WE DEAL WITH ELEMENTARY NUMERICAL METHODS AND THEIR APPLICATION IN PHYSICS AND MATHEMATICS.

EXAMPLE : CELESTIAL MECHANICS

NEWTON'S LAW : $\vec{F}_{12} = -\frac{Gm_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$

Okt 8-10:23

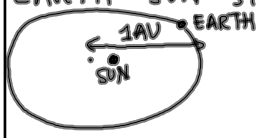
$$\begin{cases} m_1 \frac{d^2\vec{r}_1}{dt^2} = \vec{F}_{12} \\ m_2 \frac{d^2\vec{r}_2}{dt^2} = \vec{F}_{21} = -\vec{F}_{12} \end{cases}$$

2-BODY PROBLEM : EXACTLY SOLVABLE

KEPLER'S LAWS

Okt 8-10:28

EARTH - SUN SYSTEM,



SINCE $m_{EARTH} \cong 10^{-6}\, m_{SUN}$

SUN IS APPROXIMATELY STATIONARY

ORBIT OF EARTH : ELLIPSE WITH THE SUN IN ONE OF THE FOCAL POINTS

ADD A THIRD PLANET : JUPITER

$m_{JUPITER} \cong 10^{-3}\, m_{SUN}$

Okt 8-10:31

1 AU (ASTRONOMICAL UNIT)

= SEMIMAJOR AXIS OF EARTH'S ELLIPSE AROUND THE SUN

$= 1.496 \times 10^{11}$ m

Okt 8-10:36

$m_{JUPITER} = m_{SUN}$ (PICTURES FROM GIORDANO AND NAKANISHI)

TRUE 3-BODY PROBLEM

THE SUN IS NOT STATIONARY

ORBIT OF EARTH : UNSTABLE,

CAN BE COMPUTED BY NUMERICAL SIMULATIONS

CHAOS : EXTREME SENSITIVITY TO INITIAL CONDITIONS

Okt 8-10:44

MATRIX EXPONENTIAL

A nxn COMPLEX MATRIX

$$e^A \equiv \exp(A) = \sum_{k=0}^{\infty} \frac{1}{k!} A^k$$

MATLAB : expm(A)

Okt 8-11:04

ARITHMETIC OPERATIONS: * ^ /
(MULTIPLICATION *, EXPONENTIATION ^, DIVISION /)

ELEMENTWISE OPERATIONS: .* .^ ./
(ACT ELEMENT BY ELEMENT)

Okt 8-11:13

PREDEFINED FUNCTIONS (exp, log, ...)
ON MATRICES THEY ACT
ELEMENT BY ELEMENT

$$\exp(A) \neq \text{expm}(A)$$

Okt 8-11:19

FUNCTIONS IN MATLAB:

- VARIABLES CAN BE PASSED AS ARGUMENTS:

$$[\underbrace{a,b}_{\text{OUTPUT}}] = \text{meandiff}(\underbrace{x,y}_{\text{INPUT}})$$

- VARIABLES INSIDE FUNCTION ARE LOCAL (DO NOT EXIST OUTSIDE THE FUNCTION)
- WHAT MATTERS IS THE FILENAME OF A FUNCTION

Okt 8-11:37