Running coupling constant of ten-flavor QCD with the Schroedinger functional method

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I. Introduction
Successes of Lattice QCD

- Hadron masses and their interactions
- Physics@T≠0
- The SM parameters
- Weak matrix elements
- ...

Lattice calculations truly reliable.

Apply to Something different
LHC era

- Higgs mechanism
- Physics above the EW scale
- Among many New Physics candidates, Technicolor is attractive and best suited for Lattice Simulation

Use Lattice to explore LHC physics
Technicolor (TC) [Weinberg('79), Susskind('79)]

- Alternative for Higgs sector of SM model
- No fine tuning

→ New strong interaction $SU(N_{TC})$: technicolor
→ At $\Lambda_{TC} \sim v_{\text{weak}}$, technifermion condensate $\langle T_R T_L \rangle$
→ Gives dynamical $SU(2)_L \times U(1)_Y$ breaking

→ $\langle TT \rangle$ breaks chiral symmetry $SU(2)_L \times SU(2)_R$
→ Produce technipion $\pi_{TC}$ (NG bosons)
→ $\pi_{TC}$ become longitudinal components of W and Z
→ $M_W = M_Z \cos \theta_W = \frac{1}{2} g F_{\pi} \quad (F_{\pi} = v_{\text{weak}} = 246 \text{ GeV})$
Extended TC  
Eichten('80) Susskind('79)

How do SM fermion get mass?  ⇒  Extended TC

→ New gauge theory $\text{SU}(N_{\text{ETC}})$, $N_{\text{ETC}} > N_{\text{TC}}$

$T_{\text{ETC}} = (T_{\text{TC}}, f_{\text{SM}})$

→ Assuming SSB: $\text{SU}(N_{\text{ETC}}) \rightarrow \text{SU}(N_{\text{TC}}) \times \text{SM}$ at $\Lambda_{\text{ETC}} >> \Lambda_{\text{TC}}$

at $\Lambda_{\text{TC}}$ scale,

\[
\frac{1}{\Lambda_{\text{ETC}}^2} \overline{T T f f} \implies m_f = \frac{\left<T T\right>_{\text{ETC}}}{\Lambda_{\text{ETC}}^2} : \text{fermion mass}
\]

\[
\frac{1}{\Lambda_{\text{ETC}}^2} \overline{f f f f} : \text{FCNC}
\]
Walking TC

Holdom ('81) Yamawaki et al. ('86)

Solving tension of SM fermion mass VS. FCNC

\[ \frac{m_f}{\Lambda_{ETC}} = \frac{\langle \bar{T}T \rangle}{\Lambda_{ETC}^2} = \exp \left[ \int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu} \gamma(\mu) \right] \frac{\langle \bar{T}T \rangle}{\Lambda_{TC}^2} \]

If \( \gamma = O(1) \): Large enhancement

Classic TC or QCD-like theory

Walking TC

If \( \gamma = O(1) \): Large enhancement
Find the location of $N_f^c$ in various GT.

Conformal Window

\[ N_f \]

$N_f^{af}$

Asymptotic non-free

Conformal

$N_f^c$

$N_f^c$

$S\chiSB \& Confining$

Speculation on “Phase diagram” of GT

$\Leftrightarrow$ WTC?

$\Leftrightarrow$ QCD

Find the location of $N_f^c$ in various GT.
Strategy on the lattice

Searching phase (-2012?)
- Calculate **running coupling** and **anomalous dimension** directly on the lattice.
- Calculate **hadron spectrum** to see scaling behavior.

Prediction phase (2013?-)
- Perform large-scale lattice simulation of candidate theories to find the precise values for $f_\pi$, $m_\rho$, $(m_\sigma)$, $\Sigma$, S-parameter, ...

- Now is in **Searching phase**.
- **Prediction phase** on the Next-Generation supercomputer?
So far, the following SU($N_c$) gauge theories have been intensively studied:

<table>
<thead>
<tr>
<th></th>
<th>$N_c$</th>
<th>$N_f$</th>
<th>Rep.</th>
<th>Running $g^2$</th>
<th>spectroscopy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large $N_f$ QCD</strong></td>
<td>3</td>
<td>6~16</td>
<td>fund.</td>
<td>$8 &lt; N_f &lt; 12$</td>
<td>$N_f &gt; 12$</td>
</tr>
<tr>
<td><strong>Large $N_f$ two-color QCD</strong></td>
<td>2</td>
<td>6, 8</td>
<td>fund.</td>
<td>$N_f &lt; 6$</td>
<td>-</td>
</tr>
<tr>
<td><strong>Sextet QCD</strong></td>
<td>3</td>
<td>2</td>
<td>sextet</td>
<td>conformal</td>
<td>conformal confinement</td>
</tr>
<tr>
<td><strong>Two-color adjoint QCD</strong></td>
<td>2</td>
<td>2</td>
<td>adjoint</td>
<td>conformal</td>
<td>conformal</td>
</tr>
</tbody>
</table>

Currently, many contradictions and little consensus.
II. Lattice calculation of running coupling
Machines used

- Supercomputer@KEK (SR11K, BG/L)
- GPGPU & CPU servers@KEK
- INSAM GPU cluster@Hiroshima
- GPGPU, GCOE cluster system@Nagoya
- B-factory computer system
Schrödinger functional scheme

Luescher, Weisz, Wolff, (’91)

- SF coupling
  \[ \bar{g}_{SF}(L) = k \left\langle \frac{\partial S}{\partial (\text{background field})} \right\rangle^{-1} \]

- Standard background field
- \( \Theta=0 \)
- PCAC mass=0
- O(a) un-improved Wilson fermion
- Plaquette gauge action
Perturbation theory

Perturbative IRFP($g_{FP}^2$) for SU(3) gauge theory

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-loop universal</td>
<td>43.36</td>
<td>23.75</td>
<td>15.52</td>
<td>9.45</td>
<td>5.18</td>
<td>2.43</td>
<td>0.47</td>
</tr>
<tr>
<td>3-loop SF</td>
<td>159.92</td>
<td>18.40</td>
<td>9.60</td>
<td>5.46</td>
<td>2.70</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>3-loop MS</td>
<td>19.47</td>
<td>10.24</td>
<td>5.91</td>
<td>2.81</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-loop MS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Anomalous dimension in SF scheme to 2-loop

\[ \gamma^{SF} = \frac{8}{(4\pi)^2} g^2 \left\{ 1 + (0.1251 + 0.0046 N_f) g^2 \right\} \]

With $g_{FP}^2$ for 3-loop $\beta$-function in SF scheme,

\[ \gamma^{SF}_{FP} = \begin{cases} 2.76183 & \text{for } N_f = 8 \\ 1.25265 & \text{for } N_f = 10 \\ 0.50772 & \text{for } N_f = 12 \end{cases} \sim O(1) \]

Perturbation is not reliable => Use Lattice method!
8 & 12 flavor QCD

Appelquist, Fleming, Neil, ('08)

$N_f = 8$

$N_f = 12$

$g^2_{\text{IRFP}} \sim 5$ consistent with PT prediction

Conclusion: $N_f=12$ is too large while $N_f=8$ is too small. (12-flavor QCD is still under debate.)
10 flavor QCD (This work)

Raw data of each L/a is close to each other.
⇒ slow running
Step scaling function

• Taking the continuum limit
  \[ \sigma(u,s) = \lim_{a/L \to 0} \Sigma(u,s,a/L), \; u = g_{SF}^2 \]
  is not easy since O(a) improvement is not implemented

• Reducing discretization errors as much as possible before taking the limit is crucial.
Deviation at 2-loop:

\[
\delta(u, s, a/L) = \frac{\Sigma(u, s, a/L)}{1 + \delta^{(1)}(s, a/L)u} - \sigma(u, s) = \delta_2(s, a/L)u^2 + O(u^3)
\]

- Use weak coupling region to estimate the two-loop improvement coefficients.

\[
\Sigma_{\text{imp}}(u, s, a/L) = \frac{\Sigma(u, s, a/L)}{1 + \delta^{(1)}(s, a/L)u + \delta^{(2)}(s, a/L)u^2}
\]
Discrete beta function

\[ B(u, s, a/L) = \frac{1}{\Sigma_{\text{imp}}(u, s, a/L)} - \frac{1}{u} \]

Y. Shamir, B. Svetitsky and T. DeGrand, (‘08)

Extrapolation to the continuum limit shows sign-flip before \( g_{\text{SF}}^2 \) reaches about 10.
Result suggests the existence of IRFP at $g_{FP}^2 = 3.3 \sim 9.35$. Where is IRFP?
Anomalous dimension

$\sigma_P = \frac{Z_P(sL)}{Z_P(L)}$

- Non-zero BG field
- For $3.3 < g_{FP}^2 < 9.35$, $0.28 < \gamma_{FP} < 1.0$
- Precise value of $g_{FP}^2$ is important

Preliminary
III. Summary
Summary

- Lattice technique can be used to search for realistic WTC models and to see whether the long-standing (~30 yrs) problems in TC are really resolved by WTC.

- As a first step, we started with the study of running coupling of 10-flavor QCD to identify conformal window in SU(3) GT.

- The result shows an evidence of IRFP in $3.3 < g_{\text{FP}}^2 < 9.4$.
  \[ 8 < N_f^c < 10 \]

- $0.28 < \gamma_m < 1.0$ is obtained from preliminary analysis. Pinning down $\gamma_m$ requires precise value of the IRFP.

- Next important task is to calculate S-parameter.