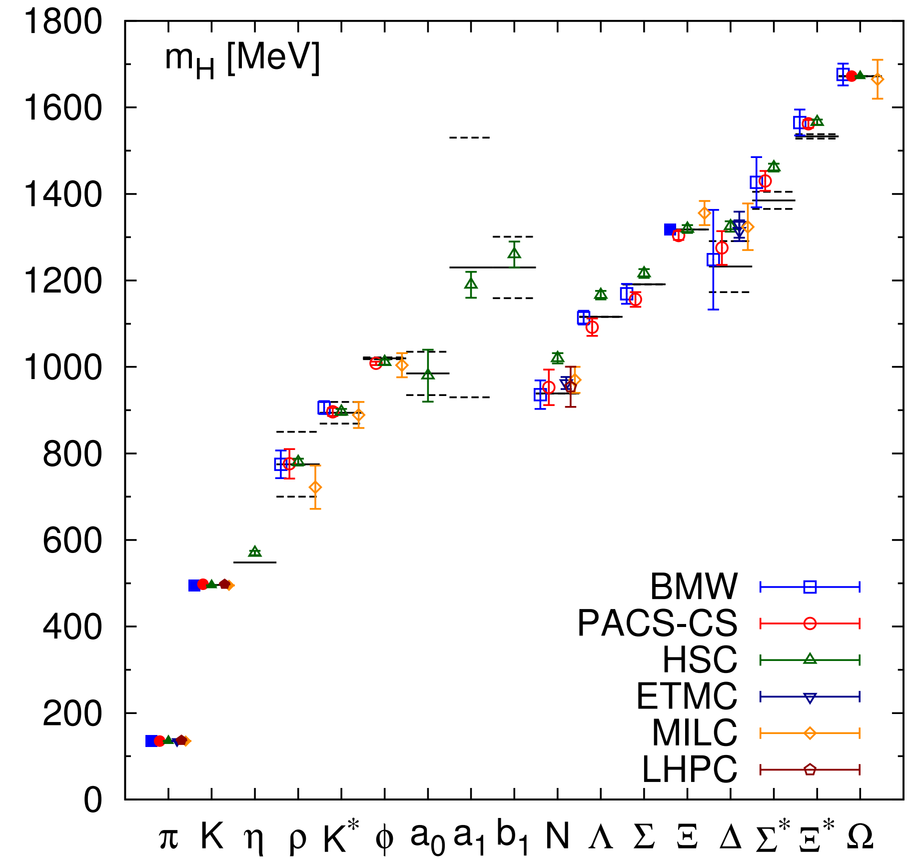


I. Introduction

Successes of Lattice QCD

- Hadron masses and their interactions
- Physics@T≠0
- The SM parameters
- Weak matrix elements
- ...

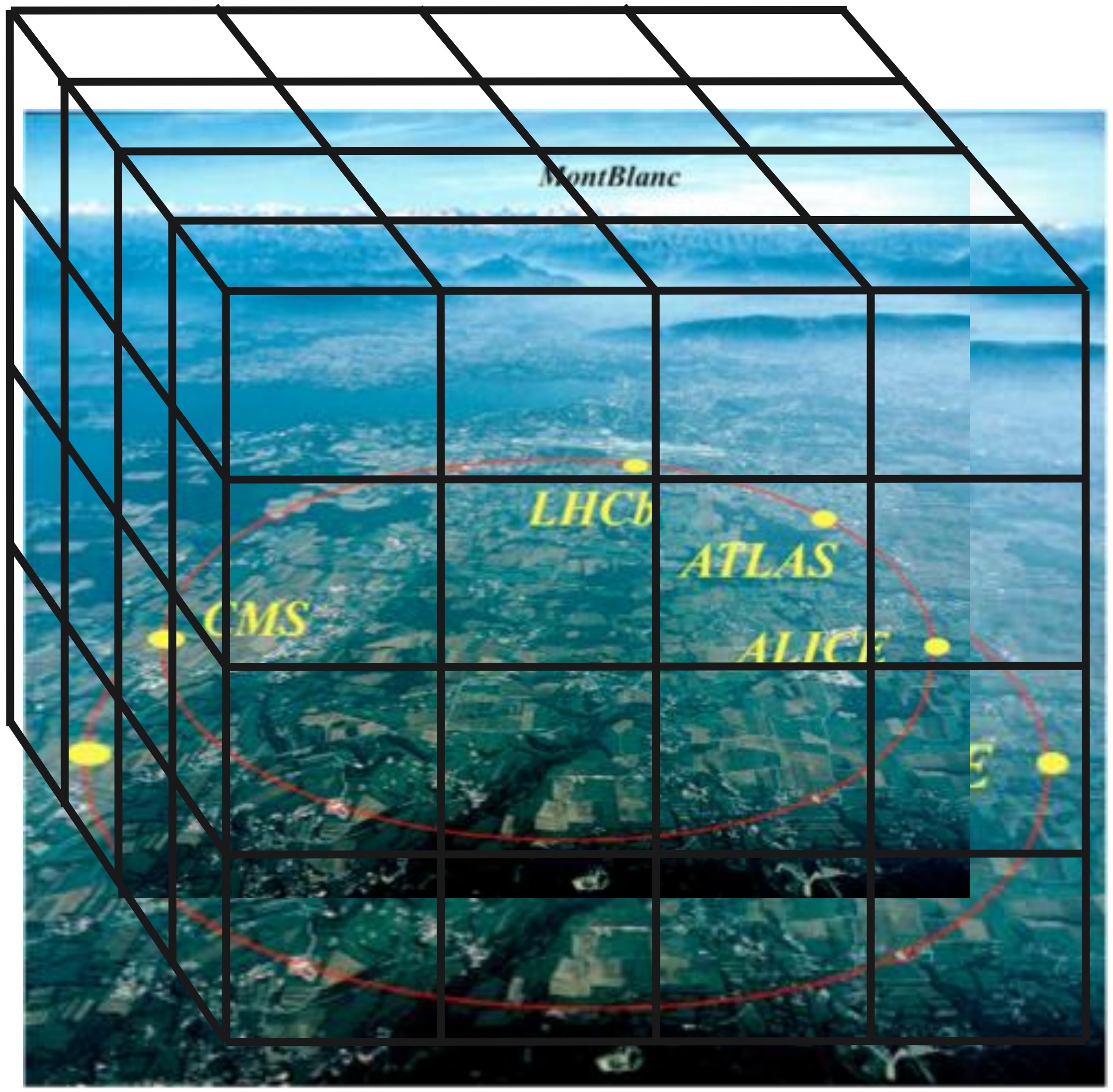
Scholz lattice 2009



Lattice calculations truly reliable.

Apply to Something different

LHC era



- Higgs mechanism
- Physics above the EW scale
- Among many New Physics candidates, Technicolor is attractive and best suited for Lattice Simulation

Use Lattice to explore LHC physics

Technicolor (TC) [Weinberg('79), Susskind('79)]

- Alternative for Higgs sector of SM model
- No fine tuning

→New strong interaction $SU(N_{TC})$: technicolor

→At $\Lambda_{TC} \sim v_{weak}$, technifermion condensate $\langle T_R T_L \rangle$

→Gives dynamical $SU(2)_L \times U(1)_Y$ breaking

→ $\langle TT \rangle$ breaks chiral symmetry $SU(2)_L \times SU(2)_R$

→Produce technipion π_{TC}(NG bosons)

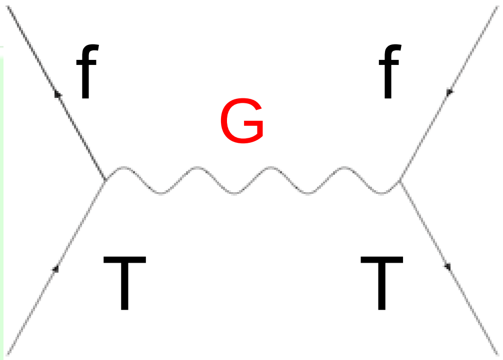
→π_{TC} become longitudinal components of W and Z

→$M_W = M_z \cos\theta_W = \frac{1}{2} g F_\pi$ ($F_\pi = v_{weak}$=246 GeV)

Extended TC

Eichten('80) Susskind('79)

How do SM fermion get mass? ⇒ Extended TC



➜ New gauge theory SU(N_{ETC}), N_{ETC}>N_{TC}

T_{ETC}=(T_{TC},f_{SM})

➜ Assuming SSB: SU(N_{ETC}) ➔ SU(N_{TC})xSM at $\Lambda_{ETC} >> \Lambda_{TC}$

at $\Lambda_{\rm TC}$ scale,

$$\frac{1}{\Lambda_{ETC}^2}\bar{T}T\bar{f}f \Longrightarrow m_f = \frac{\langle TT\rangle_{ETC}}{\Lambda_{ETC}^2} \quad : \text{fermion mass}$$

$\langle TT\rangle_{ETC} \to \Lambda_{\rm TC}^3$

$$\frac{1}{\Lambda_{ETC}^2}\bar{f}f\bar{f}f \quad : \text{FCNC}$$

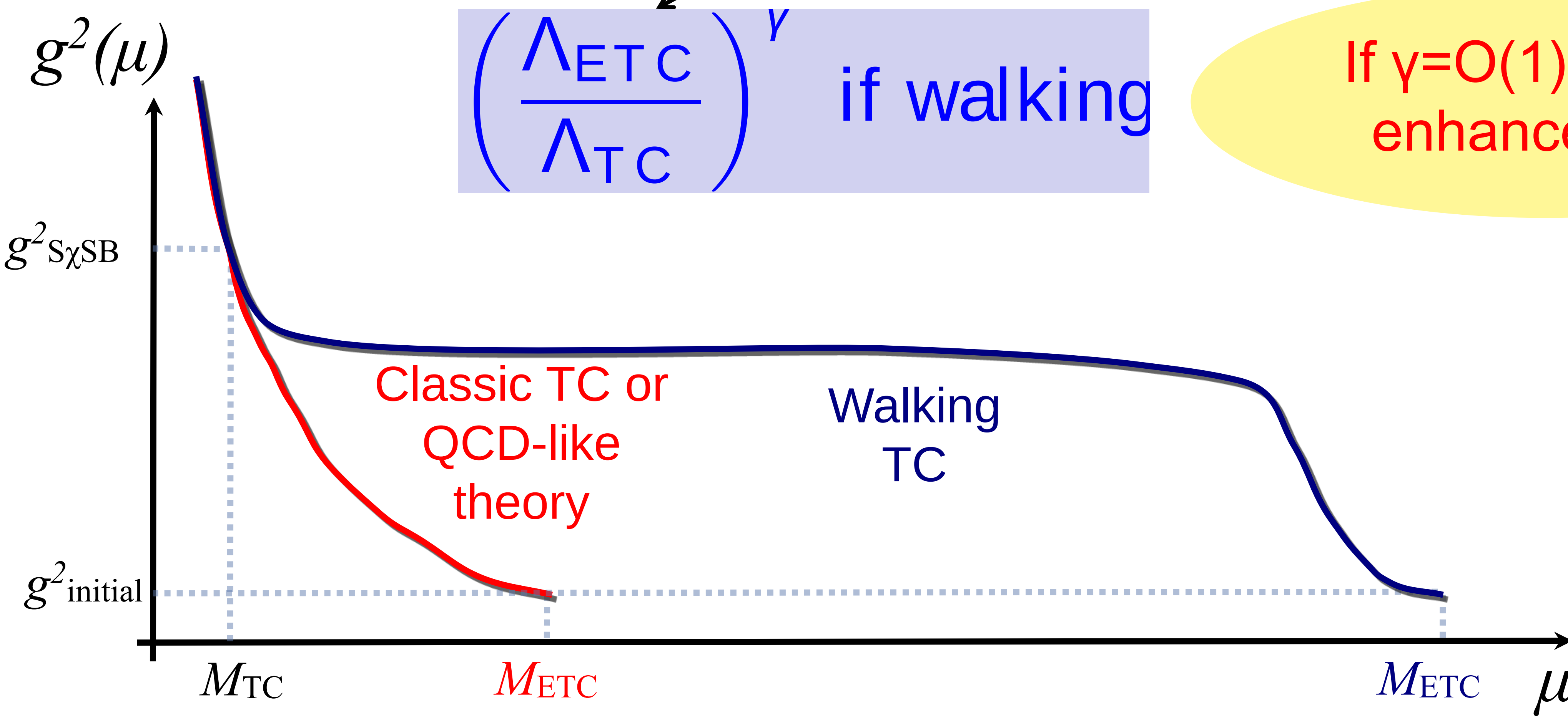
Walking TC Holdom('81) Yamawaki et al. ('86)

Solving tension of SM fermion mass VS. FCNC

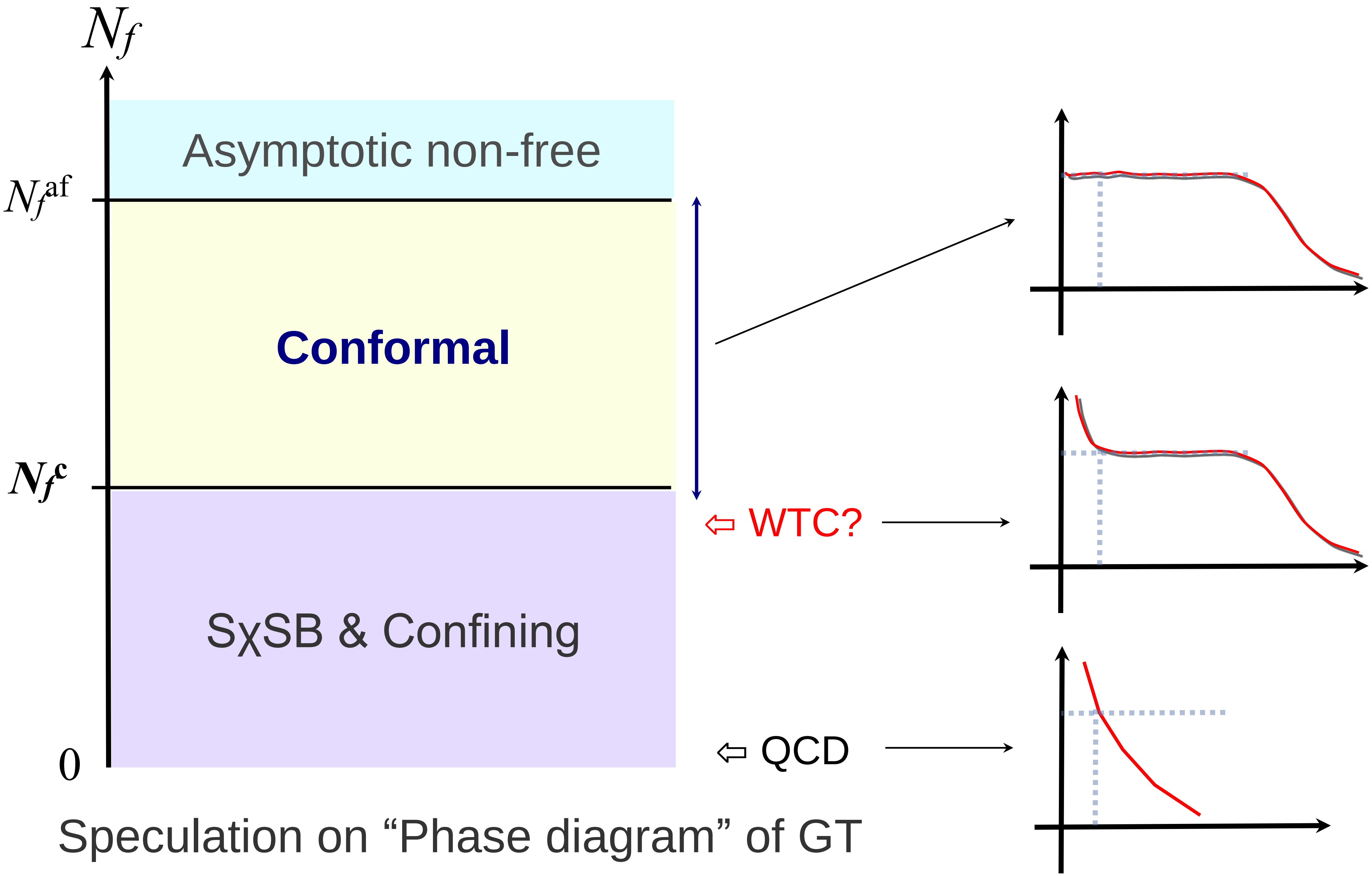
$$m_f\,|_{\Lambda_{ETC}} = \frac{\langle \bar{T}T\rangle|_{\Lambda_{ETC}}}{\Lambda_{ETC}^2} = \frac{\exp\left[\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu}\gamma(\mu)\right]\langle \bar{T}T\rangle|_{\Lambda_{TC}}}{\Lambda_{ETC}^2}$$

$$\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma} \quad \text{if walking}$$

If γ=O(1): Large enhancement



Conformal Window



Speculation on "Phase diagram" of GT

Find the location of $N_f^{\rm c}$ in various GT.

Strategy on the lattice

Searching phase (-2012?)

Calculate **running coupling** and **anomalous dimension** directly on the lattice.

Calculate **hadron spectrum** to see scaling behavior

⇓

Prediction phase (2013?-)

Perform large-scale lattice simulation of candidate theories to find the precise values for f_π, m_ρ, (m_σ), Σ, S-parameter, ...

- Now is in Searching phase.
- Prediction phase on the Next-Generation supercomputer?

Candidates for WTC

So far, the following SU(Nc) gauge theories have been intensively studied

	N_c	N_f	Rep.	Running g^2	spectroscopy
Large N_f QCD	3	6~16	fund.	$8 < N^c{}_f < 12$	$N^c{}_f > 12$ $N^c{}_f < 12$
Large N_f two-color QCD	2	6, 8	fund.	$N^c{}_f < 6$	-
Sextet QCD	3	2	sextet	conformal	conformal confinment
Two-color adjoint QCD	2	2	adjoint	conformal	conformal

Currently, many contradictions and little consensus

II. Lattice calculation of running coupling

Machines used

- Supercomputer@KEK (SR11K, BG/L)
- GPGPU & CPU servers@KEK
- INSAM GPU cluster@Hiroshima
- GPGPU, GCOE cluster system@Nagoya
- B-factory computer system

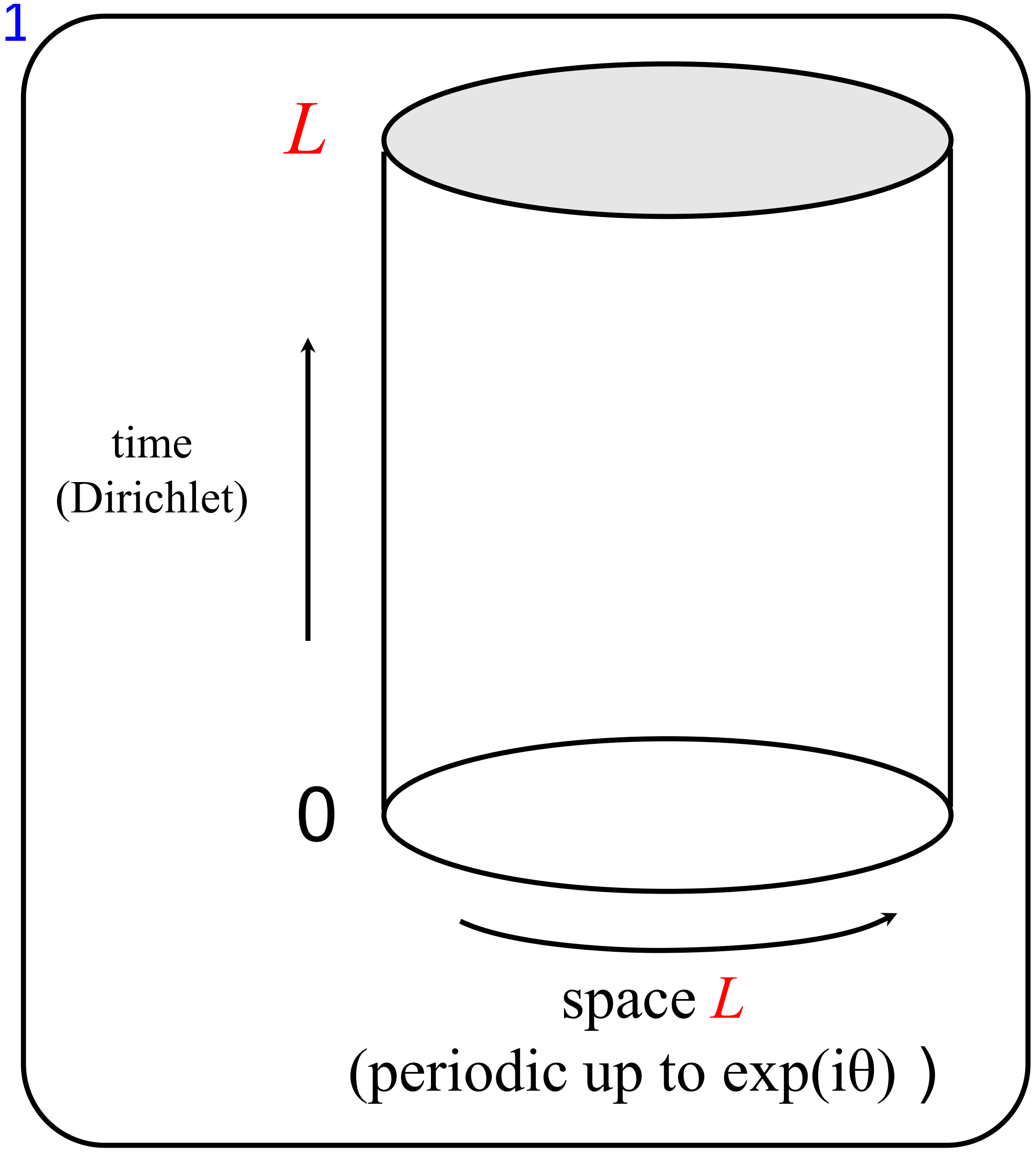
Schrödinger functional scheme

Luescher, Weisz, Wolff, ('91)

- SF coupling

$$\overline{g}^2_{SF}(L) = k \left\langle \frac{\partial S}{\partial(\text{background field})} \right\rangle^{-1}$$

- Standard background field
- Θ=0
- PCAC mass=0
- O(a) un-improved Wilson fermion
- Plaquette gauge action



Perturbation theory

Perturbative IRFP(g_{FP}^2) for SU(3) gauge theory

N_f	4	6	8	10	12	14	16
2-loop universal				27.74	9.47	3.49	0.52
3-loop SF	43.36	23.75	15.52	9.45	5.18	2.43	0.47
3-loop $\overline{\text{MS}}$		159.92	18.40	9.60	5.46	2.70	0.50
4-loop $\overline{\text{MS}}$			19.47	10.24	5.91	2.81	0.50

Anomalous dimension in SF scheme to 2-loop

$$\gamma^{\rm SF} = \frac{8}{(4\pi)^2} g^2 \left\{1 + (0.1251 + 0.0046\, N_f)\, g^2\right\}$$

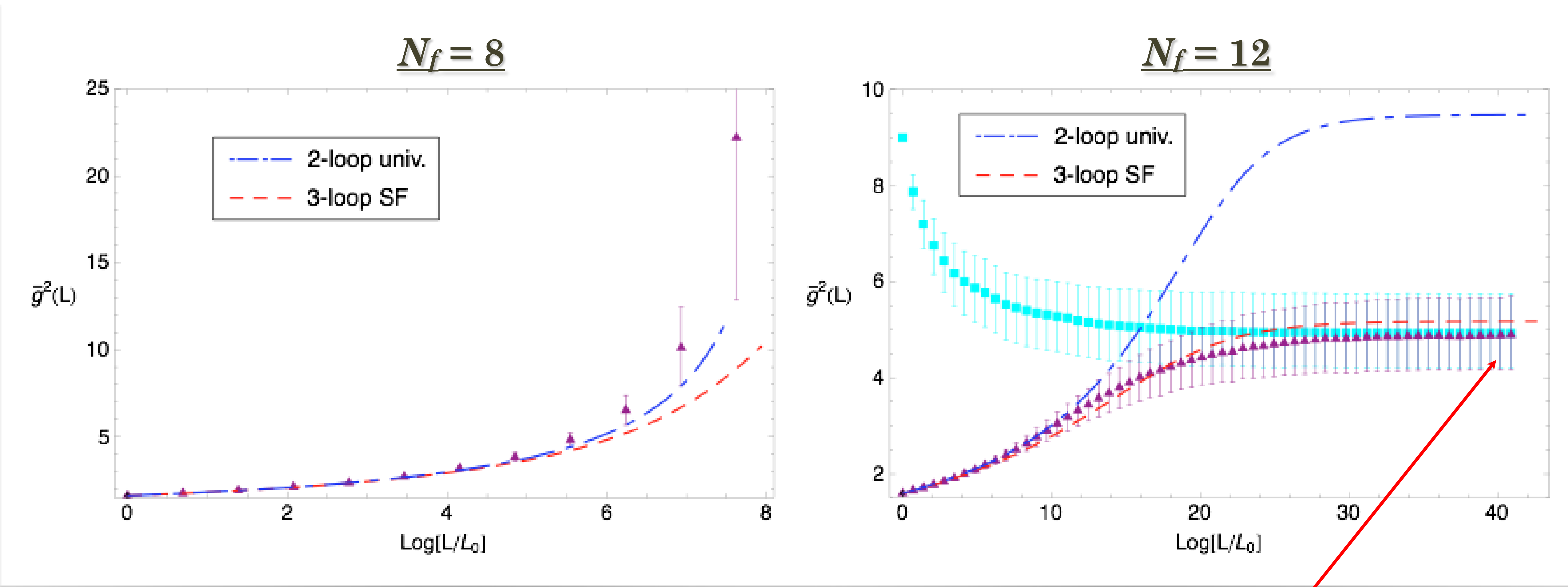
With g_{FP}^2 for 3-loop β-function in SF scheme,

$$\gamma^{\rm SF}_{\rm FP} = \begin{cases} 2.76183 & \text{for } N_f = 8 \\ 1.25265 & \text{for } N_f = 10 \quad \sim O(1) \\ 0.50772 & \text{for } N_f = 12 \end{cases}$$

Perturbation is not reliable =>Use Lattice method!

8 & 12 flavor QCD

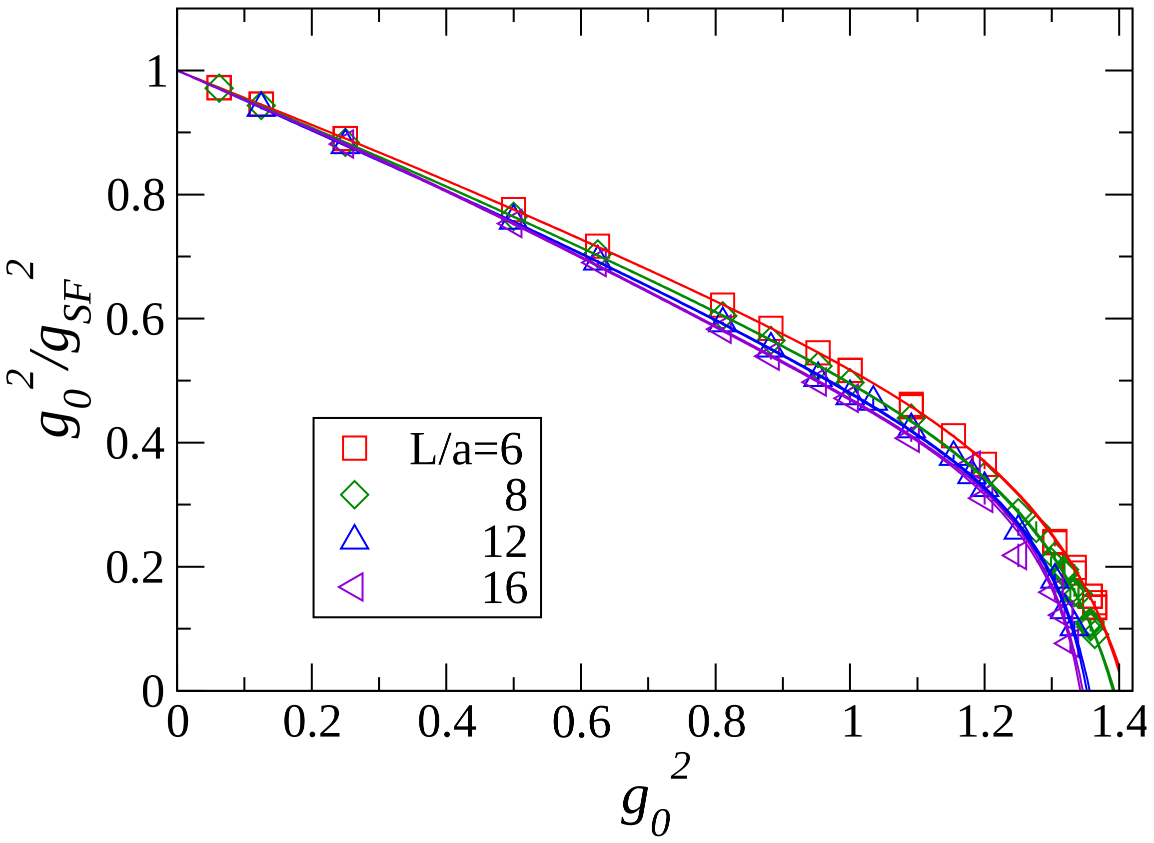
Appelquist, Fleming, Neil, ('08)

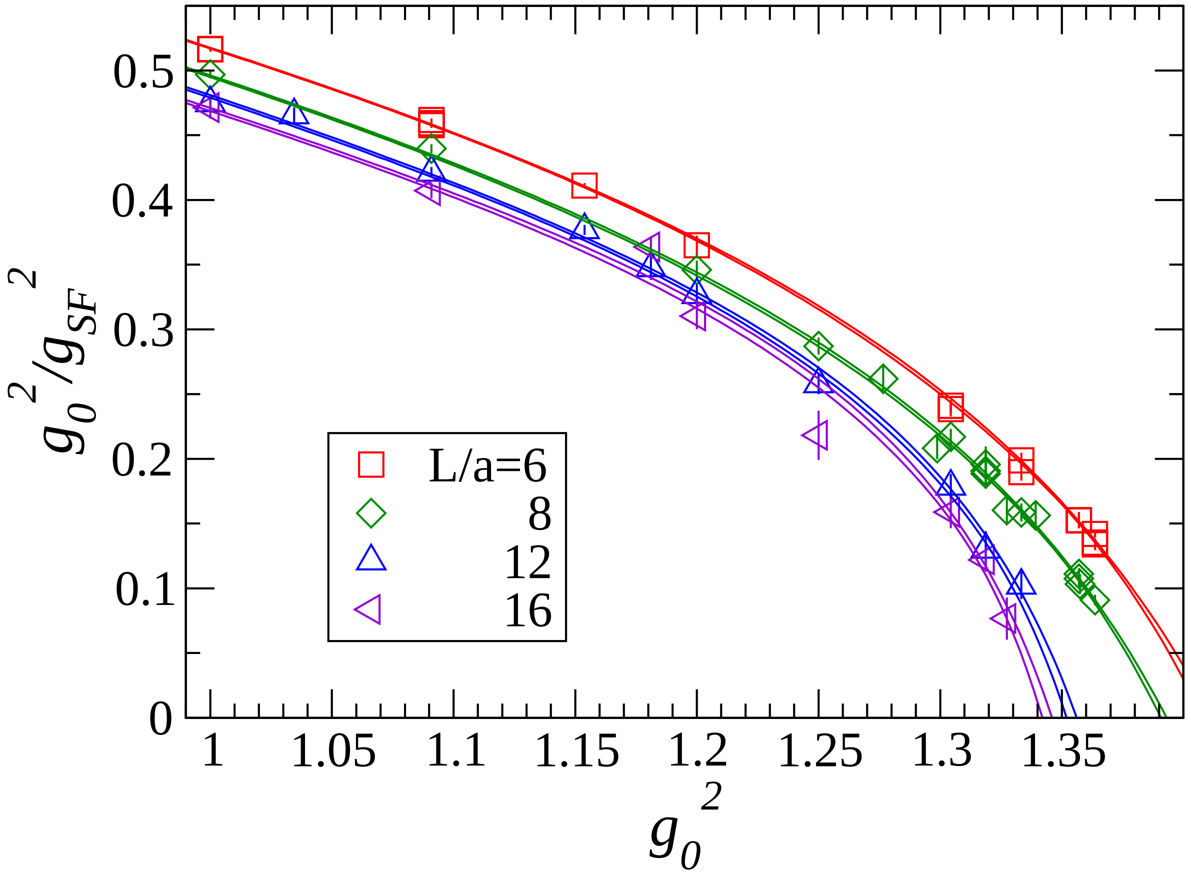


$g^2_{IRFP} \sim 5$ consistent with PT prediction

Conclusion: N_f=12 is too large while N_f=8 is too small.
(12-flavor QCD is still under debate.)

10 flavor QCD (This work)





Raw data of each L/a is close to each other.
⇒ slow running

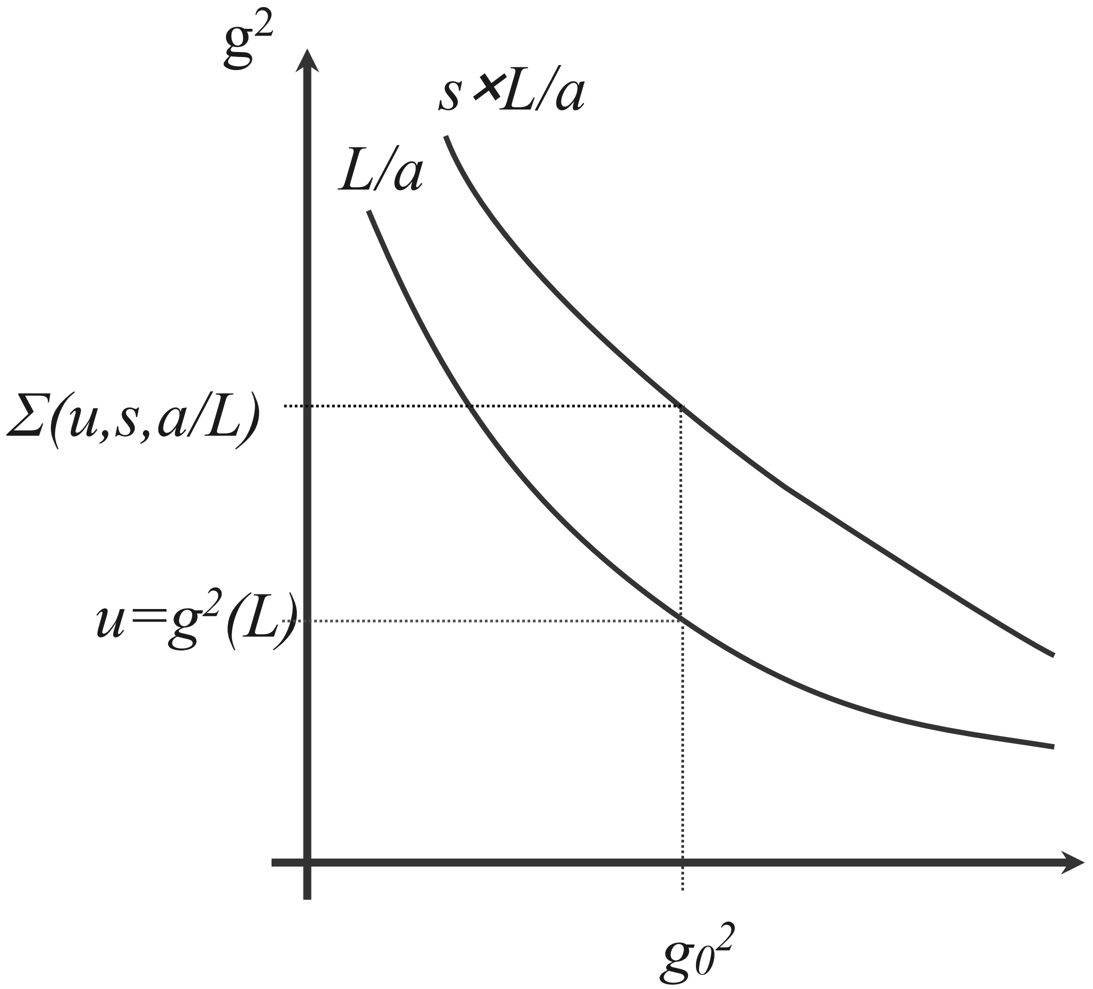
Step scaling function

- Taking the continuum limit

$$\sigma(u,s) = \lim_{a/L\to 0} \Sigma(u,s,a/L),\ u=g_{SF}^2$$

is not easy since O(a) improvement is not implemented

- Reducing discretization errors as much as possible before taking the limit is crucial.



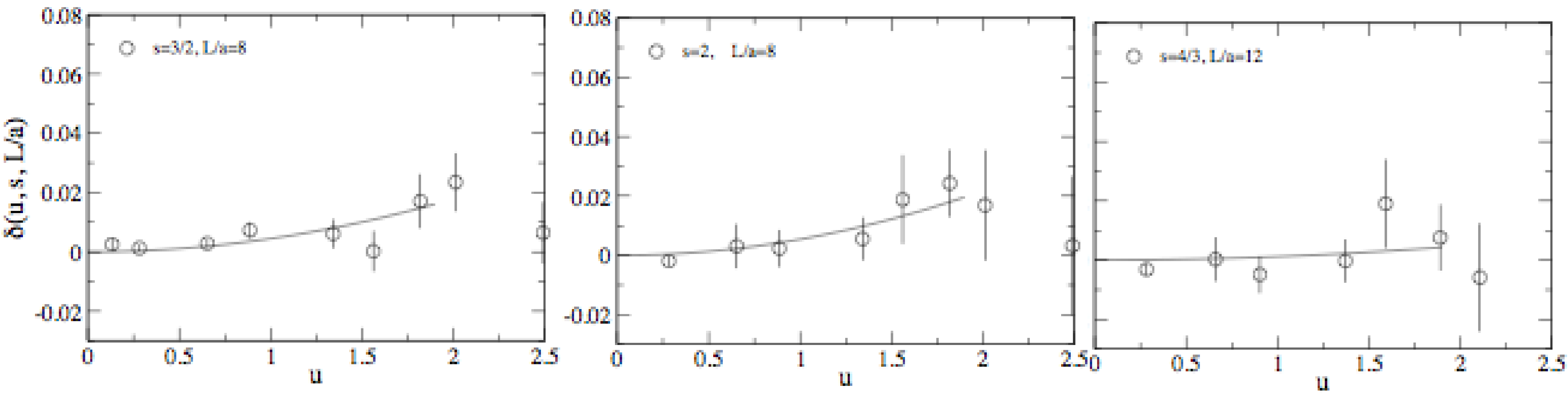
"2-loop improvement"

PACS-CS Collaboration ('09)

Deviation at 2-loop:

$$\delta(u,s,a/L) = \frac{\dfrac{\Sigma(u,s,a/L)}{1+\delta^{(1)}(s,a/L)u} - \sigma(u,s)}{\sigma(u,s)} = \delta_2(s,a/L)u^2 + O(u^3)$$

- Use weak coupling region to estimate the two-loop improvement coefficients.

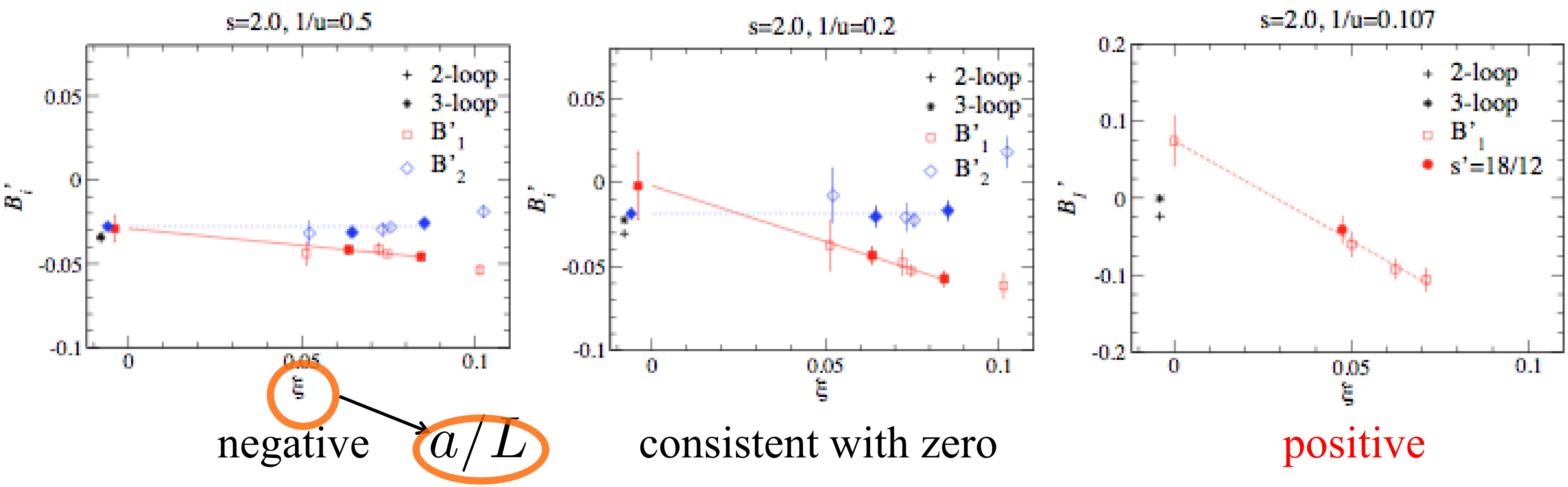


$$\Sigma_{imp}(u,s,a/L) = \frac{\Sigma(u,s,a/L)}{1+\delta^{(1)}(s,a/L)u+\delta^{(2)}(s,a/L)u^2}$$

Discrete beta function

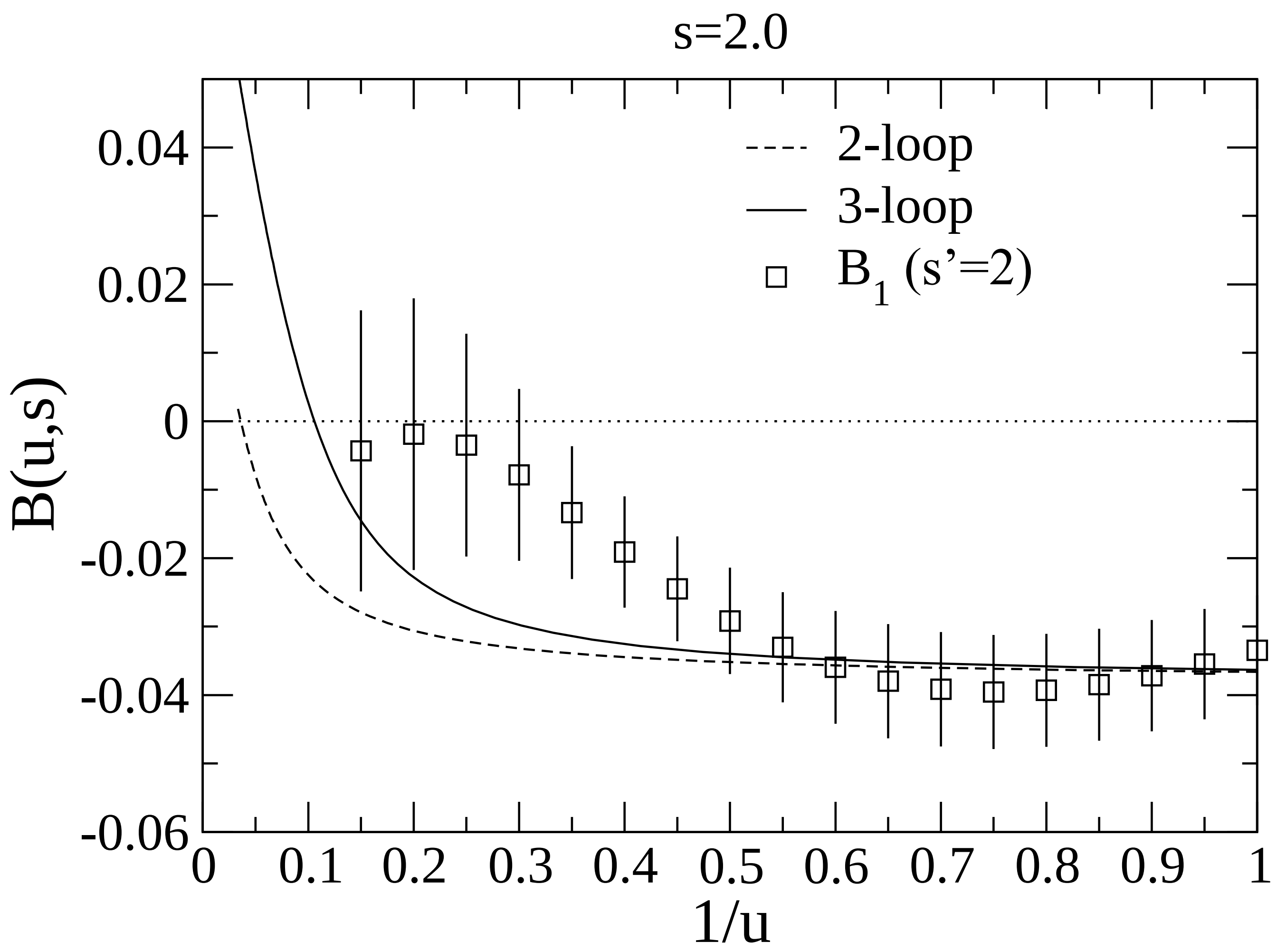
$$B(u, s, a/L) = \frac{1}{\Sigma_{imp}(u, s, a/L)} - \frac{1}{u}$$

Y. Shamir, B. Svetitsky and T. DeGrand, (‘08)



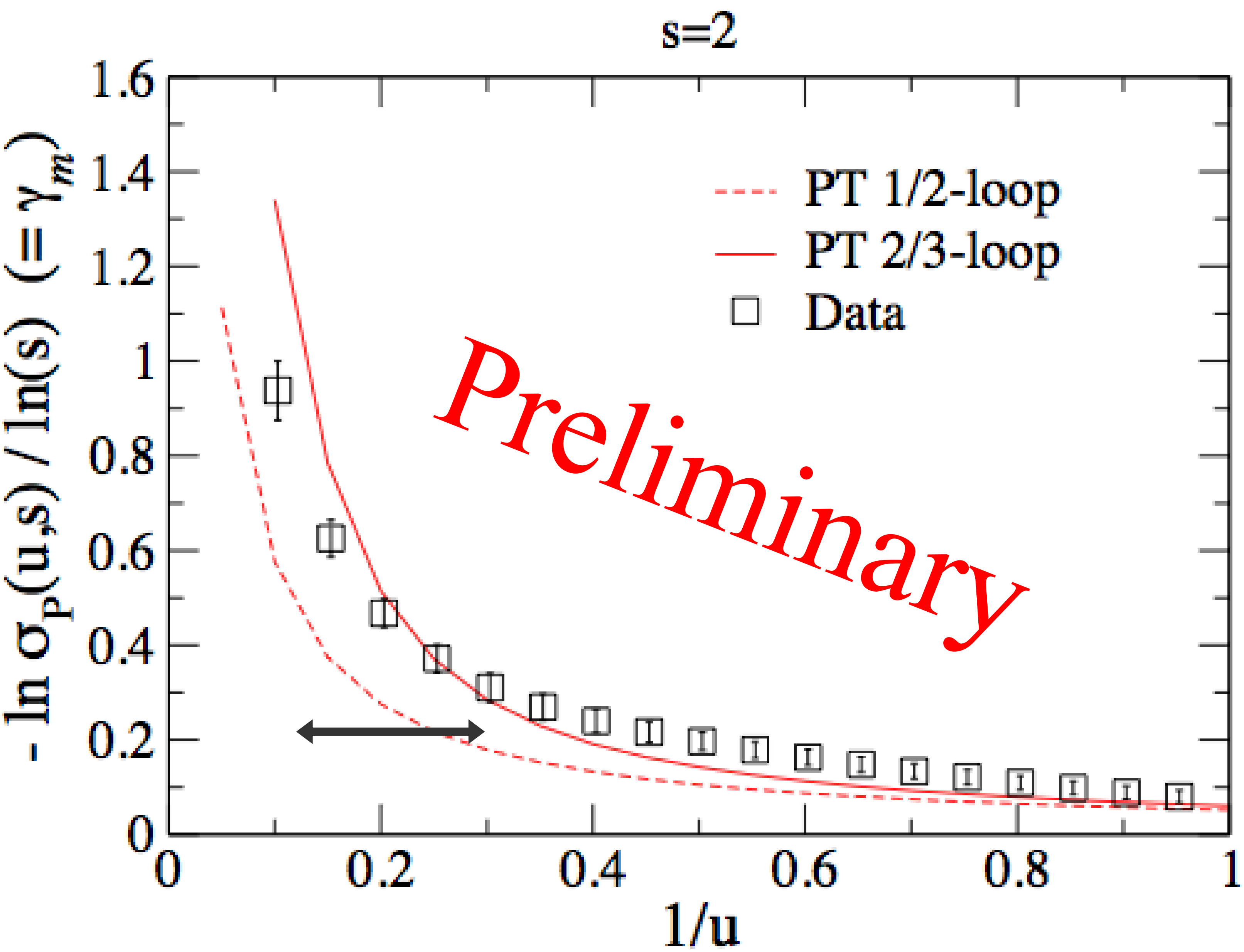
Extrapolation to the continuum limit shows sign-flip before g_{SF}^2 reaches about 10.

Where is IRFP?



Result suggests the existence of IRFP at $g_{FP}^2 = 3.3 \sim 9.35$.

Anomalous dimension



- $\sigma_P = Z_P(s\ L)\ /\ Z_P(L)$
- Non-zero BG field
- For $3.3 < g^2{}_{FP} < 9.35$, $0.28 < \gamma_{FP} < 1.0$!
- Precise value of $g^2{}_{FP}$ is important

III.Summary

Summary

- Lattice technique can be used to search for realistic WTC models and to see whether the long-standing (~30 yrs) problems in TC are really resolved by WTC.
- As a first step, we started with the study of running coupling of 10-flavor QCD to identify *conformal window* in SU(3) GT.
- The result shows an evidence of IRFP in $3.3 < g^2_{FP} < 9.4$.

  $\Rightarrow 8 < N_f^c < 10$

- $0.28 < \gamma_m < 1.0$ is obtained from preliminary analysis. Pinning down γ_m requires precise value of the IRFP.
- Next important task is to calculate S-parameter.